A STUDY ON CONSTITUTIVE EQUATION OF WALTERS LIQUID

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Abstruct

In this paper, we have made an attempt to give an introduction to the development of non-Newtonian fluid from the classical theory of Newtonian liquid. The emphasis has been given to Walters liquid $B^{/}$. Rheological model of constitutive equations of the same liquid has been discussed. An attempt has been made to deduce the the equation of motion of Walters liquid $B^{/}$. The tensor form of the constitutive equation is taken and then the equation of motion is found out in Cartesian form.

1. Introduction:

The study of the mechanics of all liquids and gases is classified as the Fluid mechanics. The ideas built up in mechanics and thermodynamics are extended in this branch to the study of motion and equilibrium of fluids. One of the indispensable tools to study the mechanics of fluids is the equation of motion of fluid which was first given by Euler in 1755 in his paper "General Principles of the Motion of Fluids", which was for perfect fluids. This was a turning point in making rapid strides altering the very nature of engineering applications. This followed the concept of perfect fluid with the assumption of linearity of relation between the stress and the rate of strain tensors and the normal direction of stress vector. A magnificent breakthrough came in 1904 with the mathematician Prandtl proposing his assumption of boundary layer. Mathematicians have begun to study and analyse different problems using models of different kinds of non- Newtonian fluids. Seth, Mishra and Tripathi [1] recently studied convective heat transfer in Walter's liquid. Charyulu, V. N. and Ram, M. S [2] studied the flow of a visco-elastic second order fluid through porous medium.

Unfortunately, the prevailing theory was of little significance in explaining some special problems in case of the chemical and process industries.

Newton [3] in 1687 suggested the constitutive equations of isotropic, viscous incompressible fluid as

$$\sigma_{ij} = -p\delta_{ij} + 2\mu.e_{ij}, e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right), \qquad (1)$$

which is a linear equation.

Here, σ_{ij} is stress tensor,

p is indeterminate hydrostatic pressure,

δ_{ij} is kronecker delta,

μ is the coefficient of viscosity,

v_i are velocity components.

2. Visco-elastic Fluids

The fluids exhibiting a certain degree of elasticity are called visco-elastic fluids. In the motion of such fluids, certain amount of energy is stored up as strain energy while a part of energy is lost due to viscous dissipation. With the removal of the stress it recovers its original state and possibly a reverse flow. Strain is responsible for these phenomena. As a consequently, the strain cannot be neglected in case of consideration of flow phenomena of visco-elastic fluid, however small it may be. The concept of "memory", i.e. the measure of elasticity of the fluid comes to the picture at this point. During the flow of such a fluid, its natural state changes constantly and it continuous endeavors to attain the instantaneous state of the deformed state, but without complete success. This lag is the so called "memory" of the fluid.

3. Walters Fluid

A special kind of visco-elastic fluid, which we are going to deal with is the Walters liquid B/ [4,5]. Its constitutive equation is

$$\sigma_{ij} = -pg_{ik} + \tau_{ij} \qquad (2)$$

where

σ_{ij} is the stress tensor,

p is an arbitrary isotropic pressure,

g_{ij} is the metric tensor of fixed coordinate system.

In case of liquids with short memories, this equation takes the simplified form

$$\tau^{ij} = 2\eta_0 e^{(1)ij} - 2K_0 \frac{\delta}{\delta t} e^{(1)ij} \qquad (3)$$

Where,

η_0 is the limiting viscosity at small rate of shear and its value is obtained by integrating the distributive function $N(\tau)$ of the relaxation times τ within the limits from 0 to $\propto$, being the distributive function of the relaxation times τ .

$K_0 = \int_0^\infty \tau N(\tau) d\tau$ and

$\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor quantity, which for any contravariant tensor b^{ij} is given by

$$\frac{\delta}{\delta t} b^{ij} = \frac{\partial b^{ij}}{\partial t} + v^m \frac{\partial b^{ij}}{\partial x^m} - \frac{\partial v^j}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mj},$$

where

v_i is the velocity vector.

x^i, x'^i are the position at the time t and $t^/$ of the element which is instantaneously at the point x^i at time t and $e_{ij}^{(1)}$ the rate of strain tensor.

Researchers while going to analyse problems relating to the flow of visco-elastic fluid taking the rheological model of Walters Fluid (Model B$^/$), needs some primary tools, the main one being the equations of motion. For deducing such equations one needs constitutive equation of the fluid in consideration, in whatever coordinate system it is required. We now go for such a process to deduce the dynamical equations of Walters liquid B$^/$ in Cartesian system of coordinates.

