FFT based Interval Arithmetic Analysis for Signal Processing System

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Abstract : Signal processing is become a dominant role in the area of engineering. Very large scale integration made a rapid advance in the research area of Digital Signal Processing in terms of high speed VLSI architecture for real time applications. DFT is one of the important operations in digital signal processing. DFT converts a signal in discrete time domain to discrete frequency domain. Fast Fourier Transform (FFT) is a form of DFT. It is used in reducing the complexity of computations. We propose interval arithmetic CORDIC architectures to efficiently support and identify the accuracy to accomplish interval trigonometric functions, to guarantee the correct possible option of the bounds of the final value with minimum error, to analyze and to measure signals for the use of DSP (Digital signal processing) applications. This paper presents interval arithmetic Cordic algorithm based FFT for improvement in performance and signal processing implementation using MATLAB toolbox INTLAB. Interval arithmetic Cordic algorithms and its FFT based approach is a suitable method for improving the performance, accuracy and reducing latency.

***IndexTerms* - DSP, Interval arithmetic, Cordic, FFT, DFT, MATLAB and INTLAB.**

I. INTRODUCTION

Signal processing is a technique of processing a signal to separate or to attain the more useful information. Digital signal processing works on frequency domain and is more advantageous when compared to time domain and this is surpassed by Discrete Fourier transform (DFT) [1]. DFT converts a signal in discrete time domain to discrete frequency domain. DFT is one of the important operations in digital signal processing. It finds application in filtering, spectral analysis, Video processing etc. DFT cannot be implemented directly because of its complexity in computation. Fast Fourier Transform (FFT) is used in reducing the complexity of computations. In FFT, butterfly operation is most computationally challenging phase. Butterfly unit consists of adders and multipliers. The CORDIC algorithm is a replacement procedure to recognize the butterfly operation without using multiplier hardware [2]. Rather than storing the twiddle factors in a ROM the Cordic based FFT design stores the twiddle factors angles in a ROM for butterfly operation. Hence this algorithm is a flexible and hardware effective making it more acceptable for butterfly operations in FFT [3]. The well grounded replacement to the typical floating point arithmetic is the interval arithmetic. The perspective to bounce rounding error in mathematical method of computing is interval arithmetic[11][12]. It is a very robust approach with plenty of applications in mathematics, computer science and engineering [4][5]. It corresponds to two values which are represented by lower and higher bound of values of the interval to guarantee, such that the accurate value lies on this interval [6]. The accuracy of the result will be given by the width of the interval. Inaccurate results cannot be solved in a well organised and competent way by means of floating point arithmetic computations [7] but these could be solved in a well organised method called interval arithmetic that produces a lower and upper bound of value for very calculation [8]. The method also allows the interval hardware to take supremacy of floating-point hardware and VLSI technology[10]. Interval arithmetic cordic algorithm and its FFT based approach is a suitable method for improving the performance, accuracy and reducing latency[17].

II. FFT BASED INTERVAL ARITHMETIC CORDIC

The lower bound and higher bound of values of the interval X are denoted in the way as xl and xu. An enclosed interval X[xl, xu] comprises of real values between and including the two bound of values xl and xu (i.e., X={x: xl<x< xu}. And outward rounding is used for the mathematical calculations of the interval endpoints[12]. The lower bound is rounded to nearest negative infinity (∇) for outward rounding, and positive infinity (Δ) for rounding upper bound. Then the resulting interval consists the accurate result from outward rounding.

2.1. Interval Arithmetic Cordic Algorithm

A rectangle is pointed by a set of vectors which corresponds to initial interval vector. These vectors corresponds to a new rectangle, if a rotation is performed which as well rotated with respect to its axis. As the real rectangle cannot be represented, so the current set of vectors is enclosed using a larger rectangle. In order to rotate, the vectors that points to the area of the rectangle have to be rotated, because the rotation action is a linear operation[13]. On final solution, to guarantee correct bounds of the errors associated in the mathematical calculations has to be considered [14]. The equations to carry out the design are derived from the rotation of transformation and modified as shown in the following equations.

$$x_{i+1} = x_i - \sigma i.2^{-i}.y_i$$
$$y_{i+1} = y_i + \sigma i.2^{-i}.x_i$$
$$z_{i+1} = z_i - \sigma i.\tan^{-1}2^{-i}$$

(1)

The N-point discrete Fourier transform is given by the expression

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad k = 0, 1, \ldots, N-1, W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$$

(2)

