ON $\delta\hat{g}$ HOMEOMORPHISM IN GRILL TOPOLOGICAL SPACES

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Abstract: This article is focused to introduce and explore the idea of a new class of homeomorphism called $\zeta\delta\hat{g}$ homeomorphism and $\zeta\delta\hat{g}c-$ homeomorphism using $\zeta\delta\hat{g}$ closed sets and study their basic properties.

IndexTerms- $\zeta\delta\hat{g}$ closed, $\zeta\delta\hat{g}$ open, $\zeta\delta\hat{g}$ continuous, $\zeta\delta\hat{g}$ irresolute**.**

I. INTRODUCTION

The idea of generalized homeomorphism and gc-homeomorphism was introduced and studied by H. Maki et.al [5].The concept of δ homeomorphism, semi homeomorphism, α -homeomorphism, $\delta\hat{g}$ homeomorphisms were published in [3],[2], [6],[4].We introduce $\zeta\delta\hat{g}$ homeomorphism and $\zeta\delta\hat{g}c$ -homeomorphism. The association of $\zeta\delta\hat{g}$ homeomorphism with various existing homeomorphism is also studied.

II. PRELIMINARIES

Definition 2.1

A subset A of (X,τ,ζ) is $\zeta\delta\hat{g}$ closed set [1] if $\varphi_\delta(A)\subseteq U$ whenever A$\subseteq$U and U is $\zeta\omega$ open in (X,τ,ζ).

Definition 2.2

A function $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ is $\zeta\delta\hat{g}$ continuous ($\zeta\delta\hat{g}$ irresolute) if $f^{-1}(V)$ is $\zeta\delta\hat{g}$ open($\zeta\delta\hat{g}$ open) in (X,τ,ζ) for every open set ($\zeta\delta\hat{g}$ open) V in (Y,σ,ζ).

$\zeta\delta\hat{g}$ Homeomorphism

Definition 3.1

A function $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ is called an $\zeta\delta\hat{g}$ closed (resp $\zeta\delta\hat{g}$ open) map if for every closed subset A of X, f(A) is $\zeta\delta\hat{g}$ closed (resp $\zeta\delta\hat{g}$ open) in (Y,σ,ζ).

Definition 3.2

A bijection $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ is said to be $\zeta\delta\hat{g}$ homeomorphism if f is both $\zeta\delta\hat{g}$ continuous and $\zeta\delta\hat{g}$ closed.

Theorem 3.3

Every δ -homeomorphism is $\zeta\delta\hat{g}$ homeomorphism and the reverse is false.

Proof:

Proof is direct by "every δ continuous map is $\zeta\delta\hat{g}$ continuous and every δ open map is $\zeta\delta\hat{g}$ open.

Example 3.4

Let X = Y = {u,v,w} with $\tau=\{\phi,X,\{u\},\{v,w\}\}$ $\sigma=\{\phi,Y,\{u\},\{w\},\{u,w\},\{v,w\}\}$ $\zeta=\{X,\{u\},\{w\},\{u,v\},\{v,w\},\{u,w\}\}$.Let $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ be an identity map. Clearly f is $\zeta\delta\hat{g}$ homeomorphisms but not δ homeomorphisms. For the closed set {u,v} in Y $f^{-1}\{u,v\}=\{u,v\}$ is not δ closed in X. This implies f is not δ continuous.

Remark 3.5

$\zeta\delta\hat{g}$ homeomorphisms is independent of $\zeta\alpha$ homeomorphism, semi- homeomorphism, α -homeomorphism and $\zeta\omega$ homeomorphism.

Theorem 3.6

The composition of two $\zeta\delta\hat{g}$ homeomorphism fails to be a $\zeta\delta\hat{g}$ homeomorphism and is given in the example.

Example 3.7

Let X=Y=Z= {u,v,w} with topologies $\tau=\{\phi,X,\{v\},\{u,v\},\{v,w\}\}$ $\sigma=\{\phi,Y,\{v\},\{w\},\{v,w\}\}$ $\eta=\{\phi,Z,\{w\},\{u,v\}\}$, $\zeta=\{X,\{v\},\{v,w\},\{u,w\}\}$ $\zeta=\{X,\{v\},\{w\},\{v,w\},\{u,w\}\}$ .Let $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ and $g:(Y,\sigma,\zeta)\to(Z,\eta,\zeta)$ be an identity map. Clearly f and g are $\zeta\delta\hat{g}$ homeomorphisms .For the closed set {w} in X $g\circ f(\{w\})=\{w\}$ is not $\zeta\delta\hat{g}$ closed in Z, so $g\circ f$ is not $\zeta\delta\hat{g}$ homeomorphism.

