RIEMANN $\zeta(s)$ HYPOTHESIS

(-Proved by Sudhanshu Gaud)

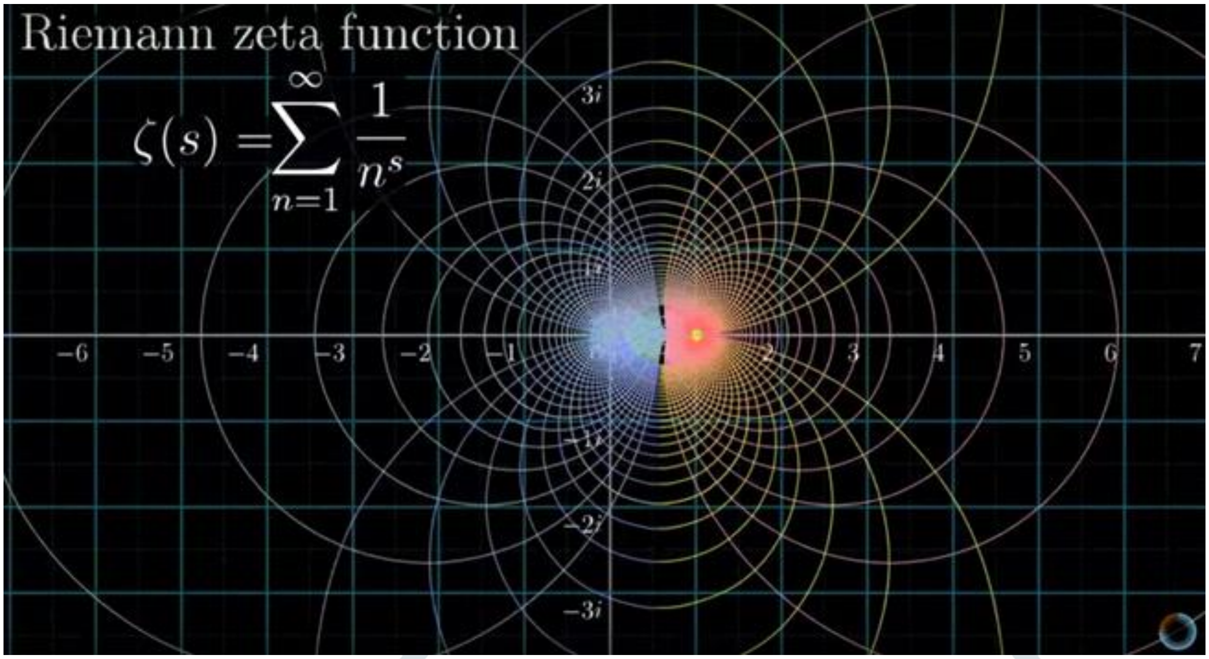
ABSTRACT: Beginning from the statement of Riemann sixth hypothesis which is unproved. Using Euler formula, hyperbolic trigonometric function formulae, Riemann integral formula of converting infinite summation into a definite integral and other functions with some logical ideas for series summation. The author proved the world's mathematical problem Riemann hypothesis raised by German mathematician B. Riemann in 1859.

1)Introduction:-The study of the non-trivial zeros of ζ(s) has been the subject of myriad investigations over the years and is of ongoing interest in number theory. It has also recently attracted the attention of the physics community .German mathematician Bernhard Riemann had raised a very important hypothesis,today known as Riemann hypothesis and which is still unsolved. The hypothesis is very important for number theory in mathematics since number theory is now densely populated with results that begins "if RH is true, then...".If it turns out false ,quite large parts of number theory will have to be rewritten .Analogs of Von Koch's results,which do not depend on RH being true ,are much uglier!

However ,they do have the advantage that we know they are true,unconditionally. The latest result posted by Wedeniwski on dated August 1st 2002, and the reports that the number of non-trivial zeros with real part one-half has now been carried to 100 billion. There is a formula for the number N(T) of zeros up to a given height T: namely , it is approximately $(T/2\pi)\log(T/2\pi)-(T/2\pi)$.There is a rule for the average spacing of zeros at height T in the critical strip: it is approximately 2π/log(T/2π). RH states that all the non-trivial zeros of the zeta function lie on the critical line. In 1914,each of Hardy and Littlewood came out with results of major importance for RH. Hardy:infinitely many of the non -trivial zeros of the zeta function have real part (1/2).Hardy,an eccentric,wrote an essay-A Mathematician. Hardy's stories include "Six new -year wishes"; and "I proved the riemann hypothesis." Hardy is best known for two great collaborations,with great Ramanujan,and with Littlewood. The great mathematician Ramanujan also found a lot of theorems based on Riemann Hypothesis. So ,the Riemann Hypothesis is very much important for us . Mathematicians define a lot of equillencse of Riemann zeta function to prove it but that way to prove it may give us several new functions without getting the proof of Riemann hypothesis.

