Convex Optimization based Sliding Mode Control Algorithm of Perturbed MAGLEV using Attractive Ellipsoid Scheme

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Abstract : This paper proposes LMI based robust and optimal sliding mode control (SMC) approach for stabilization of a perturbed Magnetic Levitation System (MAGLEV). This system levitates an object with a strong magnetic force in space against gravity and thus, requires a robust controller strategy which rapidly regulates the position of an object to equilibrium in-spite of various mismatched uncertainties such as model parametric variations and external disturbances. Therefore, this paper employs an attractive ellipsoid control strategy which is based on optimizing the size of an ellipsoid containing equilibrium while ensuring the convergence of actual position of an object in quasi-minimal zone of equilibrium in finite time. It also guarantees an effective reduction of chattering and faster convergence of system dynamics over the prescribed sliding surface. The optimal sliding surface and sliding motion are computed by solving LMIs through YALMIP and SEDUMI toolboxes in MATLAB software. Simulations and results are also compared with the dynamic sliding mode control (DSMC) contributed in literature.

***IndexTerms* - Convex Optimization, Sliding Mode Control (SMC), Linear Matrix Inequalities (LMI), Attractive Ellipsoid Methodology (AEM), Magnetic Levitation System (MAGLEV).**

I. INTRODUCTION

Magnetic Levitation system (MAGLEV) is a highly unstable system that allows a ferromagnetic object to suspend in space with a strong magnetic force, against the pull of gravitational force. It has a practical importance in various engineering applications such as rail transportation, vibration isolation, suspension of metal in induction furnaces, frictionless bearings etc. In addition to this, it also moves object from distances without any need of contact with surface, eliminates frictionless energy losses and dampens vibrations. Analytically, this system is characterized by the nonlinear differential equations and requires a suitable robust control action for regulation of position of suspended object with accurate precision [1].

Many research proposals have been bestowed in published works for stabilizing the MAGLEV system namely, adaptive [1,2], sliding mode control (SMC) [3], linearizing control method [4], robust feedback linearization [5,6], two degree of freedom PID control [7], backstepping approach [8], Linear Quadratic Regulator (LQR) [9] and higher order SMC [10]. In [11], the linear and non-linear state space controllers are employed for high precision positioning and non-linear observer is incorporated for its system states estimation. However, these linear controllers may not guarantee stability in the presence of large perturbations. Moreover, most of these methods are not concerned with assurance of robustness of closed loop system against time varying complex mismatched uncertainties which is the principal consideration of this paper.

SMC is considered as the best robust strategy for eliminating the impacts of complex modelling uncertainties such as un-modelled dynamics, model parametric variations, etc. due to its variable control structure. It has proved its effectiveness in various kinds of systems such as non-linear, linear, chaotic systems etc. It comprises of two parts [12]: the first part describes the continuous control for the sliding phase on the prescribed sliding surface. Second part reveals the discontinuous control needed to force the system dynamics present in state space onto the pre-defined sliding surface. Therefore the most important step in designing a control law is to choose a suitable sliding surface. It signifies the desired dynamics which can be tracking or regulation objectives in the sense of control theory [13,14]. Nevertheless, few issues faced by this robust technique have intensely attracted the large section of control research community. Firstly, the conventional SMC is not efficient in alleviating the impacts of mismatched uncertainties which do not satisfy the matching condition. This means that it can successfully deal with the matched uncertainties which penetrate through the input matrix of system dynamics. Moreover, the sliding phase is not at all robust to such mismatched uncertainties. Secondly, high frequency oscillations known as chattering in SMC caused due to high gain and finite switching frequency develop heating losses and excessive wear and tear of actuators and sensors [15]. Approaches such as higher order SMCs and Quasi-SMC have been utilized in literature which either enhance the complexity of the controller or reduce the robustness of the closed loop system, respectively.

