

Natural Frequencies of The Hollow Cantilever Beam With Tip Mass.

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Abstract-

In general for calculation of natural frequencies discrete system model is used. In case of cantilever beam with tip mass, transverse stiffness of the beam and tip mass is used to calculate natural frequency of the beam. This method may generate wrong value of 1st natural frequency. In addition to this it is not possible to calculate higher natural frequencies of the beam with 1 DOF model. In this paper natural frequencies of the cantilever beam with tip mass are calculated using Euler-Bernoulli beam theory. Mathematical model to calculate natural frequencies is given. 1st natural frequencies of the beam for various mass ratios are obtained using ANSYS modal analysis module. In addition to this natural frequencies of the beam for various mass ratios considering system as 1 DOF are calculated. Results obtained from all three methods are compared. For lower mass ratios 1 DOF model shows higher values of natural frequencies than continuous system. At higher values of mass ratio natural frequencies obtained from all the three models shows less variation.

Keywords- *Cantilever beam with tip mass, Natural frequencies, Euler-Bernoulli beam, 1 DOF.*

1. Introduction-

In machines, rotating and moving parts cause vibration problems. Vibration problems lead to excessive stresses, undesirable noise, looseness of parts and partial or complete failure of parts [4]. Every part in the machine is subjected to vibration due to primary mover. There have been many cases of systems not meeting performance targets because of resonance, fatigue and excessive vibration of a component. Each part in the machine has tendency to oscillate with large amplitude at certain frequencies, these frequencies are known as natural frequencies [6].

It is therefore necessary to study these natural frequencies and find ways to avoid resonance. In general for calculation of natural frequencies discrete system model is used. In case of cantilever beam with tip mass, transverse stiffness of the beam and tip mass is used to calculate natural frequency of the beam. This method may generate wrong value of 1st natural frequency. In addition to this it is not possible to calculate higher natural frequencies of the beam with 1 DOF model [4].

In order to have correct values of natural frequencies system should be considered continuous one. Euler-Bernoulli beam theory can be used to model the beam as continuous system. After modeling system as continuous one system will have infinite natural frequencies. But as amplitude of vibration will be lower at higher natural frequencies so that they can be neglected. Lower natural frequencies are important in almost all applications.

Erturk et al.[1] have given analytical modal analysis of linear transverse vibrations of an undamped Euler-Bernoulli beam with clamped-free boundary conditions and a tip mass rigidly attached at the free end. Authors started with giving classical equation of Euler-Bernoulli beam for undamped case. Then boundary conditions for free and clamped end are given. While giving boundary conditions authors have considered effect of tip mass. Values of shear force and moment at boundaries included terms involving effect of tip mass. For solving governing equation of beam author used method of separation of variable. By this method authors separated terms of space and time response. For finding the values of eigen values and convective term of response author used differential eigenvalue problem method. By using solution natural frequencies and modal response (space term) can be calculated. **Ghodge et al.[2]** have designed cantilever type of absorber considering beam as single degree of freedom system. Absorber used is mass tunable. Test setup was designed for cantilever and simply supported beam. Authors also calculated various modes of vibration using ANSYS modal analysis solver.

In this study, natural frequencies are calculated using three methods- Analytical model, 1 DOF model and ANSYS modal analysis solver. Results obtained from all these three methods are compared.

2. Mathematical modeling

In this chapter, Mathematical model for modal analysis of cantilever beam with tip mass considering Euler-Bernoulli beam is given for calculation of natural frequencies.

2.1. Modal analysis of cantilever beam with tip mass.

The schematic diagram of cantilever beam with tip mass is given below in figure (2.1),

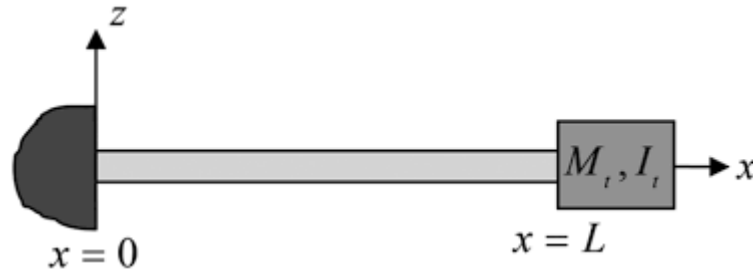


Figure2.1. - Cantilever beam with attached tip mass [1]

