MAGNETOPOLAR STRATIFIED FLOW THROUGH A POROUS MEDIUM IN SLIP FLOW REGIME

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ABSTRACT

This paper is concerned with steady MHD stratified polar flow through a porous medium in slip flow regime over a moving infinite porous plate. Exact solutions have been obtained for velocity distribution, mean angular velocity and skin-friction. Effects of different parameters entering into the problem are calculated numerically and are shown graphically.

Key Words : Angular Velocity, Magnetic Field, Moving Plate, Porous Medium, Slip Velocity.

1. INTRODUCTION

In recent years, considerable attention has been given to the polar fluid. Aero et al. [1]. D'ep [3] and Raptis [9] derived and solved flow equations of the fluids in which angular velocity of the fluid particles was considered. These fluids are known as polar fluids in the literature. Recently Jain and Taneja [6,7] solved the problems of magnetopolar flow through porous medium on porous plates.

Fluid motion influenced by the density and viscosity variations in the fluid is characterised as stratified flow. Channabasappa and Ranganna [2] studied the flow of a viscous stratified fluid past a porous bed with the idea that of stratification may provide a technique for studying the pore size in a porous medium. Singh [10] , Soundalgekar [11] Gupta and Gupta [4], Gupta and Goyal [5] and Kumar et al. [8] have discussed the problems of viscous stratified MHD flows. Recently Vyas and Mathur [12] have studied the viscous stratified flow due to an oscillating plate at the bottom of a porous medium in presence of a magnetic field.

In above referred works no attempt has been made for stratified magnetopolar fluids which prevail in nature inside the earth crust. In the present paper an analysis has been made for it by considering MHD steady viscous stratified polar flow through porous medium due to moving porous plate with slip boundary condition for velocity. The velocity of the porous plate is constant and suction velocity is normal to the plate. The expressions for the velocity distribution, mean angular velocity and skin-friction are obtained. The effects of different parameters entering into the problem viz. K (permeability parameter), M (magnetic field parameter), R (plate velocity parameter), h_1 (velocity slip parameter), α and β (rotational and couple stress parameters) and δ(stratification parameter) on the velocity distribution and skin-friction are shown graphically and discussed numerically.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider a two dimensional steady MHD flow of stratified polar fluid through a porous medium over a porous flat plate with slip boundary condition. The constant suction velocity ($-v_0$) is taken normal to the plate. Also the plate is moving in its own plane with the velocity U. The x-axis taken along the plate and y-axis normal to it. A variable magnetic field is considered fixed relative to the fluid.

We assume

$$\rho = \rho_0 e^{-\delta y}, \mu = \mu_0 e^{-\delta y}, \mu_r = \mu_{0r} e^{-\delta y} \text{ and } B = B_0 e^{-\delta y/2} \text{ for } y \geq 0 \qquad \text{------- (1)}$$

where, δ being assumed to be small positive number, ρ the density of the fluid, μ the viscosity of the fluid, μ_r the rotational viscosity, B the strength of the magnetic field and μ_0 and B_0 are the density, viscosity, rotational viscosity and strength of magnetic field respectively at y = 0.

Here magnetic Reynolds number is assumed to be small, so that the induced magnetic field due to the flow may be neglected in comparison to the applied magnetic field. Under these conditions the equations which govern the flow are:

$$\frac{\partial v}{\partial y}=0 \qquad \text{-------} \ (2)$$

$$\rho\left[v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left[(\mu+\mu_r)\frac{\partial u}{\partial y}\right]+2\mu_r\frac{\partial \omega}{\partial y}-\frac{\mu}{K}u-\sigma B^2 u \qquad \rho_0,\ \mu_0,\ \mu_{0r} \qquad \text{-------} \ (3)$$

$$v\frac{\partial \omega}{\partial y}=\frac{\gamma}{I}\frac{\partial^2 \omega}{\partial y^2} \qquad \text{-------} \ (4)$$

where μ_r is the rotational viscosity, ω the mean angular velocity of rotation of the particles, K the permeability of the porous medium, I a scalar constant of dimension equal to that of moment of inertia of unit mass and

$$\gamma=\frac{C_a+C_d}{I}$$

where C_a and C_d are coefficients of couple stress viscosities. Remaining symbols have their usual meanings.

