SOME NEW RESULTS ON NON-NEGATIVE MAJORITY TOTAL DOMINATION IN GRAPHS

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Abstract: For a simple graph$G = (V; E)$, a two valued function $h: V \to \{-1, 1\}$ is called a non-negative majority total dominating function if the sum of its function values over at least half the open neighborhoods is at least zero. A non-negative majority total domination number of a graph G is the minimum value of $\sum_{v \in V(G)} h(v)$ over all non-negative majority total dominating functions f of G and it is denoted by $\gamma_{maj}^{nt}(G)$. In this paper, we have obtained $\gamma_{maj}^{nt}(G)$ of some classes of graphs.

Index Terms: **Non-negative majority total domination number, majority total domination number, majority domination function, majority domination number.**

I. INTRODUCTION

All graphs considered here are simple, finite and undirected graphs. For basic definition and notation we follow [1,2].

The study of domination is one of the well-studied areas within graph theory. A subset L of vertices is said to be a *dominating set* of G if every vertex in V either belongs to L or is adjacent to a vertex in L. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. An excellent survey of advanced topics on domination parameters are given in the book edited by Haynes et al.

For a real valued function $h: V \to R$ on V, weight of h is defined to be $w(h) = \sum_{v \in V} h(v)$. Further, for a subset S of V we let $h(S) = \sum_{v \in S} h(v)$. Therefore $w(h) = h(V)$.

A two valued function $h: V \to$ {-1, 1} is called a signed majority dominating function if the sum of its function values over at least half the closed neighbourhoods is at least one. A non-negative majority total domination number of a graph G is the minimum value of $\sum_{v \in V(G)} f(v)$ over all *non-negative majority total dominating functions* f of G and it is denoted by $\gamma_{maj}^{nt}(G)$. Broere et al. introduced Majority domination in [3] and this concept is further studied in [4]. Later, Hua-ming xing et al. [5] introduced and studied the following concept. A function $h: V \to$ *{-1, 1}* is called a signed majority total dominating function if $h(N(v)) \geq 1$ for at least half of the vertices in graph G. The signed majority total domination number of G, is denoted by $\gamma_{maj}^{t}(G)$, and is defined as

$$\gamma_{maj}^{t}(G) = \{w(h) | h \text{ is a signed majority total dominating} \text{function of } G\}.$$

In 2017, Sahul Hamid and S. Anandha Prabhavathy [6] introduced non-negative majority total domination of a graph G which is defined as follows: a two valued function $h: V \to \{-1, 1\}$ is called a *non-negative majority total dominating function* if the sum of its function values over at least half the open neighbourhoods is at least zero. A non-negative majority total domination number of a graph G

is the minimum value of $\sum_{v\in V(G)} h(v)$ over all *non-negative majority total dominating functions f* of *G* and it is denoted by $\gamma_{maj}^{nt}(G)$, see Figure 1. So for exact values of $\gamma_{maj}^{nt}(G)$ are known only for complete graph, complete bipartite graph, path, cycle and star.

Before we get into results, we define some notation which we have used throughout the paper. If h is a minimum non-negative majority total dominating function, then

- $V_+ = \{v \in V(G): h(v) = +1\}$;
- $V_- = \{v \in V(G): h(v) = -1\}$; and
- $N_h = \{v \in V(G): N(h(v)) \geq 0\}$.

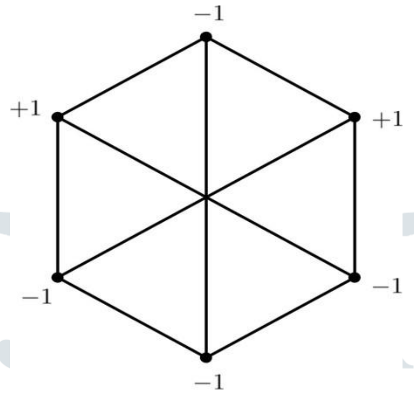

Figure 1: The graph G with $\gamma_{maj}^{nt}(G) = -2$.

II. NON-NEGATIVE MAJORITY TOTAL DOMINATION NUMBER OF COMPLETE GRAPH MINUS A PERFECT MATCHING.

Theorem 1 Let $G = K_{2a} - M$, where M is a perfect matching in a complete graph K_{2a}. Then $\gamma_{maj}^{nt}(G) = -2$.

