SOLUTION OF PROBLEM OF LONGITUDINAL DISPERSION OF MISCIBLE FLUID FLOW THROUGH POROUS MEDIA

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Abstract: In this paper we are discussing particularly longitudinal dispersion phenomenon which is the process by miscible fluids in laminar flow mix in the direction of the flow. This phenomenon is discussed by observing the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of this phenomenon yields a non-linear partial differential equation which is transferred into ordinary differential equation by separation of variables technique. Also, this partial differential equation is solved by the method of Crank Nicolson

***Index Terms* -** Longitudinal dispersion, Miscible fluid flow, Porous media, Crank Nicolson.

I. INTRODUCTION

We are discussing particularly longitudinal dispersion phenomenon which is the process by miscible fluids in laminar flow mix in the direction of the flow (Saffman P.G., (1959)). This phenomenon is discussed by observing the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of this phenomenon yields a non-linear partial differential equation which is transferred into ordinary differential equation by separation of variables technique. Also, this partial differential equation is solved by the method of Crank Nicolson.

II. FORMATION

The hydrodynamic dispersion is the macro scope outcome of the actual movement of individual tracer particles through the pore and various physical and chemical phenomenon simultaneously occurs due to molecular diffusion and convection. Several authors have discussed this same problem in different view points for miscible displacement is nothing but one type of double phase flow in porous media in which the two phases are completely soluble in each other (Harriage R.E. and Greenkorn R.A., (1970)). Thus a capillary force between these two fluids is ineffective. The miscible displacement idea could be described in a very simple form at first Darcy's law, followed by the mixture under condition of complete miscibility, and could be through to check, as a single phase fluid (Scheidegger A.E., (1959)). The change of concentration would be caused by diffusion along the flow channels and thus be governed by the bulk coefficient of diffusion of the fluid in the other. In this form, one comes at a heuristic description of miscible displacement which looks very proper.

The problem is to describe the Grow of the mixed region i.e. to find concentration as a function of time t and position x, as the two miscible fluids flow through porous media. Outside of the mixed zone (on either side) the single fluid equations describe the motion. The problem is more complicated, even in one dimension with fluids of equal properties, since the mixing takes place both longitudinally and transversely. Suppose at t=0, we inject a 'dot' of traced fluid of concentration Co rather than over the entire face. This situation is sketched in the following Fig. The dot moves from left to right it will spread in the direction of flow and perpendicular to the flow. At the right the dot has transformed into an ellipse with concentration varying from C to Co across it.

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids, is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) = 0 \tag{1}$$

where ρ is the density for the mixture. $\overline{V}$ is the pore seepage velocity vector. The equation of diffusion for a fluid flow through homogeneous porous medium, without increasing the dispersing material is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(c\,\vec{v}) = \nabla.\left[\rho \overline{D} \nabla \left(\frac{c}{\rho}\right)\right] \tag{2}$$

where C is the concentration of the fluid A in to other fluid B (Host). (i.e. c is the mass of A per unit volume of the mixture) and D is the tensor coefficient of dispersion with nine component Dij.

In a laminar flow through homogeneous porous medium at constant temperature, ρ is constant.

$$\frac{\partial C}{\partial t} + \overline{V}\frac{\partial C}{\partial X} = D_L \frac{\partial^2 C}{\partial X^2} \tag{3}$$

Where $\bar{v}$ is the component of velocity along the x-axis which is time dependent and $D_L > 0$.

Appropriate conditions in longitudinal direction are,
Initial condition $C(x,0)=\epsilon<<1$
Boundary conditions $C(0,t)=C_0; t>0,\ C(1,t)=C_1; t>0$

II. SOLUTION BY CRANK NICOLSON METHOD

Crank-Nicolson scheme is then obtained by taking average that is (J. Crank and P. Nicolson (1996))
The first order derivative by central difference and second order derivative by average at (i, j)th & (i+1, j+1)th levels,
From equation (3), we have

$$\frac{C_i^{j+1}-C_i^j}{\Delta t}=\frac{(C_{i+1}^{j+1}-2C_i^{j+1}+C_{i-1}^{j+1})+(C_{i+1}^j-2C_i^j+C_{i-1}^j)}{2(\Delta x)^2}-\bar{V}\frac{C_{i+1}^{j+1}-C_{i-1}^{j+1}+C_{i+1}^j-C_{i-1}^j}{4(\Delta x)}$$

Take $r=\dfrac{\Delta t}{4(\Delta x)}$

$$\left(-r\bar{V}-\frac{2r}{\Delta x}\right)C_{i-1}^{j+1}+\left(1+\frac{4r}{\Delta x}\right)C_i^{j+1}+\left(r\bar{V}-\frac{2r}{\Delta x}\right)C_{i+1}^{j+1}=\left(r\bar{V}+\frac{2r}{\Delta x}\right)C_{i-1}^j+\left(1-\frac{4r}{\Delta x}\right)C_i^j+\left(-r\bar{V}+\frac{2r}{\Delta x}\right)C_{i+1}^j \tag{4}$$

In equation (4) put $\Delta x=0.1,\ \Delta t=0.0025,\ \bar{v}=0.4,\ N=9,\ J=0$, we have

$$\left(-r\bar{V}-\frac{2r}{\Delta x}\right)=-0.1275,\left(r\bar{V}-\frac{2r}{\Delta x}\right)=0.1225,\left(1+\frac{4r}{\Delta x}\right)=1.25$$

$$\left(r\bar{V}+\frac{2r}{\Delta x}\right)=0.1275,\left(-r\bar{V}+\frac{2r}{\Delta x}\right)=0.1225,\left(1-\frac{4r}{\Delta x}\right)=0.75$$