The attractive ellipsoid methodology (AEM) is one of the powerful methods proposed for stabilizing the system against mismatched and matched uncertainties [16,17]. It minimizes the size of an ellipsoid that contains the equilibrium (origin). The system states find it attractive as they will be enforced to converge to a quasi-minimal region of equilibrium [18,19].

Influenced by the above deliberations, the aim of this paper is to utilize AEM based SMC for stabilization of an actual position of MAGLEV system and also to find an optimal sliding surface and chatter-free control law for finite time system states' convergence with high precision in the presence of mismatched uncertainties. To our best knowledge, such a control strategy has not been investigated for the MAGLEV system in literature. This convex optimization problem is solved by Linear Matrix Inequalities (LMIs) and Bilinear Matrix Inequalities (BMIs) through YALMIP [20,21] and SEDUMI toolboxes in MATLAB software. Also, the simulation results have been compared with the dynamic SMC (DSMC) presented in [3].

This paper is framed as: In Section 2, the dynamics of MAGLEV system is shown. The procedure for SMC based on LMI is described in Section 3. The results of implementation with MATLAB software are shown in Section 4. The conclusion is stated in Section 5.

II. MATHEMATICAL MODELING OF MAGLEV SYSTEM

Consider a block diagram of MAGLEV system presented in Fig. 1. It consists of an electromagnet wound with copper wires which produces a magnetic force F_e for holding an object by energizing the windings through coil actuator supervised by the proposed control $u(t)$. The sensor is also incorporated to sense the current position of an object and also acts as an input to controller.

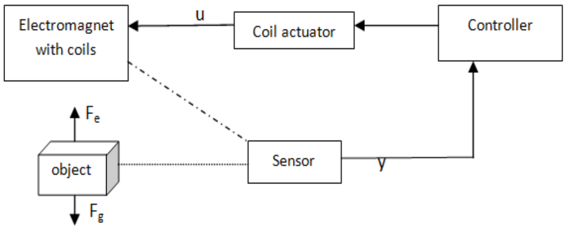


Fig. 1. Schematic diagram for MAGLEV

The non-linear differential equations describing MAGLEV system can be expressed as [22]:

$$\begin{cases} v(t) = V_r + V_L = iR + \dfrac{dL(x)i}{dt} \\ F_a = F_g - F_e \\ M\left(\dfrac{d^2x(t)}{dt^2}\right) = Mg - \dfrac{Qi}{x^2} \end{cases} \tag{2.1}$$

Here, $v(t)$ denotes the applied voltage (input), V_r is the voltage across coil's resistance R, V_L is the voltage across coil's inductance L. Whereas, i is current through the coil of electromagnet. The electromagnetic force F_e developed by varying current through electromagnet needs to be counterbalanced by gravitational force F_g. M denotes the mass of an object, x defines the position of the object, g is the gravitational constant and Q is the magnetic force constant.

Then, the non-linear equation for the coil's inductance is represented by:

$$L(x) = L_0 + \frac{2Q}{x} \tag{2.2}$$

where L_0 is a system variable denoting an electromagnet coil inductance. Assume the system state vector as: $X = [x_1 \quad x_2 \quad x_3]^T = [x \quad \dot{x} \quad i]^T$, the state space model of MAGLEV can be written as :

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \frac{Q}{M}\left(\frac{x_3}{x_1}\right)^2 \\ \dot{x}_3 &= -\frac{R}{L}x_3 + \frac{2Q}{L}\left(\frac{x_3x_2}{x_1^2}\right) + \frac{1}{L}u \end{aligned} \tag{2.3}$$

The aforementioned non-linear state space model is affine in control input *u*(*t*) which is of the form :

$$\dot{X} = f(X) + \Delta f(X) + g(X)u(t) + \Delta g(X)u(t) + d(t) \tag{2.4}$$

where $f(X), g(X) \neq 0$ are non-linear system functions and $\Delta f(X), \Delta g(X)$ and $d(t)$ represent system perturbations and external disturbances, respectively. Now, this non-linear system is linearized about equilibrium: $X_e = [0.012 \quad 0 \quad 0.75]^T$. The state space representation of the third order uncertain linear system is given as:

$$\begin{aligned} \delta\dot{X}(t) &= A\delta X(t) + Bu(t) + Eh(t, X) \\ y(t) &= C\delta X(t) \end{aligned} \tag{2.5}$$