Where $W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$ is the twiddle factor. There are $\log_2 N$ stages and every stage consists of N/2 butterfly operations. The radix -2 butterfly operation equations are

$$x_{m+1}(p) = x_m(p) + x_m(q)$$

$$x_{m+1}(q) = [x_m(p) - x_m(q)] W_N^r$$

(3)

From the above expression it is clear that it requires additions, subtractions and multiplications. A unique plan is adopted that is FFT based Cordic algorithm implementation. For the sequence of real samples, DFT is defined as

$$X(k) = [\cos 2\pi Nkn - j\sin 2\pi Nkn] \qquad (4)$$

Where cos $2\pi Nkn$ is the real part and sin $2\pi Nkn$ is the imaginary part. The above equation is rewritten as

$$X(k) = Xr(k) + Xi(k) \qquad (5)$$

The output of the FFT based CORDIC gives cosine value from the real part and the sine value from the imaginary part. Fast Fourier Transform (FFT) is used in reducing the complexity of computations[15]. In FFT, butterfly operation is most computationally challenging phase. Butterfly unit consists of adders and multipliers. Pipelined FFT is used in the multiplier stage but speed is the bottleneck in its operation [16]. The CORDIC algorithm is a replacement procedure [4] to recognize the butterfly operation without using multiplier hardware. Rather than storing the twiddle factors in a ROM the FFT based Cordic design stores the twiddle factors angles in a ROM for butterfly operation [17]. Hence this algorithm is a flexible and hardware effective making it more acceptable for butterfly operations in FFT .

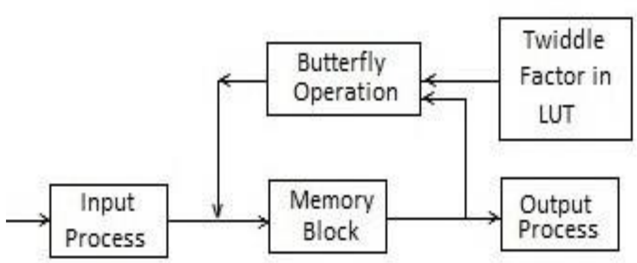

Fig.1. FFT based Cordic

Power spectrum is used for the analysis in frequency domain. Power of every frequency component of the given signal is plotted against their respective frequencies[18]. For analyzing signals converting from a two-sided power spectrum to a single-sided power spectrum is done using the FFT computation[19]. The power spectrum gives an array of values which consists of the two-sided power spectrum of a time-domain signal. The array values are proportional to the amplitude squared of each frequency component making up the time-domain signal.

III.SIMULATION RESULTS

3.1 Performance analysis

The Interval arithmetic Cordic architecture is implemented in the working platform of MATLAB (versionR2017a) Simulink toolbox INTLAB. INTLAB is a MATLAB Toolbox for validating the algorithms[21]. To measure the precision, relative error (RE) is used which is given by the ratio of the absolute error of a measured unit to the measurement being taken. It has no units and is expressed as a percentage. The analysis is found using the expression.

Relative Error=Actual value- Simulation/ Simulation*100 %

Table 3.1: Relative error for interval arithmetic cordic

Function	Degrees	Relative error
Sin	45	0.000168
Cos	45	0.000167

From Table 3.1it can be seen that the relative error is very small for sin and cosine functions for the Interval Arithmetic rotation Cordic with its pipeline architecture. The relative error achieved is very small, this indicates that the smallest relative error gives the better performance. And can be concluded that the Interval Arithmetic rotation Cordic and its FFT implementation has better performance.

3.2 Signal Analysis

FFT based CORDIC Interval Arithmetic provides precise incomes and equations to investigate the signals. The attributes of the acquired signal to be analysed are calculated using power spectral density (PSD) estimation. The number of points N in the computation is taken as power of 2 for the efficient computation with FFT. A value of N=1024 is chosen. FFT is a numeric computation of N-point DFT, there are various representations of FFT[20]. The x-axis runs from 0 to N, representing N sample values. Fig.2 represents the generated sine wave. The magnitude of the DFT abs(X) is plotted on the y-axis as shown in Fig.3 The frequency axis (x-axis) is normalized to unity by dividing the sample index on the x-axis by the length N of the FFT. This normalizes the x-axis with respect to the sampling rate as represented by Fig.4. In the frequency domain, the values take up both positive and negative frequency axis, this is shown in Fig.5. In Fig.6 plotting the DFT values on a frequency axis with both positive and negative values, the DFT value at sample index 0 is centered at the middle of the array. The x-axis runs from the points where the end points are the normalized folding frequencies with respect to the sampling rate. The normalized frequency axis is then multiplied by the sampling rate. Power Spectrum plots the power of each frequency component, absolute frequency on the x-axis Vs Power on Y-axis which is given in Fig.7.