Nomenclature for mathematical model

E	= Young's modulus of beam in MPa
I	= Moment of inertia of beam about neutral axis
m	= mass per unit length of beam
A	= cross section area of beam
L	= length of beam
w(x,t)	= transverse deflection of beam at any location 'x' and at any time 't'
Φ(x)	= special component of transverse deflection of beam
η(t)	= temporal component of transverse deflection
γ _r	= Eigen value for rth mode of vibration
I _t	= Ist moment of area about neutral axis
Ω	= frequency of excitation

2.2. Mathematical model for Euler-Bernoulli beam

Let the beam is to be prismatic with cross sectional area of A. For Forced vibrations of the cantilever beam governing equation of cantilever beam with tip mass according to Euler-Bernoulli beam theory is given below [1]. Structural damping of beam is neglected.

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + m \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \quad (1)$$

For harmonic force applied at location of ε

$$f(x,t) = F_0 \delta_{ij}(x - \varepsilon)$$

Where,

δ_{ij} = Kroneker delta

$$\delta_{ij} = 0 \text{ for } i \neq j$$

$$= 1 \text{ for } i = j$$

Boundary conditions of beam with tip mass is given by,

$$w(0, t) = 0$$

$$\left[\frac{\partial w(x, t)}{\partial x} \right]_{x=0} = 0$$

$$\left[EI \frac{\partial^2 w(x, t)}{\partial x^2} + I_t \frac{\partial^3 w(x, t)}{\partial t^2 \partial x} \right]_{x=L} = 0$$

$$\left[EI \frac{\partial^3 w(x, t)}{\partial x^3} - M_t \frac{\partial^2 w(x, t)}{\partial t^2} \right]_{x=L} = 0$$

Where, M_t = tip mass, I_t = M.I. of the tip mass.

For modal analysis of beam

Let's consider free vibrations of cantilever beam with tip mass.

Then force term on right hand side of equation (1) becomes,

$$f(x, t) = 0$$

For finding the solution of above differential equation method of separation of variable is used.

According to this method solution can be represented as

$$w(x, t) = \varphi(x)\eta(t) \tag{2}$$

Given differential equation (equation 1) can be represented by following set of ordinary differential equations.

$$\frac{EI}{m} \frac{1}{\varphi(x)} \frac{d^4 \varphi(x)}{dx^4} = - \frac{1}{\eta(t)} \frac{d^2 \eta(t)}{dt^2} \tag{3}$$

$$\frac{EI}{m} \frac{1}{\varphi(x)} \frac{d^4 \varphi(x)}{dx^4} = \gamma \tag{4}$$

$$- \frac{1}{\eta(t)} \frac{d^2 \eta(t)}{dt^2} = \gamma \tag{5}$$

Where, γ = Eigen value variable of the beam.

$$\frac{d^4 \varphi(x)}{dx^4} - \gamma \frac{m}{EI} \varphi(x) = 0 \quad (6)$$

$$\frac{d^2 \eta(t)}{dt^2} + \gamma \eta(t) = 0 \quad (7)$$

Let's first solve for the space variable equation (6),

Equation (6) is homogenous ordinary differential equation of 4th order, whose solution is given by,

$$\varphi(x) = A \cos\left(\frac{\gamma}{L}x\right) + B \cosh\left(\frac{\gamma}{L}x\right) + C \sin\left(\frac{\gamma}{L}x\right) + D \sinh\left(\frac{\gamma}{L}x\right) \quad (8)$$

For solving equation (8) that is for finding values of A, B, C, D differential eigenvalue problem approach is used. By this method we get following characteristic equation of eigenvalue,

$$1 + \cos(\gamma) \cosh(\gamma) + \gamma \frac{M_t}{mL} (\cos(\gamma) \sinh(\gamma) - \sin(\gamma) \cosh(\gamma)) = 0 \quad (9)$$

(Higher order terms are neglected in equation (9))

Equation (9) has infinite many roots for γ means system has infinite many eigenvalues means infinite values of natural frequencies. For each eigenvalue there is unique associated mode of vibration. Modes of vibration can be represented by eigenfunction $\varphi(x)$.