Proof: Let $V(K_{2a}) = \{v_1, v_2, \ldots, v_a\}$. Consider a perfect matching of K_{2a} is $M = \{v_1v_{2,}v_3v_4, \ldots, v_{2a-1}v_{2a}\}$.

Define a function $h: V \to \{-1, 1\}$ by

$$h(v_i) = \begin{cases} +1 \; if \; 1 \; \leq i \; \leq a+1 \\ -1 \; otherwise \end{cases}$$

It is easy to verify that $h(N(v)) \geq 0$ for at least half of the vertices in G. Further, $w(h) = -2$.

Thus we have $\gamma_{maj}^{nt}(G) \leq -2$.

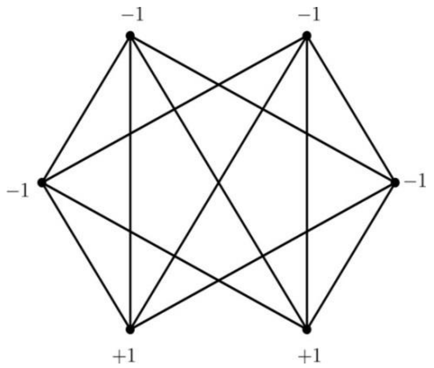

Figure 2: The Graph $K_6 - M$ with $\gamma_{maj}^{nt}(G) = -2$.

Let h be a non-negative majority total dominating function of G. Then $|V_+| + |V_-| = 2a$ and $|V_+| - |V_-| = h(V)$. Consider a vertex $v_{2i-1}, 1 \le i \le a$, of G with $N(v_{2i-1}) \ge 0$. Since G is $2a - 2$ regular,

$$h(V) = h\big(N(v_{2i-1})\big) + h(v_{2i-1}) + h(v_{2i}) \ge 0 - 1 - 1 = -2.$$

$$\text{That is, } |V_+| - |V_-| \ge -2.$$

It follows that $|V_+| \ge a - 1$ and $|V_-| \ge a + 1$ and hence we have

$$\gamma_{maj}^{nt}(G) \ge -2.$$

Thus we have $\gamma_{maj}^{nt}(G) = -2$. Hence the theorem.

III. NON NEGATIVE MAJORITY DOMINATION NUMBER OF PATH AND CYCLE RELATED GRAPHS

We start this section with definitions.

Definition 1 An $m - fan$, denoted by F_m and $V(F_m) = \{v_0, v_1, \dots, v_m\}$, is the graph that contains the path $P_m = v_1, v_2, \dots, v_m$ and the vertex v_0 is adjacent to all vertices of P_m.

Definition 2 An $m - wheel$, denoted by W_m and $V(W_m) = \{v_0, v_1, \dots, v_m\}$, is the graph that contains the cycle $C_m = (v_1, v_2, \dots, v_m)$ and the vertex v_0 is adjacent to all vertices of P_m.

Definition 3 The $(m, n) - tadpole\ graph$, denoted by $T_{m,n}$, $m \ge 3$ and $n \ge 1$, is the graph obtained by joining a cycle $C_m = (v_1, v_2, \dots, v_m)$ and a path $P_n = u_1, u_2, \dots, u_n$ by a bridge $v_m u_1$.

Definition 4 The $m - sunlet, m \ge 3$, is the graph on $2m$ vertices obtained by attaching m pendant edges to a cycle $C_{m.}$ We denote it by S_m.

We make use of the following theorem which is proved in [6].

Theorem 2 For $m \ge 3, \gamma_{maj}^{nt}(C_m) = \gamma_{maj}^{nt}(P_m) = 2\left\lceil \frac{m}{4} \right\rceil - m$.

Theorem 3[5] For ≥ 3, $\gamma_{maj}^{n}(P_m) = \begin{cases} -1\ if\ m\ is\ an\ odd\ integer \\ 0\ if\ m\ is\ an\ even\ integer \end{cases}$.

Theorem 4[5] For $m \ge 3$,

$$\gamma_{maj}^{n}(C_m) = \begin{cases} 3\ if\ m\ is\ an\ odd\ integer \\ 0\ if\ m\ is\ an\ even\ integer \end{cases}.$$

Theorem 4 For $m \ge 3$, $\gamma_{maj}^{nt}(F_m) = \gamma_{maj}^{nt}(P_m) + 1$.