Here is my point of view to prove it with a logical way, not by defining new function .

2)RH(Riemann hypothesis)-states that the non-trivial zeroes of $\zeta(\theta)$ and $\zeta(s)$ have real part (1/2) always but the zeroes should be non-trivial i.e.(except even negative integers like -2,-4,-6,-8.....).or one can say that all the non-trivial zeros of zeta function lie on the critical line .



3)PROOF:-

$\zeta(s)$ $=1^{-s}+2^{-s}+3^{-s}+4^{-s}$................. ∞ if s=(θ),then

$\zeta(s)$ = $\zeta(\theta)$,Now From Eular formula,[($e^{i\theta}=z$) or ($e^{i\theta} = \cos\theta + \imath \sin\theta$)

$\zeta(\theta)$ $=1^{-\theta}+2^{-\theta}+3^{-\theta}+4^{-\theta}$................. ∞

let $n^{-\theta}=e^{i\alpha}$ Taking natural log in both sides,we get

$-(\theta)\ln|n|=i(\alpha)$

$\alpha = i\theta \ln|n|$

$1^{-\theta}=e^{i(i\theta\ln|1|)}=\cos(i\theta\ln|1|)+i\sin(i\theta\ln|1|)$

$2^{-\theta}=e^{i(i\theta\ln|2|)}=\cos(i\theta\ln|2|)+i\sin(i\theta\ln|2|)$

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$n^{-\theta}=e^{i(i\theta\ln|n|)}=\cos(i\theta\ln|n|)+i\sin(i\theta\ln|n|)$ by adding both sides,we get

$\zeta(\theta) = \sum_{k=1}^{\infty}\cos(i\theta\ln|k|)+i\sum_{k=1}^{\infty}\sin(i\theta\ln|k|)$ for finding real part of zeroes we need to let

$\zeta(\theta)$ =0,this method of finding zeros by taking the function value =0,will give us real part only because imaginary part makes the value of function a rotation at any angle so,there may be a lot of values of such type of angles,we get

from,(formulea of hyperbolic trigonometry) that is cosh(x)=cos(ix) and sinh(x)=(-i)sin(ix)

$\sum_{k=1}^{\infty}\cosh(\theta\ln|k|) = \sum_{k=1}^{\infty}\sinh(\theta\ln|k|)$ we can write it as,

$\sum_{k=1}^{n}\cosh(\theta\ln|k|) = \sum_{k=1}^{n}\sinh(\theta\ln|k|)$ where n approaches to infinity on multiplying both sides by (2.sinh(θ ln|n^2/k|),we get

$$2.\sinh(\theta\ln|\frac{n^2}{k}|).\sum_{k=1}^{n}\cosh(\theta\ln|k|) = 2.\sinh(\theta\ln|\frac{n^2}{k}|).\sum_{k=1}^{n}\sinh(\theta\ln|k|)$$ 1)..eq

$$\because k << n \Rightarrow \therefore k << n^2$$
$$\sinh(\theta\ln|\frac{n^2}{1}|) = \sinh(\theta\ln|\frac{n^2}{2}|) = \sinh(\theta\ln|\frac{n^2}{3}|) = ... = \sinh(\theta\ln|\frac{n^2}{n}|)$$ As here k is variable but the value of above function is same for all value of k so,1)eq turn out to be

$$\sum_{k=1}^{n}2.\sinh(\theta\ln|\frac{n^2}{k}|)\cosh(\theta\ln|k|) = \sum_{k=1}^{n}2.\sinh(\theta\ln|\frac{n^2}{k}|)\sinh(\theta\ln|k|)$$ from,(formulae of hyperbolic trigonometry)we get,

$$2\sinh(A)\cosh(B) = \sinh(A+B) + \sinh(A-B)$$
$$\because \sinh(-x) = -\sinh(x)$$
$$\therefore 2\sinh(A)\cosh(B) = \sinh(A+B) - \sinh(B-A)$$
$$2\sinh(A)\sinh(B) = \cosh(A+B) - \cosh(A-B)$$
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$$\therefore 2\sinh(A)\sinh(B) = \cosh(A+B) - \cosh(B-A)$$