For rth mode of vibration eigenvalue is represented by γ_r and corresponding eigenfunction is given by $\varphi_r(x)$.

$$\varphi_r(x) = A_r \left[\cos\left(\frac{\gamma_r}{L}x\right) - \cosh\left(\frac{\gamma_r}{L}x\right) + \zeta_r \left(\sin\left(\frac{\gamma_r}{L}x\right) - \sinh\left(\frac{\gamma_r}{L}x\right) \right) \right] \quad (10)$$

Where A_r = modal constant

Value of ζ_r is given by,

$$\frac{\sin(\gamma_r) - \sinh(\gamma_r) + \gamma_r \frac{M_t}{mL} (\cos(\gamma_r) - \cosh(\gamma_r))}{\cos(\gamma_r) + \cosh(\gamma_r) - \gamma_r \frac{M_t}{mL} (\sin(\gamma_r) - \sinh(\gamma_r))} \quad (11)$$

Undamped natural frequency of free vibration for rth mode is given by,

$$\omega_r = \gamma_r^2 \sqrt{\frac{EI}{mL^4}} \quad (12)$$

For solving equation (7) which represent time dependent part in the solution,

Equation (7) is 2nd order homogeneous differential equation. Its solution is given by[1],

$$\eta(t) = E \cos(\omega t) + F \sin(\omega t) \tag{13}$$

3. Results

3.1. Beam parameters

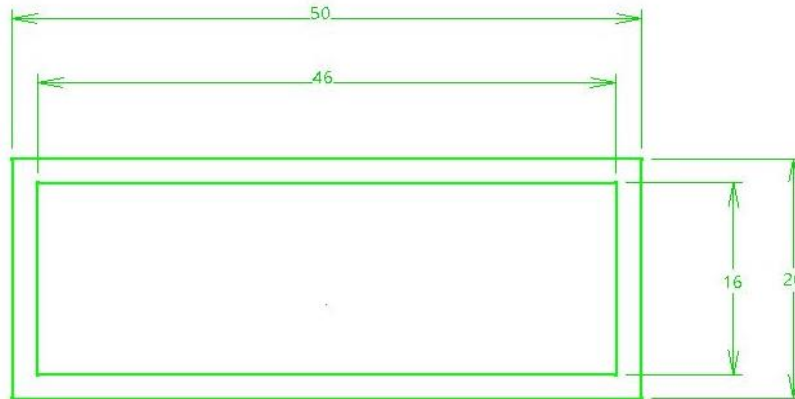


Figure3.1- cross section of the beam

Sr. No.	Parameter	Value
1	Natural frequency of beam	= Around 10 Hz
2	Cross sectional area	= 284 mm ²
3	Length of beam	= 1100 mm
4	Moment of inertia	= 29603.7 mm ⁴
5	Beam stiffness	= 13345.019 N/m
6	Beam material	= Structural steel ($\rho= 7850 \text{ kg/m}^3, E=200 \text{ GPa}$)

3.2. Calculation of eigen values.

For calculation of eigen values equation (9) is used. The equation is solved using Matlab. The code for it is attached in appendix

1. Results for eigenvalues are tabulated as follows (table (3.1)),

Table.3.1. – Eigen values for various mass ratios.

Mass ratio	Eigen values				
	1	2	3	4	5
0.5	1.419	4.111	7.190	10.298	13.421
1	1.248	4.031	7.134	10.257	13.388
1.5	1.146	3.999	7.113	10.242	13.376
2	1.076	3.983	7.103	10.234	13.370
2.5	1.023	3.972	7.096	10.229	13.366
3	0.981	3.965	7.092	10.226	13.364

For our application 1st eigen value is of importance. Particularly square of eigen value is important as equation of natural frequency involves square of eigen value term. Below graph (figure (3.2)) shows variation of square of eigen value v/s mass ratio,