The boundary conditions are:

$$\left.\begin{array}{lll} u=U+L_1\dfrac{\partial u}{\partial y}, & \dfrac{\partial \omega}{\partial y}=-\dfrac{\partial^2 u}{\partial y^2} & \text{at } \ y=0 \\ u\to U_\infty\ , & \omega\to 0 & \text{as } \ y\to\infty \end{array}\right\} \qquad \text{-------} \ (5)$$

where L being the mean free path and m_1 the Maxwell's reflexion coefficient.

$$L_1=\frac{2-m_1}{m_1}L,$$

Integration of equation (2) for constant suction gives.

$$v=-v_0 \qquad \text{-------} \ (6)$$

Now introducing the following non-dimensional quantities:

$$\left.\begin{array}{l} y^*=\dfrac{yv_0}{\nu_0},\ \alpha=\dfrac{\nu_{0r}}{\nu_0},\ \beta=\dfrac{I\nu_0}{\gamma},\ \omega^*=\dfrac{\nu_0\omega}{v_0U_\infty},\ \delta^*=\dfrac{\delta v_0}{\nu_0}\ , \\ u^*=\dfrac{u}{U_\infty},\ R=\dfrac{U}{U_\infty},\ K^*=\dfrac{v_0{}^2K}{\nu_0{}^2},\ M^2=\dfrac{\sigma B_0{}^2\nu_0}{\rho_0 v_0{}^2},\ h_1=\dfrac{v_0L_1}{\nu_0} \end{array}\right\} \qquad \text{-------} \ (7)$$

In view of equations (1), (6) and (7) for the free stream velocity, the equations of motion in non-dimensional form after dropping the asterisks over them, reduce to:

$$(1+\alpha)\frac{\partial^2 u}{\partial y^2}+\left[1-\delta(1+\alpha)\right]\frac{\partial u}{\partial y}-N^2 u=-N^2-2\alpha\frac{\partial \omega}{\partial y} \quad \text{------- (8)}$$

$$\frac{\partial^2 \omega}{\partial y^2}+\beta\frac{\partial \omega}{\partial y}=0 \quad \text{------- (9)}$$

with corresponding boundary conditions:-

$$\left.\begin{array}{lll} u=R+h_1\dfrac{\partial u}{\partial y}, & \dfrac{\partial \omega}{\partial y}=-\dfrac{\partial^2 u}{\partial y^2} & \text{at } y=0 \\ u\to 1, & \omega\to 0 & \text{as } y\to\infty \end{array}\right\} \quad \text{------- (10)}$$

where $N^2=\frac{1}{K}+M^2$

Solving the equations (8) and (9), we get the exact solution after using corresponding boundary conditions:

$$u=1+C_2 e^{-R_2 y}+\frac{2\alpha\beta C_1}{(\beta+R_1)(\beta-R_2)}e^{-\beta y} \quad \text{------- (11)}$$

$$\omega=C_1 e^{-\beta y} \quad \text{------- (12)}$$

Where,

$$R_1=\frac{-a_2+\sqrt{a_2{}^2+4a_1N^2}}{2a_1}, \qquad R_2=\frac{a_2+\sqrt{a_2{}^2+4a_1N^2}}{2a_1}$$

$$C_1=\frac{R_2{}^2(R-1)}{\left[\beta(1-a_3\beta)(1+h_1R_2)+R_2{}^2a_3(1+h_1\beta)\right]}, \quad C_2=\frac{(R-1)-a_3C_1(1+h_1\beta)}{(1+h_1R_2)}$$

$$a_2=1-\delta a_1, \quad a_3=\frac{2\alpha\beta}{(\beta+R_1)(\beta-R_2)} \qquad a_1=1+\alpha,$$