Here, state error, $\delta X = X - X_e$, A is 3×3 system matrix, E is 3×2 gains of disturbance signals, B is 3×1 input matrix, u is 1×1 sliding mode control signal, δX is 3×1 state vector and $h(t, X) = \Delta A(X) + \Delta B(X)u(t) + d(t)$ denotes 2×1 bounded and lumped mismatched Quasi-Lipschtiz uncertainties/ disturbances, whose bound is given by:

$$h^T R_h h \leq h_0 + \delta X^T R_x \delta X \tag{2.6}$$

where R_x and R_h are positive definite matrices and h_0 is a positive constant.

$$h^T \begin{bmatrix} 13 & 15 \\ 15 & 40 \end{bmatrix} h \leq 1 + \delta X^T \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix} \delta X \qquad (2.7)$$

Since, the task of non-linear system is to track the commanded signal and the linear system state vector will be composed of error states and should be regulated to zero. The values of the various parameters of the system are shown in Table 1.

Table 2.1: Values of MAGLEV system parameters

Parameters	**Value**	**Units**
M	0.04	Kg
g	9.8	m/s^2
L	0.0092	H
R	1.2	Ω
Q	0.0001	-
Initial conditions: δX_0	$[0.03 \quad -0.01 \quad 0.597]$	[m,m/s,A]

III. Controller Design

3.1 Controller Design

The main intention of this paper is to provide a robust control law which will drive all the error states including deviation in MAGLEV position, velocity and current from equilibrium to zero in the presence of external disturbances and mismatched uncertainties in finite time t_r .

$$\lim_{t \to t_r} \delta X \to 0 \qquad (3.1)$$

3.2 Preliminary Step for Application of the Proposed Method

The solution to the above problem can be initiated by decomposing B into $B_1 \in R^{n-m \times m}$, $B_2 \in R^{m \times m}$ and then, change the system dynamics (5) with a non-singular transformation matrix $T: \varsigma = T\delta X$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad T = \begin{bmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix} \qquad (3.2)$$

where $n = 3, m = 1, |B_2| \neq 0$ and this matrix will reduce the system dynamics to regular form:

$$\begin{aligned} \dot{\varsigma}_1 &= A_{11}\varsigma_1 + A_{12}\varsigma_2 + E_1 h(t,x) \\ \dot{\varsigma}_2 &= A_{21}\varsigma_1 + A_{22}\varsigma_2 + u(t) + E_2 h(t,x) \end{aligned} \qquad (3.3)$$

where

$$\varsigma = \begin{bmatrix} \varsigma_1 \\ \varsigma_2 \end{bmatrix}, \quad TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad TE = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

are the modified system matrices. The sliding surface describing the regulation objective is selected as:

$$s(t) = C_1 \varsigma = C\varsigma_1 + \varsigma_2 = 0 \qquad (3.4)$$

where $C_1 = [C \quad 1]$ should be Hurwitz, which will be found from convex optimization problem presented in next section. The general sliding mode control law is a combination of continuous and discontinuous controls. Therefore in (12), the first term denotes the equivalent control law for sliding phase computed by taking the first order derivative of sliding surface (11) and second term gives the discontinuous control meant for the reaching phase. The discontinuous control gain $K(\varsigma)$ is obtained through the same optimization process which also avoids the overestimation to counteract uncertainties and thus chattering would be negligible.