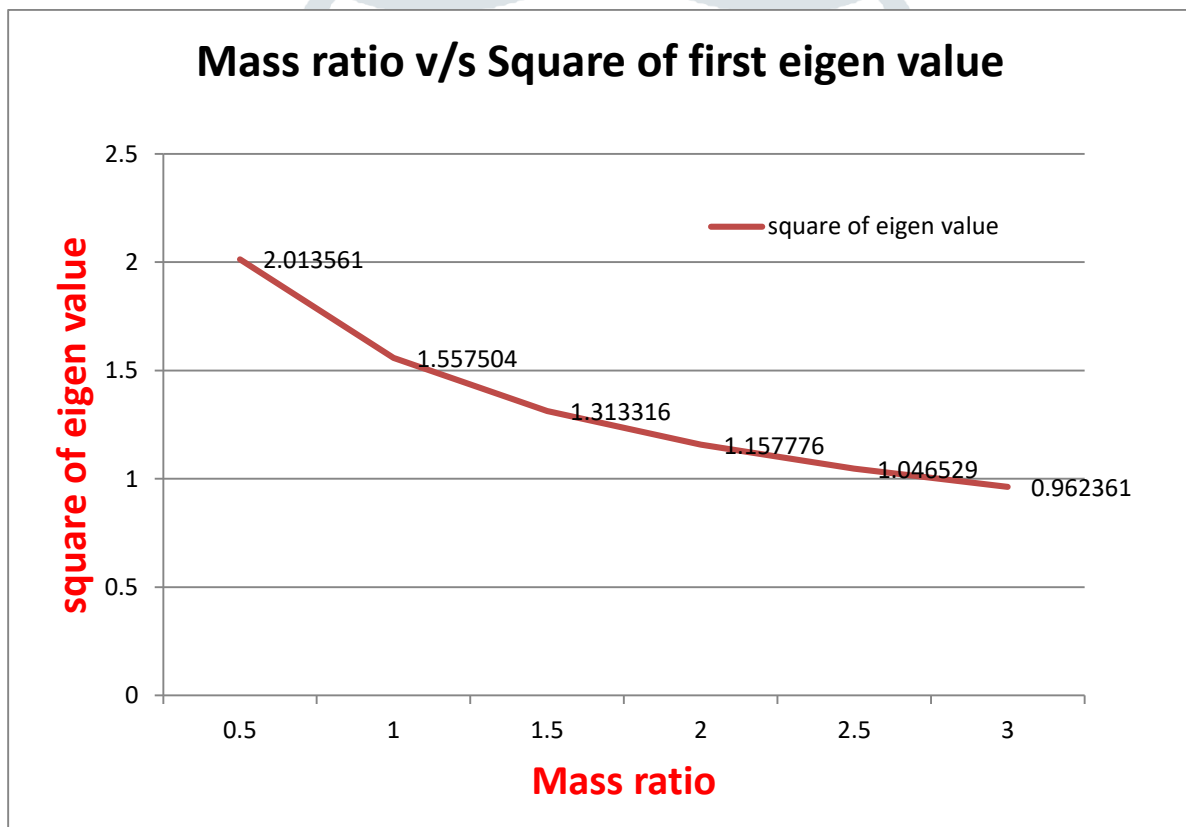


Figure3.2. - Effect of variation of mass ratio on the square of Eigen value of the beam.

3.3. Calculation of natural frequencies.

By using equation (12) natural frequencies are calculated. Table of values for corresponding values of eigen value is given below (table 3.2),

Table3.2. – Natural frequencies of the beam

Mass ratio	Natural frequency (Hz)				
	1	2	3	4	5
0.5	16.70526	140.2115	428.8904	879.821	1494.37
1	12.92164	134.8075	422.2355	872.8292	1487.031
1.5	10.89577	132.6757	419.7533	870.2781	1484.366
2	9.605347	131.6161	418.5739	868.9191	1483.035
2.5	8.6824	130.8902	417.7493	868.0703	1482.148
3	7.984111	130.4292	417.2785	867.5612	1481.704

As for our purpose 1st natural frequency is important. Its variation with respect to mass ratio is given in figure (3.3),