3. SKIN-FRICTION

The non-dimensional skin-friction τ at the plate is given by

$$\tau=\frac{\tau_\omega}{\rho U_\infty v_0}=(1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$=(1+\alpha)\left(-R_2C_2-\frac{2\alpha\beta^2C_1}{(\beta+R_1)(\beta-R_2)}\right) \quad \text{------- (13)}$$

4. DISCUSSION AND CONCLUSION

In order to understand the physical situation we have calculated the numerical values of the velocity distribution and skin-friction and are shown graphically [Fig. 1 to 4] for different values of K (permeability parameter), M (magnetic field parameter), R (plate velocity parameter), h_1 (velocity slip parameter), α and β (rotational and couple stress parameters) and δ (stratification parameter).

In figure 1 the velocity distribution is plotted against y for $h_1 = 0.4$, $\alpha = 0.2$, $\beta = 2.0$ and $\delta = 0.1$. It is being observed that when K and R are increased velocity is increased but the phenomena reverses in case of M. In case of non porous medium ($K \rightarrow \infty$) velocity is decreased when M increased. Further when R = 0 (case corresponding to stationary plate) velocity increases as y increases and tends to 1 as $y \rightarrow \infty$.

In figure 2 the velocity distribution is plotted against y for K = 1.0, M = 0.2 and R = 1.5. It is being seen that when α, β and δ are increased velocity is increased but when h_1 is increased the velocity is decreased. Also when $h_1 = 0$ (no slip condition at the plate) velocity is increased as compared to when $h_1 \neq 0$.

In figure 3 the skin-friction (τ) is plotted against M for, $\alpha = 0.2$, $\beta = 2.0$ and $\delta = 0.1$. It is being observed that when K, R and h_1 are increased skin friction is decreased. It is interesting to note that when medium is non-porous ($K \rightarrow \infty$) skin-friction is decreased in comparison to porous medium. Also when $h_1 = 0$ (no slip condition at the plate) skin-friction is increased as compared to when $h_1 \neq 0$.

In figure 4 the skin-friction (τ) is plotted against M for K = 1.0, R = 0.5 and $h_1 = 0.4$. It is being seen that when β and δ are increased skin-friction is decreased but the phenomena reverses for α.

$h_1 = 0.4, \alpha = 0.2, \beta = 2.0, \delta = 0.1$

S.No.	K	M	R
1.	1.0	0.2	1.5
2.	2.5	0.2	1.5
3.	1.0	2.0	1.5
4.	1.0	0.2	2.5
5.	∞	0.2	1.5
6.	1.0	0	1.5
7.	∞	0	1.5
8	1.0	0.2	0.0