$$u(t) = -(C_1 B)^{-1} C_1 A \varsigma(t) - K(\varsigma) sign(s(t)) \qquad (3.5)$$

where $|C_1 B| \neq 0$ to avoid control singularity problem and $K(\varsigma) = \sqrt{a + \varsigma^T R \varsigma}$ is a constant gain matrix with positive bounded constant a , and matrix $R \in R^{3\times3}$ such that:

$$0 \leq a_{\min} \leq a \leq a_{\max}, 0 \leq \|R\| \leq b \qquad (3.6)$$

3.3 Ellipsoid Method

Definition: The ellipsoid given by:

$$\varepsilon(P) = \delta X^{T} P \delta X < 1, \;\; P > 0, \; \delta X \in R^{3} \tag{3.7}$$

is said to be invariant if any state trajectory starting in ellipsoid stays in it for later time t > 0. Also, it is called as attractive if a state trajectory starting outside this ellipsoid converges to this ellipsoid. The equation (14) describes the smaller size of ellipsoid with larger P.

Therefore, the optimization problem becomes: $\max trace(P) = \min trace(P^{-1})$ minimizes the sum of squares of semi-axis of ellipsoid.

3.4 SMC Design based on AEM

Designing a SMC based on attractive ellipsoid method (AEM) requires three steps. Firstly, the parameters of sliding surface are selected for desired control objective (8). Secondly, the control law is devised to drive the closed loop system dynamics to vicinity of origin inside an attracting ellipsoid. Thirdly, the system states are made to reach the smallest invariant ellipsoid. To achieve this control objective, a convex optimization problem in two- parametric form of LMIs (and BMIs) has been solved by using Theorem 1. Readers can refer to [16,17] for Lyapunov's stability analysis. Let $Y = CP$ and the control problem becomes: minimize trace (Z) such that the inequalities in theorem 1 are satisfied.

Theorem 1 (Theorem 4 [16]) *:* LMIs obtained by S- procedure for fixed τ_1, τ_2, δ, give the solutions (a, M, Z, Y, P, R):

$$\begin{bmatrix} b^2 I_3 & R \\ R & I_3 \end{bmatrix} > 0 \quad , \quad R - \frac{a}{h_0} R_x \geq 0 \tag{3.8}$$

$$\begin{bmatrix} P & \begin{bmatrix} P & -Y^T \end{bmatrix} \\ \begin{bmatrix} P \\ -Y \end{bmatrix} & TZT^T \end{bmatrix} \geq 0 \quad , \quad \begin{bmatrix} \frac{a}{h_0} R_h & E^T T^T \\ TE & \begin{bmatrix} \lambda P & 0 \\ 0 & L \end{bmatrix} \end{bmatrix} > 0$$

$$\begin{bmatrix} \frac{P}{\lambda} & Y^T \\ Y & 1 - M \end{bmatrix} \geq 0 \quad , \quad \tau \geq 0, P \succ 0, M \succ 0, \lambda \succ 0 \tag{3.9}$$

$$\begin{bmatrix} \begin{matrix} PA_{11}{}^T + A_{11}P - Y^T A_{12}{}^T \\ - A_{12}Y + \tau P + \tau_2 E_1 R_f{}^{-1} E_1{}^T \end{matrix} & \begin{bmatrix} P & -Y^T \end{bmatrix} \\ \begin{bmatrix} P \\ -Y \end{bmatrix} & -\tau_2 R_x{}^{-1} \end{bmatrix} \leq 0 \tag{3.10}$$

Here, a, τ, λ are positive numbers, R is positive semi-definite matrix, bounded control parameters, P denotes the 2×2 positive definite matrix and Y is 3×3 positive definite matrix.

This gives the quasi-minimal attractive (invariant) ellipsoid $\varepsilon(Z^{-1})$ for the system. The sliding surface coefficient vector is given by:

$$C_1 = \begin{bmatrix} YP^{-1} & 1 \end{bmatrix} T \tag{3.11}$$

Proof: The proof of the afore-described theorem has been comprehended in [16,17].

The optimal sliding motion is given by:

$$\begin{aligned} \dot{\varsigma}_1(t) &= (A_{11} - A_{12}C)\varsigma_1(t) + E_1 h(t,x) \\ \varsigma_2(t) &= -C\zeta_1(t) \end{aligned} \tag{3.12}$$

The complete optimization algorithm is described as:

Algorithm: Optimization Process

1: Transform the model of magnetic levitation system to regular form using (9).