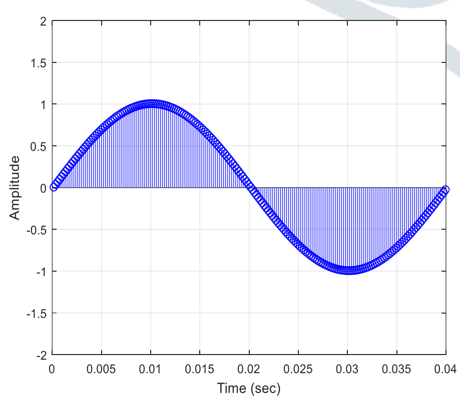

Fig .2. Representation of Sine wave

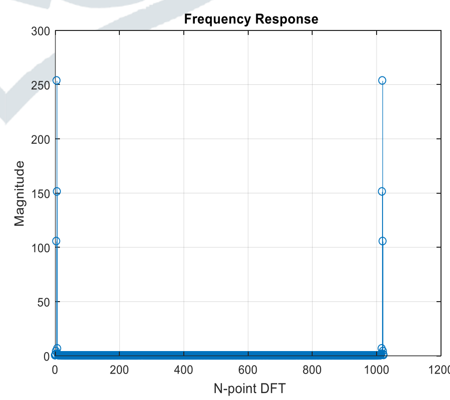

Fig.3. Magnitude values of FFT

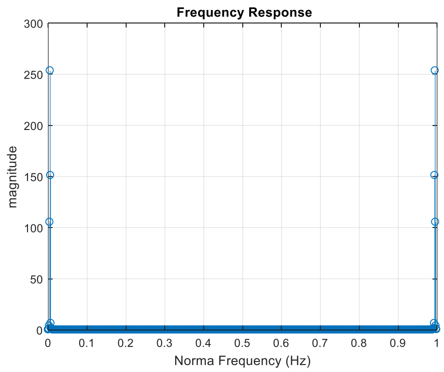

Fig.4. Magnitude values against normalised frequency

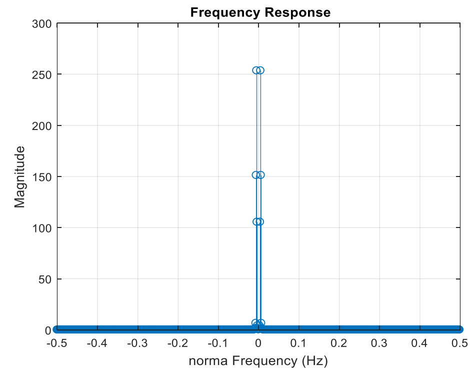

Fig.5. Magnitude values with normalised frequency (+ve and –ve)

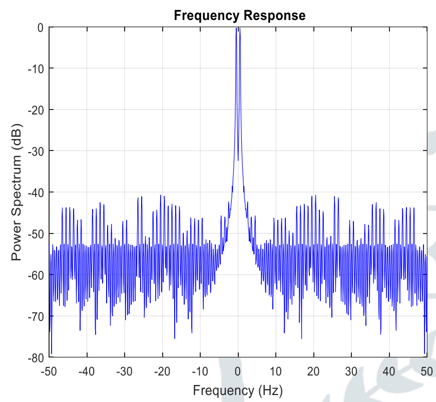

Fig.6. Power spectral density

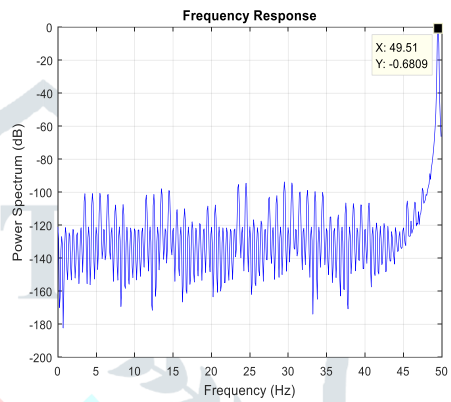

Fig.7. One sided Power spectral density

IV CONCLUSION

In this paper, we profound FTT based interval arithmetic Cordic algorithm with reduced time and error that can be implemented in signal processing applications. An approach based on FFT interval arithmetic Cordic algorithm has been presented to enhance the performance and to increase the accuracy of the result with less hardware and with the reduction in the number of iterations. The designs reveals that better performance can be achieved with the smaller number of iterations and thus avoiding longer computation time. The algorithm is a flexible and hardware effective making it more acceptable for butterfly operations in FFT.

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