Proof: Let$V(F_m) = \{v_0, v_1, \dots, v_m\}$. Define a function $h: V \to \{-1, 1\}$ by

$$h(v_i) = \begin{cases} +1\ if\ 0 \le i \le \left\lceil \frac{m}{2} \right\rceil + \left\lceil \frac{m}{4} \right\rceil\ and\ i \equiv 2(mod\ 3) \\ -1\ otherwise \end{cases}$$

One can easily verify that $h(N(v)) \geq 0$ for at least half of the vertices with weight $w(h) = 2\left\lceil\frac{m}{4}\right\rceil + 1 - m = \gamma_{maj}^{nt}(P_m) + 1$.

Hence $\gamma_{maj}^{nt}(F_m) \leq \gamma_{maj}^{nt}(P_m) + 1$.

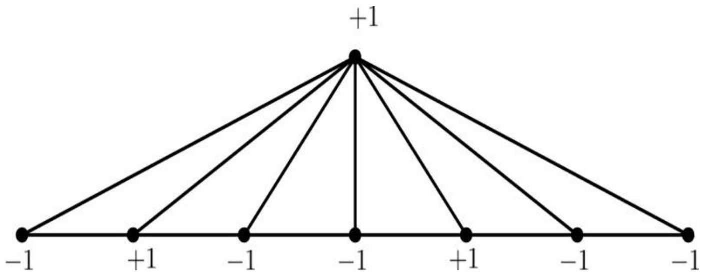

Figure 3: The graph F_7 with $\gamma_{maj}^{nt}(F_7) = -2$.

To prove reverse inequality, let h be a non-negative majority total dominating function of F_m. Then $|V_+| + |V_-| = m + 1$ and $|V_+| - |V_-| = h(V)$.

Case 1: $v_0 \in N_h$.

Since $v_0 \in N_h$ and degree of v_0 is m, $|V_+| \geq \left\lceil\frac{m}{2}\right\rceil$ and $|V_-| \leq \left\lfloor\frac{m}{2}\right\rfloor$

Hence $\gamma_{maj}^{nt}(F_m) \geq |V_+| - |V_-| - 1$

$$= \left\lceil\frac{m}{2}\right\rceil - \left\lfloor\frac{m}{2}\right\rfloor - 1.$$

As $\left\lceil\frac{m}{2}\right\rceil - \left\lfloor\frac{m}{2}\right\rfloor - 1 \geq 2\left\lceil\frac{m}{4}\right\rceil + 1 - m$,

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{nt}(P_m) + 1. ----(1)$$

Case 2: $v_0 \notin N_h$.

It is clear that either $v_0 = -1$ or $v_0 = 1$.

Let $v_0 = 1$. Since h is the minimum non-negative total dominating function of F_m and $v_0 = 1$,

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{nt}(P_m) + 1.$$

By Theorem 2, we have

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{nt}(P_m) + 1 = 2\left\lceil\frac{m}{4}\right\rceil + 1 - m. ----(2)$$

Let $v_0 = -1$ and $H = F_m - v_0 \cong P_m$. Because of the choice of v_0, a vertex $v \in N_h$ if and only if the sum of the weight of the $N(v)$ of H must be at least one. Consequently,

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{n}(P_m) + 1.$$

By Theorem 3, it is clear that

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{n}(P_m) + 1 \geq \gamma_{maj}^{nt}(P_m) + 1. ----(3)$$

From (1), (2) and (3), we have $\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{nt}(P_m) + 1$.

Thus $\gamma_{maj}^{nt}(F_m) = \gamma_{maj}^{nt}(P_m) + 1$.

Theorem 5 For $m \geq 4$, $\gamma_{maj}^{nt}(W_m) = \gamma_{maj}^{nt}(P_m) + 2$.

Proof: Let $V(F_m) = \{v_0, v_1, \dots, v_m\}$. Define a function $h: V \to \{-1, 1\}$ by

$$h(v_i) = \begin{cases} +1 \; if \; i = 1 \; and \; 0 \; \leq i \; \leq \left\lceil \frac{m}{2} \right\rceil + \left\lceil \frac{m}{4} \right\rceil \; and \; i \equiv 2(mod\; 3) \\ -1 \; otherwise \end{cases}$$

Clearly, $h(N(v)) \geq 0$ for at least half of the vertices with weight $w(h) = 2\left\lceil \frac{m}{4} \right\rceil + 2 - m = \gamma_{maj}^{nt}(P_m) + 2$.