2: Find the optimal solutions using YALMIP and SEDUMI toolboxes with objective function chosen as trace(Z) and constraints described in theorem 1.

3: Compute the control law by substituting the obtained optimal solutions using (12).

4: Determine the optimal sliding motion using (19).

IV. SIMULATIONS AND RESULTS

In this section, MATLAB simulation work is presented with the YALMIP and SEDUMI toolboxes. The differential equations (19) are solved with ode23 solver with the tolerance of 1e-2, for obtaining respective time domain responses. The efficiency of the proposed method is compared with the dynamic sliding mode control (DSMC) presented in [3] in the presence of mismatched uncertainties (20).

$$h(.) = \begin{bmatrix} 0.0028\cos(0.4t) - 0.00879\sin(0.4t) \\ 0.0499\cos(0.4t) + 0.0499\sin(0.4t) \end{bmatrix} \quad (4.1)$$

with gain vector chosen as:

$$E = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \quad (4.2)$$

Various fixed controller parameters are given in Table 2.

Table 4.1: Values of Proposed Controller parameters

Parameters	Values
τ_1	1.52
τ_2	1
λ	0.1
b	0.005
$a_{\min}$	0
$a_{\max}$	40

The problem is solved with solution $trace(Z) = 0.7153$. The coefficient vector of a sliding surface is obtained as:

$$C_1 = \begin{bmatrix} -18.0761 & -0.0507 & 0.0092 \end{bmatrix} \quad (4.3)$$

Fig. 2 shows the 3D convergence of error state trajectories to the invariant ellipsoid. Fig. 3 (a) and (b) show the comparison between the two techniques for MAGLEV object's position. It can be analyzed that the position error is regulated to zero in 0.05s with the proposed method. Whereas, DSMC takes 3s for zero position error with peak of 1.7 m due to its inability to handle the mismatched uncertainties. Fig. 4(a) and (b) give the time domain responses for object's velocity. The proposed method has rapidly stabilized the object's velocity in 0.05s.

Fig. 5 (a) and (b) show that the proposed technique has also outperformed DSMC in terms of the current tracking error profile. The peak value of current with the proposed method (0.1A) is much lower than that of DSMC (6800A). It reduces heating losses as well as actuator size. Therefore, the complexity as well as cost of the system will be reduced.

Fig. 6 (a) and (b) give the convergence of the sliding variable versus time. The proposed method allows the convergence of sliding variable to zero in finite time (0.05s) without any chattering and low peak overshoots as compared to DSMC (3s). Also, Fig. 7 (a) and (b) show the lesser overshoots and chattering free control effort with the presented method than DSMC. Hence, it has been verified that the proposed method gives high time domain performances as compared to the dynamic SMC (DSMC).

