GENERALIZIED FUZZY NUMBERS USING FUZZY LINEAR PROGRAMMING AND MULTI-ATTRIBUTE DECISION MAKING

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ABSTRACT:

In this paper proposes a decision making system of multiple attributes with generalized fuzzy number (GFN_S). In one of the method to calculate the distance between GFN_S using the Hausdorff distance. Based on maximization deviation degree, the weight of the attribute are calculated by a linear programming model. In addition, linguistic variables are using a process of alternative on qualitative attributes to fuzzy ratings and ranking alternatives rank formula which changed degree of possibility is adopted. Introduces a numerical example validate the proposed model, and the arise suggest that the proposed model offer's an efficient and realistic way to meet the different assessment criteria of decision – makers then a numerical examples is established to implement process of the technique.

Keywords: multi-attribute decision making; generalized fuzzy number; Housdorff distance modified possibility degree Fuzzy Linear Programming for fuzzy number, linguistic variables.

1.Introduction :

Multi–attribute decision making (MADM) refer to making leaning decision making by a evaluation and prioritizing a confined set of alternatives based on multiple conflict attributes. MADM has been a research area in management science for a long time period (Zanakis, Solomon, Wishart & Dubish,1998). An MADM problem can be explained with a set of attribute and finite alternatives. Multi-attribute decision making (MADM) can be used to rate the preference under various condition in specific fields of implementation(Yakowitz et al, 1993 Zavadskas et al, 2008; Sun,2010; Kou and Lin, 2014) over the course of the years several technique have been created. For examples, saaty (1980) suggested the Analytic Hierarchy Process (AHP), one of the most commonly used methods of MADM. Hwang and Yoon suggested the methodology of order choice to rate preference over multiple characteristic by comparison to ideal solution (TOPSIS) process. Gabus and Fontela (1972) suggested the Decision Taking Trial and Assessment Laboratory (DEMATE) approach for the inter relationships between device variables and visualizing the process through comparison of cause and effect. Peng *et al* (2008) suggested a MADM paradigm for credit data processing, including multi–criteria convex quadratic programming. Barker and Zabinsky suggested a reverse logistics model of the MADM utilizing AHP. Chan (2012) proposed the integrated MADM methods and the provided Inno-Qual performance system applications. In Triantaphyllou and Sanchez a responsiveness review procedure was carried out with the MADM methods in (1997).

This paper is organized as follows. Section 2 basic definitions and fuzzy number and GFN and linguistic variables are defined as well as the fuzzy distance formula and normalization method.3 Attribute weights proposed in this section. Section 4 ranking of the alternatives. Section 5 used to illustrated with a real life numerical example. Section 6 conclusion.

2. Basic concepts and definition

2.1. Definition of Fuzzy Number

The universal set of real numbers R is defined by a fuzzy set by the membership function

$\mu_{\tilde{A}}(x)$ satisfied normality convexity and piecewise continuity.

2.2. Definition of Trapezoidal Fuzzy Number

Membership function of Trapezoidal fuzzy number can be said by

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ *where* $a_1 \le a_2 \le a_3 \le a_4$

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} 0 & for\ \ x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ \ a_1 \le x \le a_2 \\ 1 & for\ \ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & for\ \ a_3 \le x \le a_4 \\ 0 & for\ \ x \ge a_4 \end{cases}$$

2.3. Generalized Fuzzy Number

A GFN $\tilde{A}$ is given by $\tilde{A}$ = $(a_1,b_1,c_1,d_1)_n$, $n > 0$, $0 \le a_1 \le a_2 \le a_3 \le a_4$if the membership function $\mu_{\tilde{A}}: R \to [0,1]$ is defined as follows

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} 0 & for\ \ x < a_1 \\ (\frac{x-a_1}{a_2-a_1})^n & for\ \ a_1 \le x \le a_2 \\ 1 & for\ \ a_2 \le x \le a_3 \\ (\frac{a_4-x}{a_4-a_3})^n & for\ \ a_3 \le x \le a_4 \\ 0 & for\ \ x \ge a_4 \end{cases}$$

Intuitively explaining a generalized fuzzy number, we have assume that, a_1=2, a_1=4, a_1=6, a_1=8, a_1=10, the GNF with the different values of n.

We can discuss GFN $\tilde{A}$ based on the GFN graphs three points of view. First, if n=1, then $\tilde{A}$ is a trapezoidal fuzzy number, and if n=1 and b=c, then the $\tilde{A}$ is positive it is a triangular fuzzy number, therefore, and the trapezoidal fuzzy number is a special form of the GFN. Second, if n>1, the contract for the left and right division the GFN's membership function. Third, $0 < n < 1$, the left and right branches extend to include the GFN with membership feature. In fact, we can get other GFN characteristics. Since, if n increases , the GFN'S blurred degrees decrease, resulting in larger variations in the results.