The crank Nicolson method can be written in matrix form (5)

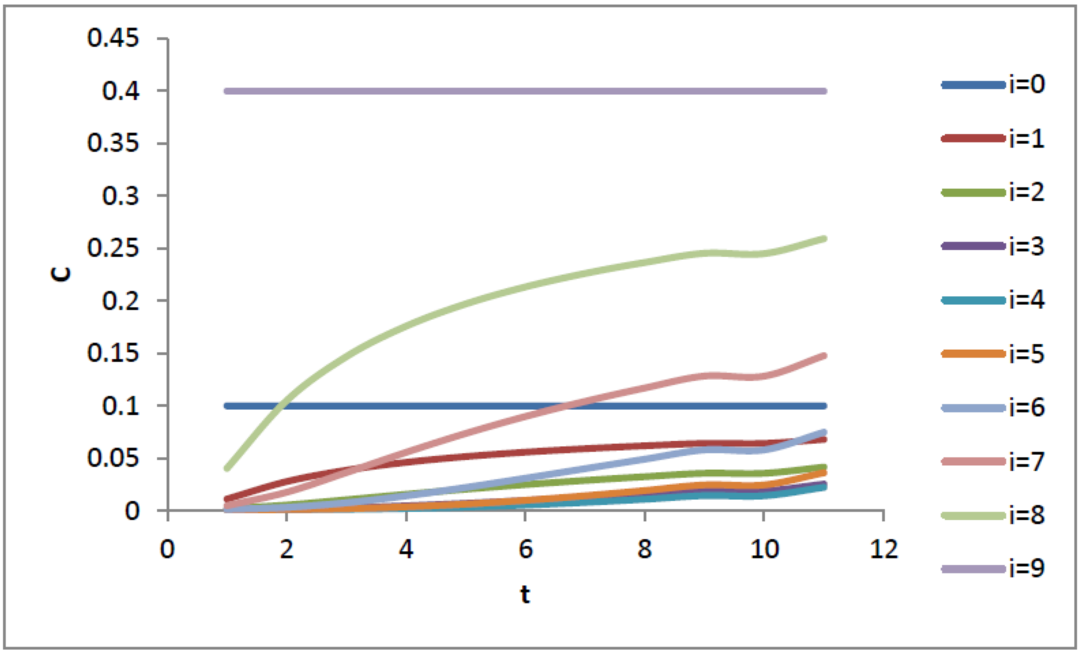
$$\begin{bmatrix} 1.25 & -0.1225 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1275 & 1.25 & -0.1225 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1275 & 1.25 & -0.1225 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1275 & 1.25 & -0.1225 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1275 & 1.25 & -0.1225 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1275 & 1.25 & -0.1225 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.1275 & 1.25 & -0.1225 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.1275 & 1.25 \end{bmatrix}\begin{bmatrix} C_1^1 \\ C_2^1 \\ C_3^1 \\ C_4^1 \\ C_5^1 \\ C_6^1 \\ C_7^1 \\ C_8^1 \end{bmatrix}+\begin{bmatrix} -0.1275C_0^1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.1225C_9^1 \end{bmatrix}$$

$$=\begin{bmatrix} 0.75 & 0.1225 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1275 & 0.75 & 0.1225 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1275 & 0.75 & 0.1225 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1275 & 0.75 & 0.1225 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.1275 & 0.75 & 0.1225 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1275 & 0.75 & 0.1225 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1275 & 0.75 & 0.1225 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1275 & 0.75 \end{bmatrix}\begin{bmatrix} C_1^0 \\ C_2^0 \\ C_3^0 \\ C_4^0 \\ C_5^0 \\ C_6^0 \\ C_7^0 \\ C_8^0 \end{bmatrix}+\begin{bmatrix} 0.1275C_0^0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.1225C_9^0 \end{bmatrix}$$

Table 1

x \ t	j=0	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8	j=9	j=10
i=0	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
i=1	0.011201	0.027859	0.038684	0.046182	0.051666	0.055863	0.0592	0.061939	0.064254	0.064253	0.068038
i=2	0.002051	0.005484	0.010521	0.015717	0.020575	0.024973	0.028925	0.032498	0.035765	0.035765	0.04164
i=3	0.001109	0.001643	0.002872	0.004762	0.007127	0.009806	0.012698	0.015741	0.018897	0.018897	0.025466
i=4	0.001015	0.001119	0.001466	0.002236	0.003555	0.005471	0.007965	0.010978	0.01443	0.01431	0.022325
i=5	0.001039	0.001299	0.002112	0.003783	0.006433	0.010004	0.014351	0.019301	0.024686	0.024686	0.036221
i=6	0.001387	0.003285	0.00762	0.014201	0.022247	0.031058	0.040149	0.049214	0.058073	0.058065	0.07484
i=7	0.004911	0.017683	0.036411	0.055713	0.073711	0.089899	0.104286	0.117057	0.128433	0.128352	0.147808
i=8	0.040501	0.105005	0.146921	0.175949	0.197171	0.213391	0.226241	0.236722	0.245473	0.245164	0.25937
i=9	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

Figure1



IV. CONCLUSION

Here, the table represents a solution of a problem of the longitudinal dispersion phenomenon of miscible fluids flow through homogeneous porous media by using some standard and existing assumptions. The solution obtained is numerical by using the Crank Nicolson Method. Further it is also concluded that the behavior of the concentration in the longitudinal dispersion is oscillatory at some time of interval for some fix values of x and it become constant after some duration of time. Thus, the result obtained is very feasible with physical problem which opens new direction and interpretation.

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