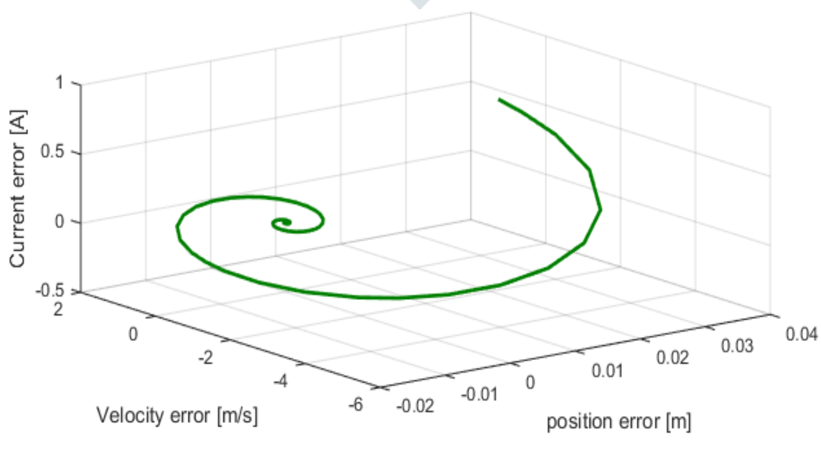
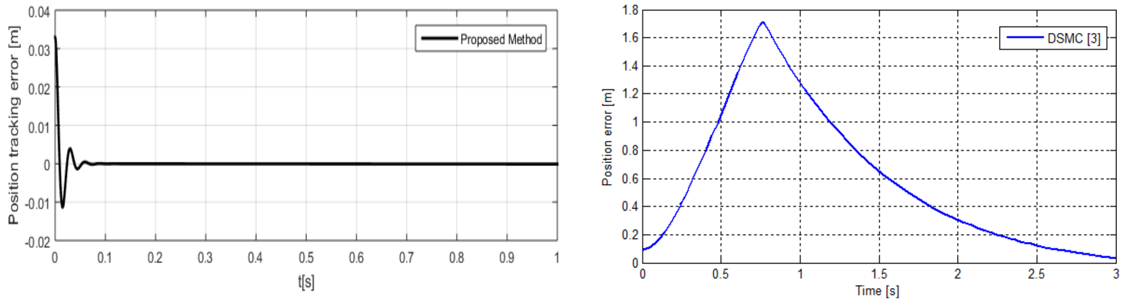
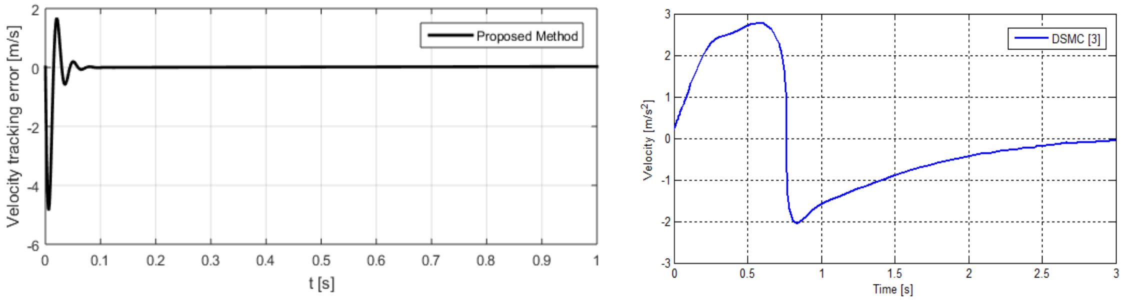


Fig. 2. 3D plot for convergence of state trajectories with the proposed method



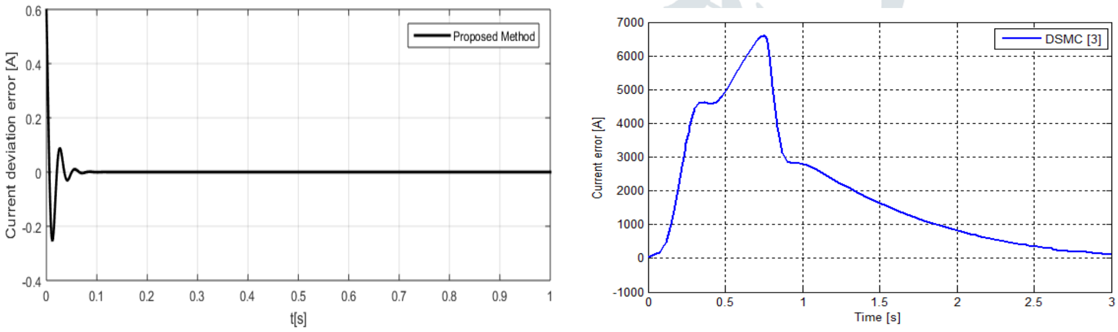
(a) (b)

Fig. 3. Position tracking error with (a) proposed method and (b) DSMC [3] with mismatched uncertainties



