ON COMMUTATIVITY FOR NON ASSOCIATIVE PRIMITIVE RINGS WITH $x(x^2 + y^2) - (x^2 + y^2)x \in z(R)$

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ABSTRACT*:*Johnson, Outcalt and Yaqub [3] proved that if a non- associative ring R satisfy the identity $x^2 y^2 = y^2 x^2$ for all x,y in R, then R is commutative. The generalization of this result proved by R. D. Giri and others [1] stoctes that if R is a non-associative primitive ring satisfies the identities $x^2 y^2 - y^2 x^2 \in z(R)$, where z(R) denotes the center, then R is commutative. A modification of Johnson's identity viz, $x^2 y^2 - y^2 x^2$ for all x,y in R, for a non - associative ring R which has no element of additive order 2, is commutative was proved by R. N. Gupta [2], R. D. Giri and others [1] generalized Gupta's result by taking $x(xy)^2 -(xy)^2 x \in z(R)$. We have to proved that if R is a non-associative ring of char≠4 satisfies then R is commutative $x(x^2 + y)^2 - (x^2 + y)^2 x \in z(R)$

KEYWORDS: Center, Commutativity, Primitive Ring.

I.INTRODUCTION:

The study of associative and non- associative rings has evoked great interest and importance. The results on associative and non-associative rings in which one does assume some identities in the center have been scattered throughout the literature. Many sufficient conditions are well known under which a given ring becomes commutative.Notable among them are some given by Jacobson, Kaplansky and Herstein. Many Mathematicians of recent years studied commutativity of certain rings with keen interest. Among the mathematicians Herstein, Bell, Johnsen, Outcalt, Yaqub, Quadri and Abu-khuram are the ones whose contributions to this field are outstanding.

II.PRELIMINARIES:

1.Non - Associative Ring:

If R is an abelian group with respect to addition and with respect to multiplication R is distributive over addition on the left as well as on the right.

For every elements x, y,z of R

$(x+y)z = xa+yz, z(x+y) = zx+zy$

Alternative rings, Lie rings and Torsian rings are best examples of these non-associative rings.

2.Commutator:

For every x, y in a ring R satisfying $[x,y]=xy-yx$ then $[x,y]$ is called a commutator.

3.Commutative Ring:

For every x, y in a ring R if $xy = yx$ then R is called a commutative ring.

Non commutative ring is split from the commutative ring, i.e., R is not commutative with respect to multiplication. i.e., we cannot take $xy = yx$ for every x,y in R as an axiom.

4.Primitive Ring:

A ring R is defined as primitive in case it possesses a regular maximal right ideal, which contains no two-sided ideal of the ring other than the zero ideal.

5.Torsion-Free ring:

If R is m-torsion free ring, then mx=0 implies x=0 for positive integer m and x is in R.

6.Center :

In a ring R, the center denoted by Z(R) is the set of all elements $x \in R$ Such that xy=yx for all $X \in R$, It is important to note that this definition does not depend on the associative of multiplication and in fact, we shall have occasion to deal with derivation of non-associative algebras.

III. MAIN RESULTS:

THEOREM 1 :

Let R be a non-associative ring of char ≠2 with unity 1 satisfying $x(x^2 + y^2) - (x^2 + y^2)x \in z(R)$ then R is commutative.

PROOF:

Form hypothesis,

$x(x^2 + y^2) - (x^2 + y^2)x \in z(R)$ (1.1)

Replacing y by y+1 in 1.1 and using 1.1 we get,

$2xy - 2yx \in z(R)$

That is,

$2(xy - y x) (R) \in z(R)$

Since R is of char ≠ 2

Hence $xy - yx \in z(R)$

and hence xy = yx

So R is commutative.

This completes the proof of the theorem

THEOREM 2:

Let R be a non-associative ring of char ≠ 4 with unity 1 satisfying $[xy - (xy)^2, x] = 0$ then R is commutative.

PROOF:

From hypothesis

$[xy - (xy)^2, x] = 0$

That is,

$x[xy - (xy)^2] - [xy - (xy)^2]x \in z(R)$ (2.1)

Replacing x by x+1 in 2.1 and using 2.1 we get,

$x[y-(xy)y-y(xy)-y^2] - [y-(xy)y-y(xy)-y^2]x \in z(R)$ (2.2)

Now replacing x by y +1 in 2.2 and using 2.2 we obtain,

$x(-3xy-yx-2y) - (-3xy-yx-2y)x \in z(R)$ (2.3) replacing x by x+1 in 2.3 and using 2.3we get,

$- 4xy + 4yx=0$

That is

$4(yx - xy) \in z(R)$

Since R is of char $\neq 4$

Hence $xy-yx \in z(R)$

and hence $xy = yx$

So R commutative.

This completes the proof of the theorem.

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