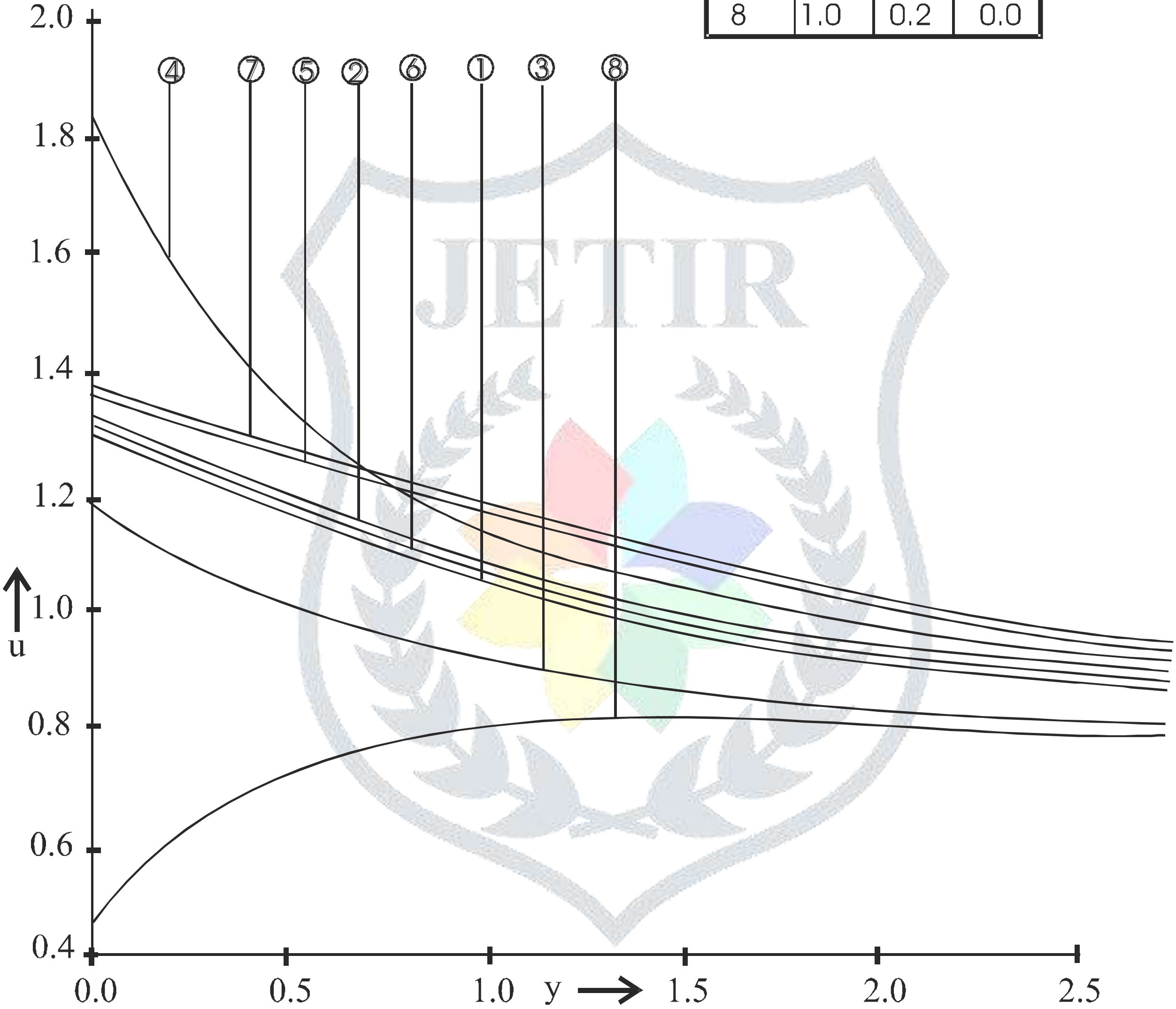

Fig. 1. Velocity distribution (u) plotted against y for different values of K, M and R.

K=1.0, M=0.2, R=1.5

S.No.	h_1	α	β	δ
1.	0.4	0.2	2.0	0.1
2.	0.7	0.2	2.0	0.1
3.	0.4	0.8	2.0	0.1
4.	0.4	0.2	5.0	0.1
5.	0.4	0.2	2.0	0.2
6.	0	0.2	2.0	0.1