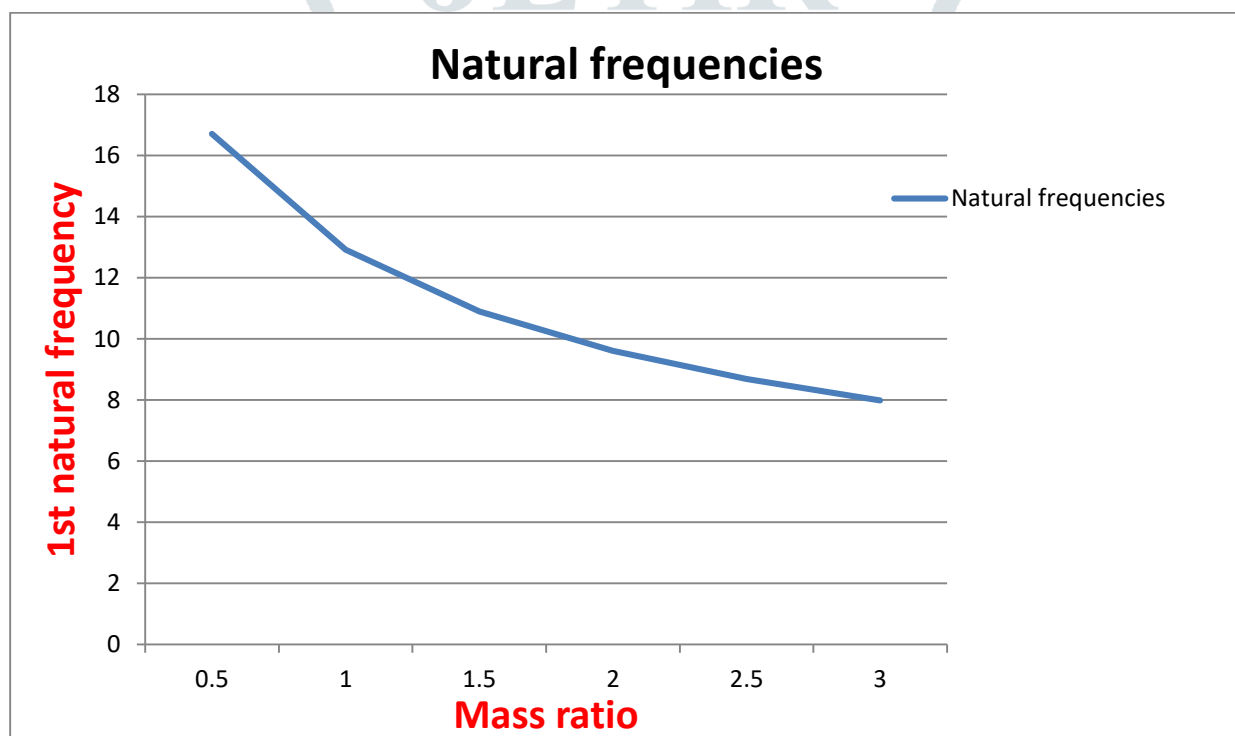
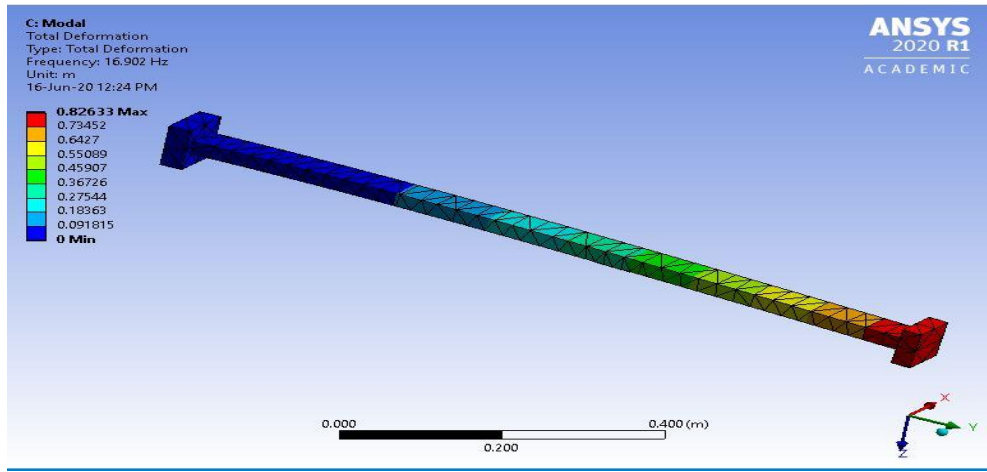