$$\text{Hence } \gamma_{maj}^{nt}(W_m) \leq \gamma_{maj}^{nt}(P_m) + 2.$$

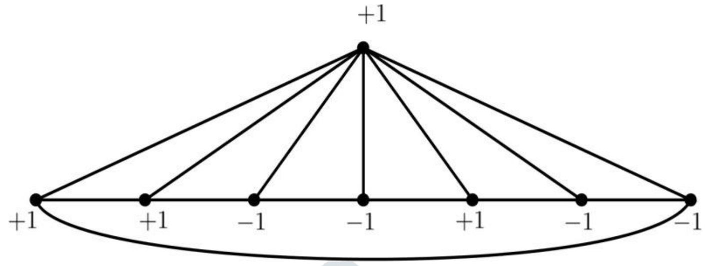

Figure 4: The graph $\gamma_{maj}^{nt}(W_7) \leq 0$.

To prove equality, let h be a non-negative majority total dominating function of F_m. Then $|V_+| + |V_-| = m + 1$ and $|V_+| - |V_-| = h(V)$.

Case 1: $v_0 \in N_h$.

Since $v_0 \in N_h$ and degree of v_0 is m, $|V_+| \geq \left\lceil \frac{m}{2} \right\rceil$ and $|V_-| \leq \left\lfloor \frac{m}{2} \right\rfloor$

Hence $\gamma_{maj}^{nt}(W_m) \geq |V_+| - |V_-| - 1$

$$= \left\lceil \frac{m}{2} \right\rceil - \left\lfloor \frac{m}{2} \right\rfloor - 1.$$

As $\left\lceil \frac{m}{2} \right\rceil - \left\lfloor \frac{m}{2} \right\rfloor - 1 \geq 2\left\lceil \frac{m}{4} \right\rceil + 1 - m$,

$$\gamma_{maj}^{nt}(W_m) \geq \gamma_{maj}^{nt}(P_m) + 2. - - - - (1)$$

Case 2: $v_0 \notin N_h$.

It is clear that either $v_0 = -1$ or $v_0 = 1$.

Let $v_0 = 1$. Since h is the minimum non-negative total dominating function of W_m and $v_0 = 1$,

$$\gamma_{maj}^{nt}(W_m) \geq \gamma_{maj}^{nt}(P_m) + 2.$$

By Theorem 2, we have

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{nt}(P_m) + 1 = 2\left\lceil \frac{m}{4} \right\rceil + 2 - m. - - - - (2)$$

Let $v_0 = -1$ and $H = W_m - v_0 \cong C_m$. Because of the choice of v_0, a vertex $v \in N_h$ if and only if the sum of the weight of the $N(v)$ of H must be at least one. Consequently,

$$\gamma_{maj}^{nt}(W_m) \geq \gamma_{maj}^{n}(C_m) + 1.$$

By Theorem 4, it is clear that

$$\gamma_{maj}^{nt}(F_m) \geq \gamma_{maj}^{n}(C_m) + 1 \geq \gamma_{maj}^{nt}(P_m) + 2. - - - - (3)$$

From (1), (2) and (3), we have $\gamma_{maj}^{nt}(W_m) \geq \gamma_{maj}^{nt}(P_m) + 2$.

$$\text{Thus } \gamma_{maj}^{nt}(W_m) = \gamma_{maj}^{nt}(P_m) + 2.$$

Theorem 6 For $m \geq 3$ and $n \geq 2$,

$$\gamma_{maj}^{nt}(T_{m,n}) = \gamma_{maj}^{nt}(C_m) + \gamma_{maj}^{nt}(P_n).$$

Proof: Let $V(C_m) = \{v_1, v_2, \dots, v_m\}$ and $V(P_n) = \{u_1, u_2, \dots, u_n\}$. Let $e = v_1u_1$ be the bridge joining C_m and P_n. Then $V(T_{m,n}) = V(C_m) \cup V(P_n)$.

Define a function $h: V \to \{-1, 1\}$ by

$$h(v_i) = \begin{cases} +1 \; if \; 0 \leq i \leq \left\lceil \frac{m}{2} \right\rceil + \left\lceil \frac{m}{4} \right\rceil \; and \; i \equiv 2(mod\ 3) \\ -1 \; otherwise \end{cases}$$ and

$$h(u_i) = \begin{cases} +1 \; if \; 0 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + \left\lceil \frac{n}{4} \right\rceil \; and \; i \equiv 2(mod\ 3) \\ -1 \; otherwise \end{cases}$$

It is not difficult to check that $h(N(v)) \geq 0$ for at least half of the vertices with weight

$$w(h) = 2\left(\left\lceil \frac{m}{4} \right\rceil + \left\lceil \frac{n}{4} \right\rceil\right) - (m+n) = \gamma_{maj}^{nt}(C_m) + \gamma_{maj}^{nt}(P_n).$$

Hence $\gamma_{maj}^{nt}(T_{m,n}) \leq \gamma_{maj}^{nt}(C_m) + \gamma_{maj}^{nt}(P_n)$.