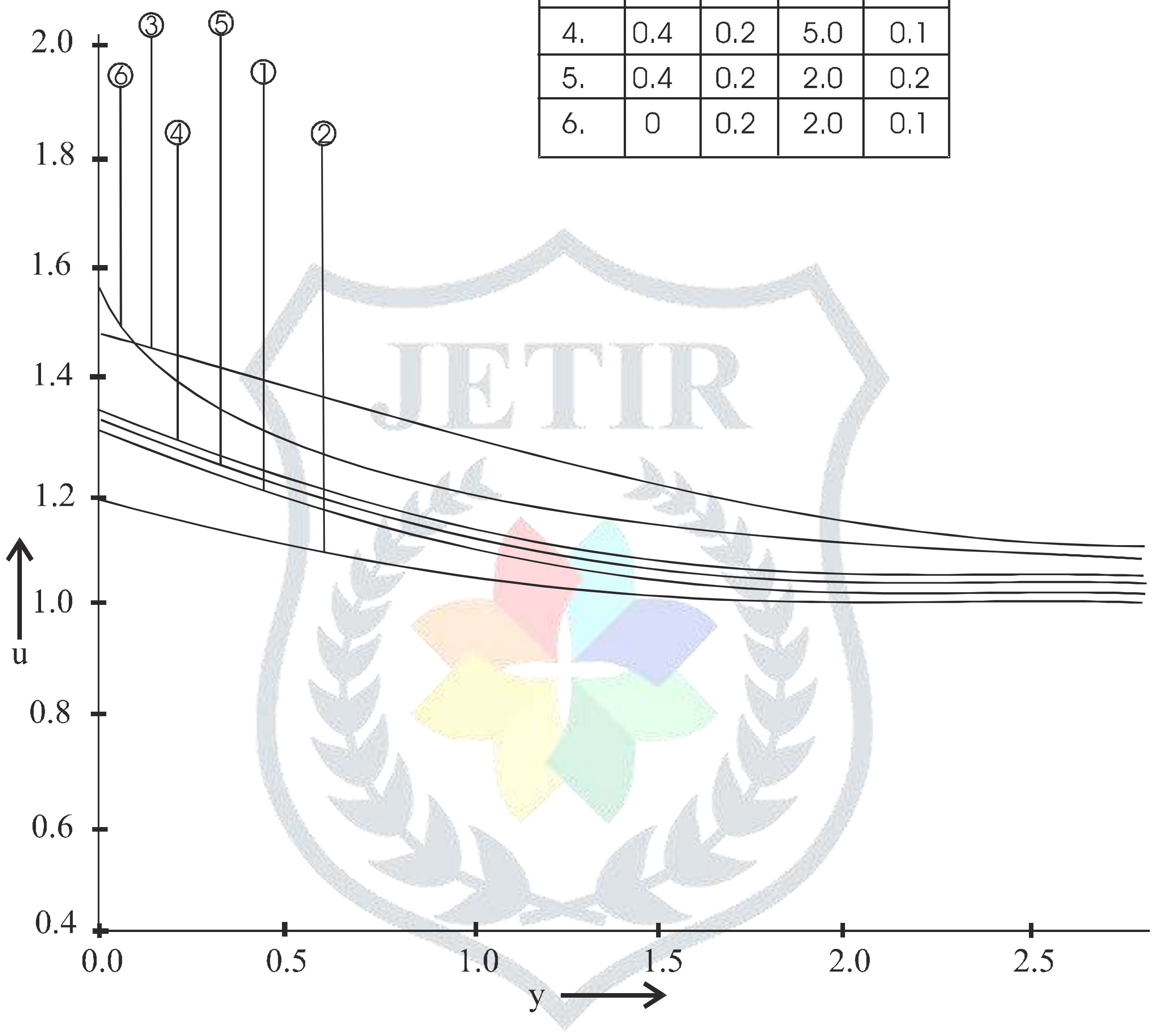

Fig. 2 . Velocity distribution (u) plotted against y for different values of h_1,α,β and δ.

$\alpha=0.2$, $\beta=2.0$, $\delta=0.1$

S.No.	K	R	h_1
1.	1.0	0.5	0.4
2.	2.5	0.5	0.4
3.	1.0	0.9	0.4
4.	1.0	0.5	0.8
5.	∞	0.5	0.4
6.	1.0	0.0	0.4
7.	1.0	0.5	0.0