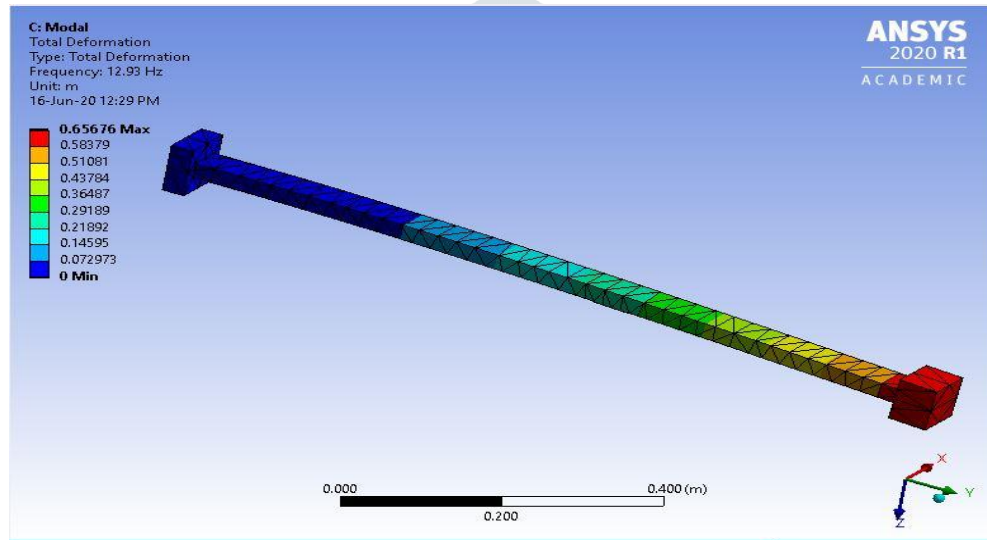
Figure3.3. - Effect of variation of mass ratio on the natural frequency of the beam.

3.4. 1st natural frequencies of the beam using ANSYS modal analysis module.

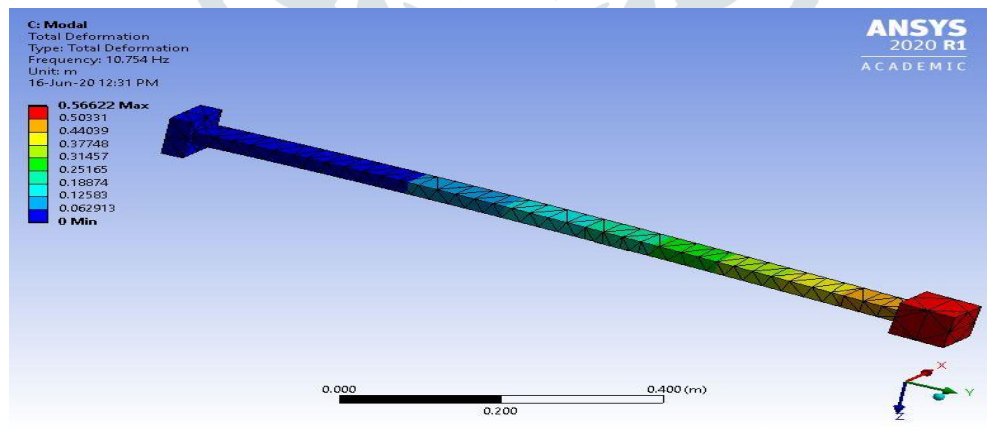
Numerical values for the natural frequencies of the beam are obtained using ANSYS modal analysis module. Numerical results and analytical results are compared in table (3.3).