4. Equations in Cartesian Coordinates

Let x, y, z be the rectangular Cartesian coordinates and u, v, w be the velocity components in these directions. Then the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4)$$

The equations of motions are

$$\rho a_x = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z},$$

$$\rho a_y = \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z},$$

$$\rho a_z = \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}, \qquad (5)$$

where

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z},$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z},$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}, \qquad (6)$$

are the acceleration components in x, y, z directions.

The components of stress for Walters fluid (Model B$^{/}$) are given by

$$\sigma_{xx} = -p + 2\eta_0 \frac{\partial u}{\partial x} -$$

$$2K_0\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\frac{\partial u}{\partial x} - \left\{2\left(\frac{\partial u}{\partial x}\right)^2\right.\right.$$

$$\left.\left. + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\frac{\partial u}{\partial y} + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\frac{\partial u}{\partial z}\right\}\right],$$

$$\sigma_{yy} = -p + 2\eta_0 \frac{\partial v}{\partial y}$$

$$-2K_0\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\frac{\partial v}{\partial y} - \left\{2\left(\frac{\partial v}{\partial y}\right)^2\right.\right.$$

$$\left.\left. + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\frac{\partial v}{\partial x} + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\frac{\partial v}{\partial z}\right\}\right],$$

$$\sigma_{zz} = -p + 2\eta_0 \frac{\partial w}{\partial z}$$

$$-2K_0\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\frac{\partial w}{\partial z} - \left\{\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\frac{\partial w}{\partial x}\right.\right.$$

$$\left.\left. + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\frac{\partial w}{\partial y} + 2\left(\frac{\partial w}{\partial z}\right)^2\right\}\right],$$

$$\sigma_{xy} = \eta_0\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$-2K_0\left[\frac{1}{2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \left\{\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\right.\right.$$

$$\left. + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\frac{\partial v}{\partial z}\right\}$$

$$-\left\{\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial u}{\partial y}\right.$$

$$\left.\left. + \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\frac{\partial u}{\partial z}\right\}\right],$$

$$\sigma_{xz} = \eta_0\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-$$

$$2K_0\left[\frac{1}{2}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\left\{\frac{\partial u}{\partial x}\frac{\partial w}{\partial x}\right.\right.$$

$$\left.+\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\frac{\partial w}{\partial z}\right\}-\left\{\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\frac{\partial u}{\partial x}\right.$$

$$\left.\left.+\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\frac{\partial u}{\partial y}+\frac{\partial w}{\partial z}\frac{\partial u}{\partial z}\right\}\right]$$

$$\sigma_{yz} = \eta_0\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)$$

$$-2K_0\left[\frac{1}{2}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right.$$

$$-\left\{\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\frac{\partial w}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial w}{\partial y}+\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\frac{\partial w}{\partial z}\right\}$$

$$-\left\{\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\frac{\partial v}{\partial x}\right.$$

$$\left.\left.+\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\frac{\partial v}{\partial z}\right\}\right]. \qquad (7)$$

Putting these stress components in the equations of motions (5) we will get the required equations.

5. Conclusion:

To facilitate the solutions of various problems of engineering and industrial importance involving the visco-elastic fluid flows, it is necessary that methods be developed for studying the properties of such fluids in configurations other than the classical viscometric flow fields. Out of the different rheological models, we have chosen Walters fluid (Model B$^{/}$). The main characteristic of such rheological models is the presence of parameters or indexes defining the nature of the fluid. If we put the value of this parameter equal to zero, we get the results for Newtonian fluid. Many commonly found examples of non-Newtonian fluids are many salt solutions, ketchup, custard, starch suspensions, molten polymer, blood, commonly used paints, shampoo etc. Framing and solving different problems involving such types of fluids has wide scope of applications in different fields like chemical industries, paint industries, medical treatments, aviation and maritime industries. Here the ground has been prepared for further investigation of the flow patterns and analyse their behavior under different valid and suitable boundary conditions. The results reveal various aspects of the additional terms in the constitutive equation as compared to Newtonian fluid.

Major portion of the earth consists of liquid. Our day to day life is highly dependent on the use of liquids and their characteristics from crude oil to blood. These are not generally perfect liquids which is almost impossible to find in normal conditions. As such, the study of the characteristics of various models of non-Newtonian fluids, including visco-elastic fluids is important from the point of industrial importance. The scope of this field is ever-increasing.

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