III. $\zeta\delta\hat{g}c$ -HOMEOMORPHISM

Definition 4.1

A bijection $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ is said to be $\zeta\delta\hat{g}c$ -homeomorphisms if both f and f^{-1} are $\zeta\delta\hat{g}$ irresolute.

Theorem 4.2

Every $\zeta\delta\hat{g}c$ homeomorphism is an $\zeta\delta\hat{g}$ homeomorphism.

Proof:

Let $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ be a $\zeta\delta\hat{g}c$ homeomorphism. Then f , f^{-1} are $\zeta\delta\hat{g}$ irresolute and f is bijection. Thus f and f^{-1} are $\zeta\delta\hat{g}$ continuous. Hence f is $\zeta\delta\hat{g}$ homeomorphism.

Remark 4.3

The converse is false is shown in the example.

Example 4.4

Let X=Y={u,v,w} with topologies $\tau=\{\phi,X,\{v\},\{w\},\{v,w\}\}$ $\zeta=\{X,\{v\},\{v,w\},\{u,w\}\}$, $\sigma=\{\phi,Y,\{w\},\{v,w\}\}$, $\zeta=\{X,\{v\},\{w\},\{v,w\},\{u,w\}\}$. Let $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ be an identity map. Then f is $\zeta\delta\hat{g}$ homeomorphism but f is not $\zeta\delta\hat{g}c$ homeomorphism because the subset {w} is $\zeta\delta\hat{g}$ closed in X but f{w} ={w} is not $\zeta\delta\hat{g}$ closed in Y.

Theorem 4.5

If $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ be $\zeta\delta\hat{g}c-$homeomorphism then $\zeta\delta\hat{g}-cl[f^{-1}(B)]=f^{-1}[\zeta\delta\hat{g}-cl(B)]$ $for\ all\ B\subset Y$.

Corollary:4.6

Let $f:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ is $\zeta\delta\hat{g}c$ -homeomorphism then $\zeta\delta\hat{g}-cl[f(B)]=f[\zeta\delta\hat{g}-cl(B)]$ $for\,all\ B\subset X$.

Theorem 4.7

The composition of two $\zeta\delta\hat{g}c-$ homeomorphism is $\zeta\delta\hat{g}c-$ homeomorphism.

Proof:

Let $g:(X,\tau,\zeta)\to(Y,\sigma,\zeta)$ and $h:(Y,\sigma,\zeta)\to(Z,\eta,\zeta)$ be $\zeta\delta\hat{g}c-$ homeomorphism. Let S be a $\zeta\delta\hat{g}$ open set in (Z,η,ζ) .Since h is a $\zeta\delta\hat{g}$ -irresolute,

$h^{-1}(\{S\})$ is $\zeta\delta\hat{g}$ open set in (Y,σ,ζ) .Since g is a $\zeta\delta\hat{g}$ -irresolute and bijective map,

$g^{-1}(h^{-1}\{S\})$ is $\zeta\delta\hat{g}$ open set in (X,τ,ζ) .That is $(h\circ g)^{-1}(V)=g^{-1}(h^{-1}\{V\})$ is $\zeta\delta\hat{g}$ open in (X,τ,ζ) .This implies that $h\circ g$ is $\zeta\delta\hat{g}$ -irresolute. Let T be a $\zeta\delta\hat{g}$ open set in (X,τ,ζ) .Since g^{-1} is $\zeta\delta\hat{g}$ -irresolute, $(g^{-1})^{-1}$ (T) is $\zeta\delta\hat{g}$ open in (Y,σ,ζ). That is g (T) is $\zeta\delta\hat{g}$ open in (Y,σ,ζ) .Since h^{-1} is $\zeta\delta\hat{g}$ -irresolute, $(h^{-1})^{-1}(f(T))$ is $\zeta\delta\hat{g}$ open in (Z,η,ζ) .That is h(g(T)) is $\zeta\delta\hat{g}$ open in (Z,η,ζ). Therefore $(h\circ g)(T)$ is $\zeta\delta\hat{g}$ open in (Z,η,ζ) .This implies that $((h\circ g)^{-1})^{-1}(T)$ is $\zeta\delta\hat{g}$ open in (Z,η,ζ) .Thus the mapping $(h\circ g)^{-1}:(Z,\eta,\zeta)\to(X,\tau,\zeta)$ is $\zeta\delta\hat{g}$ -irresolute. Hence the composition of map $h\circ g$ is a $\zeta\delta\hat{g}c-$ homeomorphism.

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