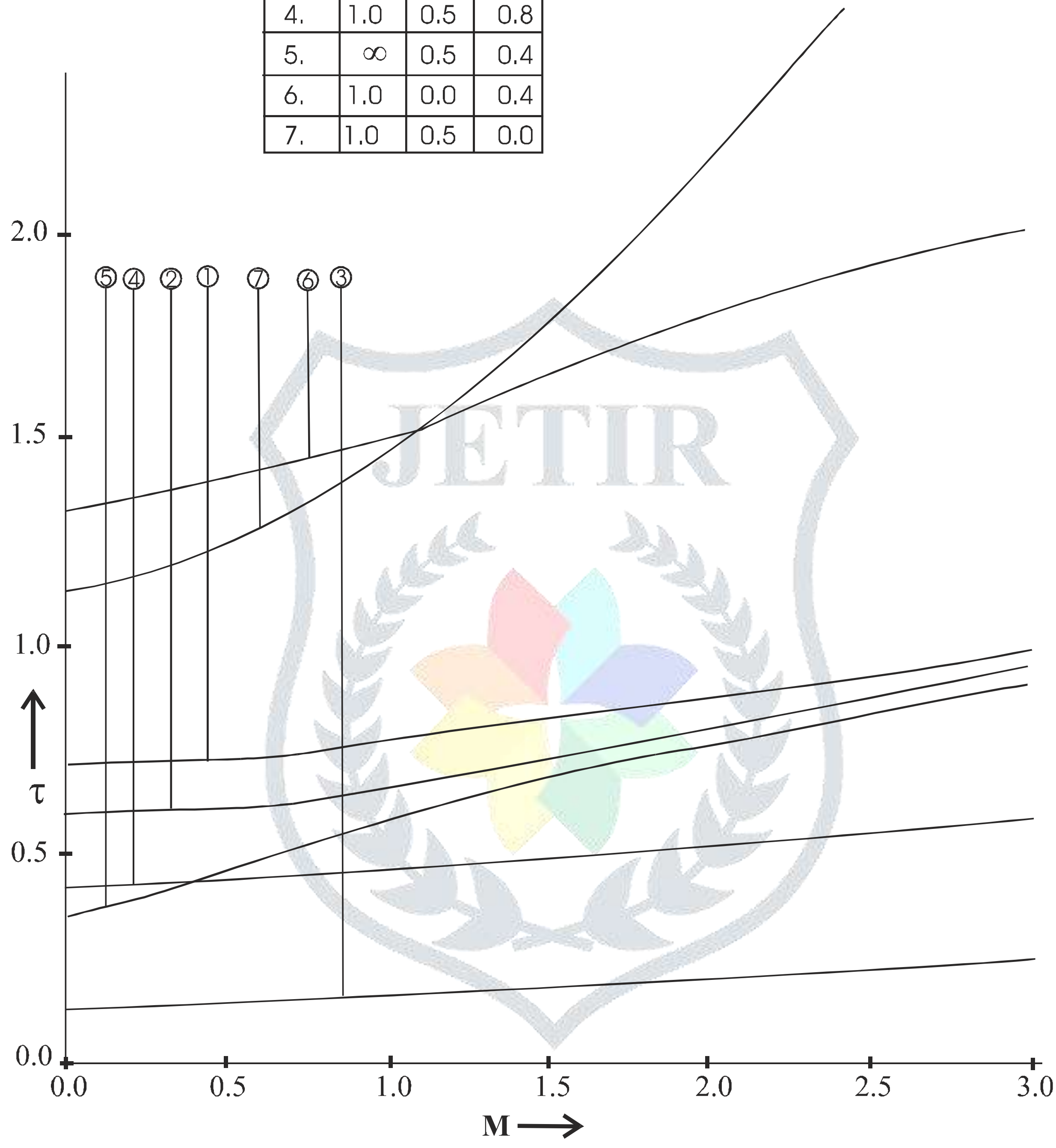

Fig. 3. Skin-friction (τ) plotted against M for different values of K, R and h_1.