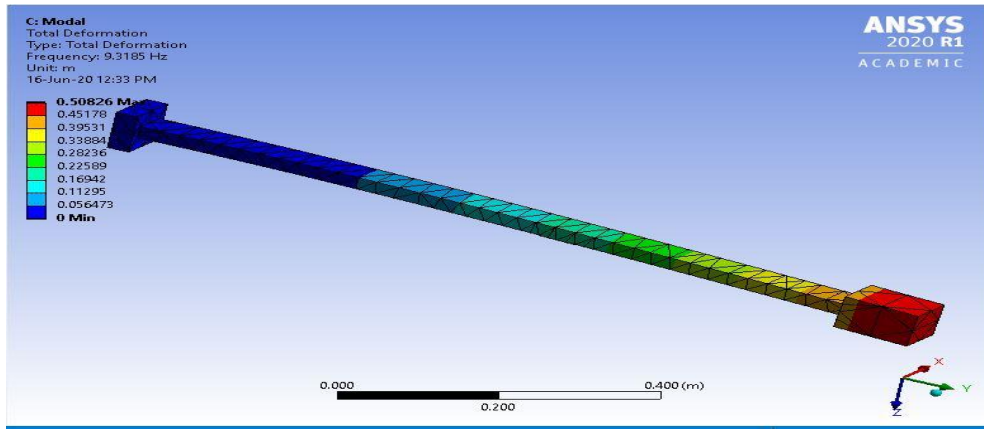
(a)



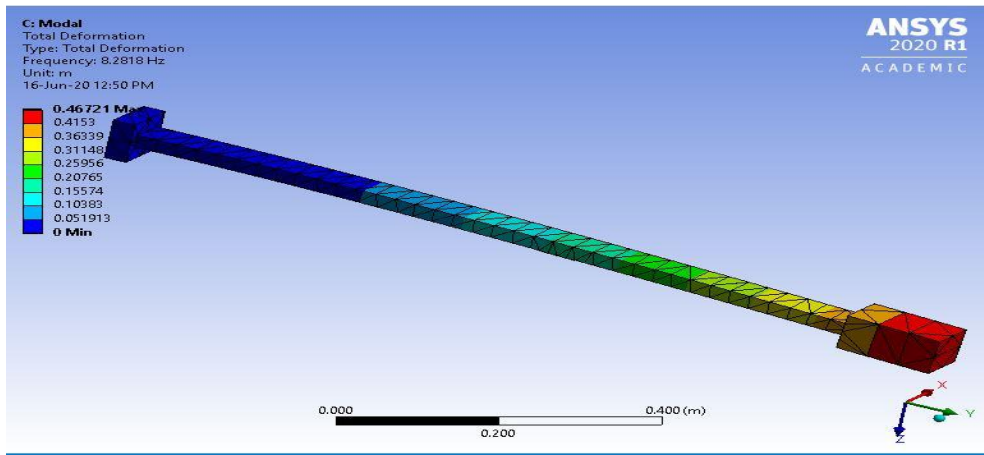
(b)



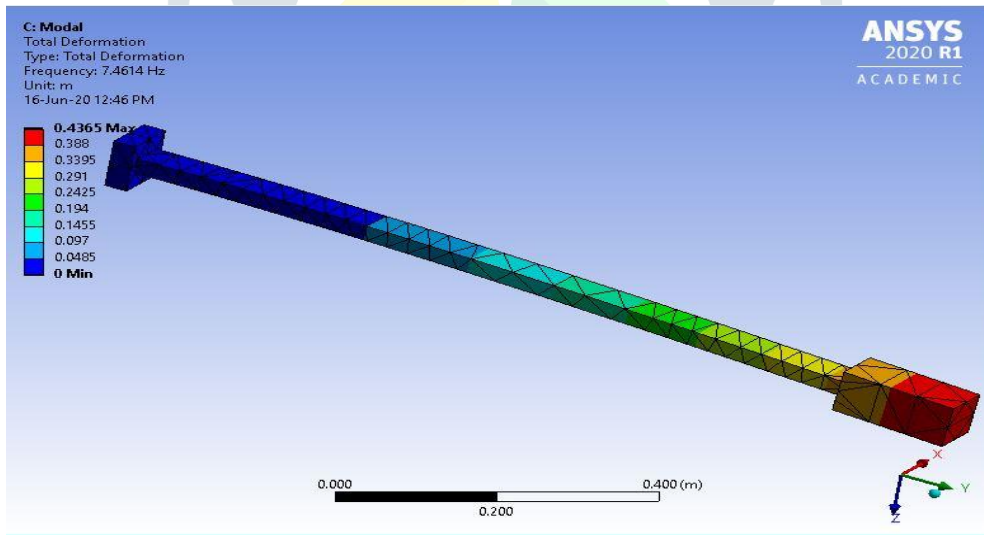
(c)