As here A=($(\theta\ln|\frac{n^2}{k}|)$)and B=($(\theta\ln|k|)$)from,formulae of log function,we get

$$\ln|A| + \ln|B| = \ln|A.B|$$
$$\ln|A| - \ln|B| = \ln|A/B|$$ we get,

$$\sum_{k=1}^{n}\{\sinh(\theta\ln|\frac{n^2(k)}{k}|) - \sinh(\theta\ln|\frac{(k)k}{n^2}|)\} = \sum_{k=1}^{n}\{\cosh(\theta\ln|\frac{n^2(k)}{k}|) - \cosh(\theta\ln|\frac{(k)k}{n^2}|)\}$$ if we cancel out equal terms then we get,

$$\sum_{k=1}^{n}\sinh(\theta\ln|\frac{k^2}{n^2}|) = \sum_{k=1}^{n}\cosh(\theta\ln|\frac{k^2}{n^2}|)$$ On multiplying both sides by (1/n),we get

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{k=1}^{n}cosh(\theta\,ln|\frac{k^2}{n^2}|) = \lim_{n\to\infty}\left(\frac{1}{n}\right)\sum_{k=1}^{n}sinh(\theta\,ln|\frac{k^2}{n^2}|)$$ (2)eq let (k/n)=x and from(formula of Riemann integral ,we get)

$$\lim_{n\to\infty}\sum_{k=1}^{k=n}\left(\frac{1}{n}\right)f\left(\frac{k}{n}\right)=\int_0^1 f(x)dx$$

so,(2)eq. turns out to be

$$\int_0^1 cosh(2\theta\, ln\,|\,x\,|)dx=\int_0^1 sinh(2\theta\, ln\,|\,x\,|)dx$$

from(ln|a^2|=2.ln|a|)

from(formulae of trigonometric and hyperbolic trigonometric functions we get)

cosh(x)=cos(ix) and sinh(x)=(-i)sin(ix)

$$\int_0^1 cos(2i\theta\, ln\,|\,x\,|)dx=\int_0^1(-i).\{sin(2i\theta\, ln\,|\,x\,|)\}dx$$

let u=2i θ ln|x| $\Longrightarrow x=e^\wedge(u/2i\theta)$

$$du=\left(\frac{2i\theta}{x}\right)dx\Rightarrow dx=\left(\frac{x}{2i\theta}\right)du$$

$$dx=e^\wedge(u/2i\theta)\frac{du}{2i\theta}$$

here (from Euler formula($cos(u)+i\,sin(u)=e^\wedge(iu)$),

when x=0,then u= $\lim_{x\to 0}$ 2i θ ln|x|

u=2i θ $\lim_{x\to 0}$ ln|x| (3)eq

when x=1 then u=0 ,let

$$e^\wedge p=e^\wedge(ia)=cos(a)+i\,sin(a)$$

if e^(ia)=0 then,

$$cos(a)=-i\,sin(a)$$
$$tan(a)=i$$

$$\tan^{-1}(i)=a$$

$$p = ia$$

$$\Rightarrow p = i\tan^{-1}(i)$$

then , e^(p)=0 and p= $\lim_{x \to 0}$ ln|x| by putting the value in (3)eq ,we get

u=2i θ (i $\tan^{-1}(i)$)

u=-2 $\theta \tan^{-1}(i)$ so,now the above integral equation turn out to be

$\int_{-2\theta\tan^{-1}(i)}^{0} e^{\left(\frac{u}{2i\theta}+iu\right)}$ =0 $\Longrightarrow \int_{-2\theta\tan^{-1}(i)}^{0} e^{ua}$ =0 & let a=(i+1/2i θ)

now from($\int e^\wedge(ua)du = \frac{e^\wedge(ua)}{a}$ and $\int_a^b f(x)dx = F(b) - F(a)$)we get ,

$$e^\wedge(0)/(i+1/2i\theta) - \{e^\wedge(i+1/2i\theta)(-2\theta\tan^{-1}(i))\}/(i+1/2i\theta) = 0$$

if we cancel out denominator and compare power we get real value of(θ)

$$(-2\theta\tan^{-1}(i))(i+1/2i\theta) = 0$$

if we add (0+0+0.......... ∞)=0 ,it means we can say that (0* ∞)=0 $\Rightarrow (-2\theta\tan^{-1}(i))$ = ∞ then,

$i + \frac{1}{2i\theta} = 0 \Longrightarrow \frac{1}{2i\theta} = -i \Longrightarrow \frac{1}{2\theta} = 1$ so, we get real part of (θ),i.e θ =1/2

hence, if { $\zeta(\theta)$ =0 },then { $\theta = \left(\frac{1}{2}\right) \pm iy$ } proved

Acknowledgements

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References:-

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