$K=1.0$, $R=0.5$, $h_1=0.4$

S.No.	α	β	δ
1.	0.2	2.0	0.1
2.	0.4	2.0	0.1
3.	0.2	5.0	0.1
4.	0.2	2.0	0.2
5.	0.0	2.0	0.1
6.	0.2	2.0	0.0

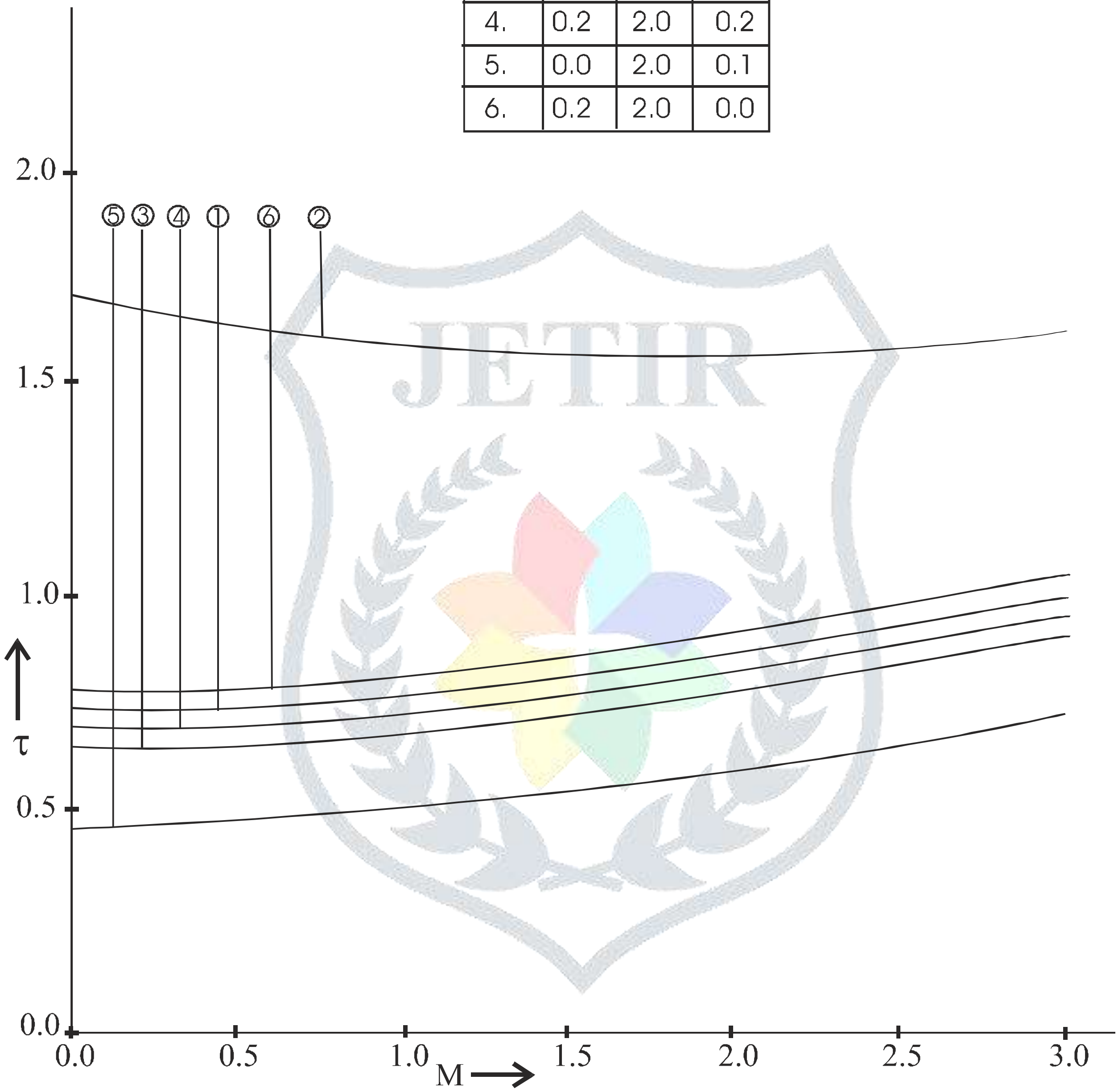

Fig. 4. Skin-friction (τ) plotted against M for different values of α, β and δ.

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