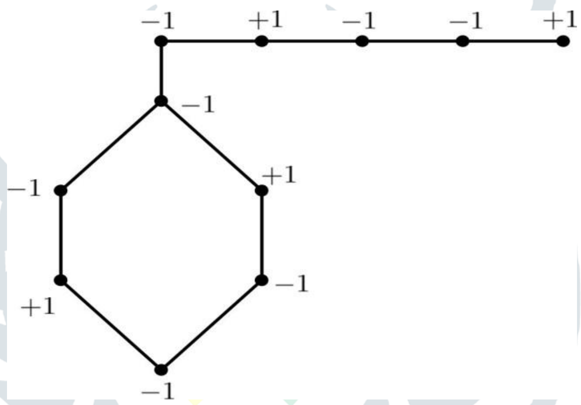

Figure 5: The graph $T_{6,5}$ with $\gamma_{maj}^{nt}(T_{6,5}) = -7$

On the other hand, let h be a non-negative majority total dominating function of $T_{m,n}$. Let $G = T_{m,n} - e = C_m \cup P_n$. By Theorem 2, we have $\gamma_{maj}^{nt}(T_{m,n}) \geq \gamma_{maj}^{nt}(C_m) + \gamma_{maj}^{nt}(P_n)$. Hence $\gamma_{maj}^{nt}(T_{m,n}) = \gamma_{maj}^{nt}(C_m) + \gamma_{maj}^{nt}(P_n)$.

We make use of the following theorem which is proved in [].

Theorem 7 If G has m vertices, then

$$\gamma_{maj}^{nt}(G) \geq \begin{cases} \dfrac{\delta m - 2\Delta m}{\delta + \Delta} \; if \; m \; is \; even \\ \dfrac{\delta m + \Delta(1-2m)}{\delta + \Delta} \; if \; m \; is \; odd \end{cases}$$

Theorem 8 For $m \geq 3, \gamma_{maj}^{nt}(S_m) = \left\lceil \frac{m}{2} \right\rceil - 2m$.

Proof: Let $V(S_m) = \{v_1, v_2, \dots, v_m\} \cup \{u_1, u_2, \dots, u_m\}$, where degree of $u_i = 1$.

Define a function $h: V \to \{-1, 1\}$ by

$$h(v_i) = \begin{cases} +1\ if\ i \equiv 1(mod\ 2) \\ -1\ otherwise \end{cases} \text{ and}$$

$$h(u_i) = -1\ for\ all\ i.$$

From the above function, it guarantee that at least half the vertices of S_m has $h(N(v)) \geq 0$.

Hence, we have $\gamma_{maj}^{nt}(S_m) \leq \left\lceil \frac{m}{2} \right\rceil - 2m. - - - - (1)$

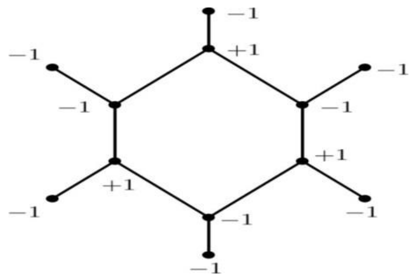

Figure 6: The graph S_6 with$\gamma_{maj}^{nt}(S_6) = -6$

For equality, let h be a non-negative majority total dominating function of S_m.

Let $N_h = U \cup V$, where U contains the vertex of degree one.

Clearly, $\delta = 3, \Delta = 3$ and $|V(S_m)| = 2m$.

By Theorem 7 we have

$$\gamma_{maj}^{nt}(S_m) \geq \begin{cases} -m\ if\ m\ is\ even \\ \dfrac{1-2m}{2}\ if\ m\ is\ odd \end{cases} - - - - (2)$$

From (1) and (2), we have

$$\gamma_{maj}^{nt}(S_m) = \left\lceil \frac{m}{2} \right\rceil - 2m.$$

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