(d)



(e)



(f)

Figure-3.4.(a,b,c,d,e,f) – Modal analysis of the cantilever beam for various mass ratios (0.5 to 3).

3.5. Natural frequencies of the beam using 1 DOF model.

Transverse stiffness of the cantilever beam is given by [4],

$$K = \frac{3EI}{L^3}$$

Material used for beam is structural steel. Material has young modulus of 200 GPa. We used rectangular hollow cross section as shown in figure (3.1). Moment of inertia of the beam is $29603.7mm^4$.

Natural frequency of the 1 DOF system is given by [4],

$$f = \frac{1}{2\pi} \sqrt{\frac{K}{m_{tip}}}$$

By using above equation natural frequencies are calculated for various mass ratios. Results are shown in table (3.3).

Table3.3. – 1st natural frequencies of the beam.

Mass ratio	Tip mass (Kg)	Analytical value (Hz)	Numerical value (Hz)	1 DOF Hz
0.5	0.891	16.705	16.902	19.478
1	1.782	12.921	12.930	13.773
1.5	2.675	10.895	10.754	11.241
2	3.564	9.605	9.318	9.739
2.5	4.455	8.682	8.282	8.712
3	5.346	7.984	7.461	7.952

4. Conclusion

1. For lower mass ratios (i.e. from 0.5 to 1.5) values of 1st natural frequencies obtained from 1 DOF model shows large deviation from analytical and numerical results
2. For higher mass ratios (i.e. from 2 to 3) values obtained from 1 DOF model are nearly same as that of analytical results.
3. For lower mass ratios, results shows that if we consider beam and tip mass system as 1 DOF model we would generate wrong values of 1st natural frequencies.

References

- [1] Alper Erturk and Daniel J. Inman. "Modal Analysis of a Uniform Cantilever with a Tip Mass." Piezoelectric Energy Harvesting, First Edition. John Wiley & Sons, Ltd, published-2011
- [2] Vaibhav Ghodge, A. P. Bhattu, S. B. Patil, "Design and Validation of a Mass Tuned Dynamic Vibration Absorber." International Journal of Advance Research, Ideas and Innovations in Technology, published-2018
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- [4] Grover G.K., "Mechanical Vibrations", Nem Chand & Bros, Eighth Edition, 2009.
- [5] S.S Rao, "Vibration of continuous systems", John Wiley & Sons, 2007
- [6] Rao S.S., "Mechanical Vibrations", Wesley, Third Edition, 1995.

Appendix

Matlab code for calculation of eigen values and natural frequencies.

clc;

close all;

```
clear all;

syms x;

f=1+cos(x)*cosh(x)+x*2*(cos(x)*sinh(x)-sin(x)*cosh(x)); %Enter the Function here

g=diff(f); %The Derivative of the Function

n=input('Enter the number of decimal places:');

epsilon = 5*10^-(n+1); %x0 = input('Enter the intial approximation:');

for xit=1:15

x0=xit;

for i=1:100

f0=vpa(subs(f,x,x0)); %Calculating the value of function at x0

f0_der=vpa(subs(g,x,x0)); %Calculating the value of function derivative at x0

y=x0-f0/f0_der; % The Formula

err=abs(y-x0);

if err<epsilon %checking the amount of error at each iteration

break

end

x0=y;

end

y

xit=xit+1;

end

y = y - rem(y,10^-n);
```