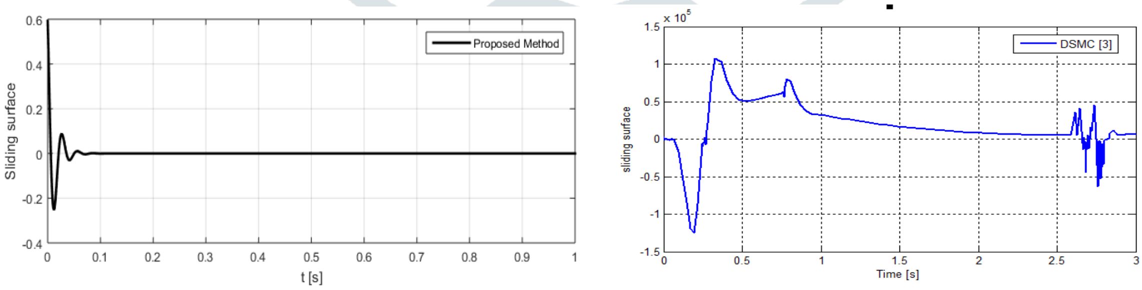
(a) (b)

Fig. 4. Velocity tracking error with (a) proposed method and (b) DSMC [3] with mismatched uncertainties



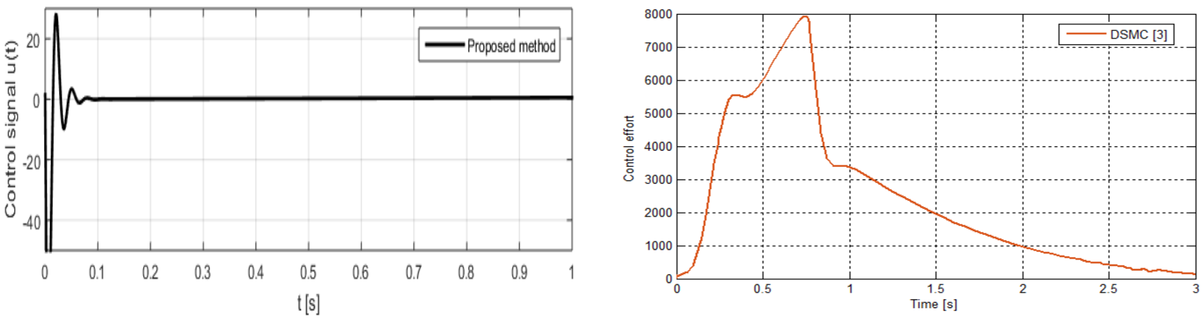
(a) (b)

Fig. 5. Current tracking error with (a) proposed method and (b) DSMC [3] with mismatched uncertainties



(a) (b)

Fig. 6. Sliding surface convergence with (a) proposed method and (b) DSMC [3] with mismatched uncertainties



(a) (b)

Fig. 7. Control effort with (a) proposed method and (b) DSMC [3] with mismatched uncertainties

V. CONCLUSIONS

An attractive ellipsoid based sliding mode control has been implemented for the magnetic levitation system. MATLAB simulations are carried out with YALMIP and SEDUMI toolboxes for solving convex optimization based LMIs. The time domain responses with the proposed method are compared with the dynamic sliding mode control. It has also been verified that the high performance specifications such as faster convergence, zero steady state error and chatter-free control in the presence of mismatched uncertainties are achieved with the proposed method. In future, the proposed methodology can also be applied to various other kinds of electrical and mechanical systems for their stabilization and tracking control objectives.

REFERENCES

[1] Boonsatit, N. and Pukdeboon, C., 2016. Adaptive fast terminal sliding mode control of magnetic levitation system. Journal of Control, Automation and Electrical Systems, 27(4): 359-367.

[2] Lin, C.M., Lin, M.H. and Chen, C.W., 2011. SoPC-based adaptive PID control system design for magnetic levitation system. IEEE Systems journal, 5(2): 278-287.

[3] Al-Muthairi, N.F. and Zribi, M., 2004. Sliding mode control of a magnetic levitation system. Mathematical problems in engineering, 2004(2): 93-107.

[4] Trumper, D.L., Olson, S.M. and Subrahmanyan, P.K., 1997. Linearizing control of magnetic suspension systems. IEEE Transactions on control systems technology, 5(4): 427-438.

[5] Pradhan, S.K. and Singh, R., 2014, May. Nonlinear control of a Magnetic Levitation System using feedback linearization. In 2014 IEEE International Conference on Advanced Communications, Control and Computing Technologies (pp. 152-156). IEEE.

[6] El Hajjaji, A. and Ouladsine, M., 2001. Modeling and nonlinear control of magnetic levitation systems. IEEE Transactions on industrial Electronics, 48(4) : 831-838.

[7] Singh, B.K. and Kumar, A., 2019. Stabilization and Control of Magnetic Levitation System Using 2-Degree-of-Freedom PID Controller. In Recent Trends in Communication, Computing, and Electronics (pp. 569-579). Springer, Singapore.

[8] Hasirci, U., Balikci, A., Zabar, Z. and Birenbaum, L., 2010. A novel magnetic-levitation system: design, implementation, and nonlinear control. IEEE Transactions on Plasma Science, 39(1) :492-497.

[9] Kumar, E.V. and Jerome, J., 2013. LQR based optimal tuning of PID controller for trajectory tracking of magnetic levitation system. Procedia Engineering, 64: 254-264.

[10] Goel, A. and Swarup, A., 2016, October. A novel high-order sliding mode control of magnetic levitation system. In 2016 IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS) (pp. 1-4). IEEE.

[11] Barie, W., Chiasson, J., 1996: Linear and nonlinear state-space controllers for magnetic levitation. International Journal of systems science 27(11):1153-63.

[12] Young, K.D., Utkin, V.I. and Ozguner, U., 1999. A control engineer's guide to sliding mode control. IEEE transactions on control systems technology, 7(3): 328-342.

[13] Utkin, V.I., 1992. Sliding modes in optimization and control problems.

[14] Gao, W. and Hung, J.C., 1993. Variable structure control of nonlinear systems: A new approach. IEEE transactions on Industrial Electronics, 40(1): 45-55.

[15] Utkin, V., Guldner, J. and Shi, J., 2017. Sliding mode control in electro-mechanical systems. CRC press.

[16] Polyakov, A. and Poznyak, A., 2011. Invariant ellipsoid method for minimization of unmatched disturbances effects in sliding mode control. Automatica, 47(7): 1450-1454.

[17] Poznyak, A., Polyakov, A. and Azhmyakov, V., 2014. Attractive ellipsoids in robust control. Switzerland: Springer International Publishing.

[18] Gonzalez-Garcia, S., Polyakov, A. and Poznyak, A., 2008, November. Output linear feedback for a class of nonlinear systems based on the invariant ellipsoid method. In 2008 5th international conference on electrical engineering, computing science and automatic control (pp. 7-12). IEEE.

[19] Deepika, Narayan, S. and Kaur, S., 2019. A Simulation Study: LMI Based Sliding Mode Control with Attractive Ellipsoids for Sensorless Induction Motor. In Progress in Advanced Computing and Intelligent Engineering (pp. 493-503). Springer, Singapore.Poznyak, A.: Advanced Mathematical Tools for Control Engineers: Volume 1: Deterministic Systems. Elsevier (2010).

[20] Lofberg, J., 2004, September. YALMIP: A toolbox for modeling and optimization in MATLAB. In 2004 IEEE international conference on robotics and automation (IEEE Cat. No. 04CH37508) (pp. 284-289). IEEE.

[21] Kumar, T., Shimi, S.L., Karanjkar, D. and Rana, S., 2014, March. Modeling, simulation and control of single actuator magnetic levitation system. In 2014 Recent Advances in Engineering and Computational Sciences (RAECS) (pp. 1-6). IEEE.