Now, any two generalized fuzzy number $\tilde{A}$ = $(a_1,b_1,c_1,d_1)_n$, $\tilde{B}$ = $(a_2,b_2,c_2,d_2)_n$ and some positive real numbers $\boldsymbol{\lambda}$ then the operation of the fuzzy number $\tilde{A}$ and $\tilde{B}$ can be represented as follows:

1. $\widetilde{A} \oplus \widetilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)_n$,
2. $\widetilde{A} * \widetilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2)_n$,
3. $\boldsymbol{\lambda}\tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)_n$,
4. $\tilde{A}/\tilde{\boldsymbol{B}} = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2)_n$,
5. Manhattan distance between GFNs

$$d_M(\tilde{A},\tilde{B}) = \int_0^1 |\tilde{A}_\lambda^- - \tilde{B}_\lambda^-| + |\tilde{A}_\lambda^+ - \tilde{B}_\lambda^+|$$

$$d\lambda = \left|\frac{n}{n+1}(b_1-b_2) + \frac{1}{n+1}(a_1-a_2)\right|$$

$$+ \left|\frac{n}{n+1}(c_1-c_2) + \frac{1}{n+1}(d_1-d_2)\right|$$

Manhattan distance between the main operation in generalized fuzzy number is convoluted and the Euclid distance between generalized fuzzy numbers even more on. The estimation method is the simplified by the paper suggests the Hausdorff distance between GFNS. In Nadler (1978), the distance to Hausdorff is a measure of the degree of A and B in metric space S is identical in terms of their roles. It can also measure the maximum degree of mismatch between two sets and a be more robust; some problems can be referred by the Hausdorff distance

2.4. Linguistic variable

A linguistic variable is a variable whose values are in linguistic terms.

The notion of the linguistic variable which can be useful in the situation where determination problem are too difficult or too ill-defined to be described suitably by using the traditional quantitative expressions.

For example, the performance category of the preference on qualitative attribute can be intimate using linguistic variable such as extremely high, very high, high, medium, low, very low, extremely low, etc. Also linguistic values can be represented and determined by a positive trapezoidal fuzzy numbers. For examples "high" and "low" can be proposed by a positive trapezoidal fuzzy number (0.2,0.3,0.4,0.5) and (0.7,0.8,0.9,0.10), respectively.

2.5 Distance between two trapezoidal fuzzy number

Let us consider the value $\tilde{m}$ = (m_1,m_2,m_3,m_4) and $\tilde{n}$ = (n_1,n_2,n_3,n_4) be two trapezoidal fuzzy numbers. Then the vertex method can be calculated by the distance between as follows;

$$d(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{4}[(m_1-n_1)^2 + (m_2-n_2)^2 + (m_3-n_3)^2 + (m_4-n_4)^2]} \quad \ldots\ldots\ldots(1)$$

Eq. (1) simple method to calculate the distance between two trapezoidal fuzzy number [2,9].

If the real numbers are $\tilde{m}$ and $\tilde{n}$ are distance measurement $d(\tilde{m},\tilde{n})$ is the identical to be Euclidean distance.

Suppose the values $\tilde{m}$ = (m_1,m_2,m_3,m_4) and $\tilde{n}$ = (n_1,n_2,n_3,n_4).

$$d(\tilde{m},\tilde{n}) = \sqrt{\frac{1}{4}[(m_1-n_1)^2 + (m_2-n_2)^2 + (m_3-n_3)^2 + (m_4-n_4)^2]}$$

$$= \sqrt{\frac{1}{4}[(m-n)^2 + (m-n)^2 + (m-n)^2 + (m-n)^2]}$$

$$= \sqrt{\frac{1}{4}[(m-n)^2}$$

In fact, the two trapezoidal fuzzy number $\tilde{m}$ and $\tilde{n}$ are identical if and only if the distance measurement $d(\tilde{m},\tilde{n})$ = 0.

2.6 The normalization method

Attribute can generally be classified into two types: benefit attributes and attributes on cost. this is, the set of attributes C can be split into two subgroups: C^1 and C^2, where C^k (K=1,2) ia a class of advantages and parameters of changes, respectively. In addition that, $C = C^1 \cup C^2$ *and* $C = C^1 \cap C^2 = \emptyset$,where $\emptyset$ is empty set.Since the m objectives may be measured in different ways, the decision matrix $\breve{D}^p$ needs to be normalized. In this papr, we can choose the following normalization formula

$$\tilde{r}^p{}_{ij} = \left(\frac{a_{ij}^p}{c_{ij}^{max}}, \frac{b_{ij}^p}{b_{ij}^{max}}, \frac{c_{ij}^p}{a_{ij}^{max}}, \hat{}1\right) \quad for\ j \in C^1 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

And

$$\tilde{r}^p{}_{ij} = \left(\frac{a_{ij}^{max}}{c_{ij}^p}, \frac{b_{ij}^{max}}{b_{ij}^p}, \frac{c_{ij}^{max}}{a_{ij}^p}, \hat{}1\right) \quad for\ j \in C^2 \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

3. Attribute weights:

In this section, according to the maximum degree of deviation , the weight of the variable may be adjusted to regular system design. To represent the alternatives, let us X={x_1,x_2,x_3……….. x_m} and C={c_1,c_2,c_3………c_n} to represent the attribute of the evaluation. This is the attribute which are additionally independent. $\widetilde{X_{ij}}$ in the value of the value assessed C_j attribute of alternatives x_i, and is expressed in the GFN on the paper. The different values of $\widetilde{X_{ij}}$ can be expressed by a matrix $\tilde{V} = (\widetilde{x_{ij}})_{max}$, which is considered by the decision making matrix. The vector of the attribute weight is W = {W_1,W_2,W_3..............W_n} to replace the difference of attribute directory on the dimension, each attribute directory is standard.

$$\widetilde{x'}_{ij} = \begin{cases} \dfrac{\tilde{x}_{ij}{}^1}{\widetilde{x^+}_j} & \hat{}1\forall i \in M, j \in I_1 \\ \dfrac{\widetilde{x^-}_j{}^1}{\tilde{x}_{ij}} & \hat{}1\forall i \in M, j \in I_2 \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Where I_1 is correlated with a series of gain parameters, and I_2 is linked to a set of criteria and M = {1,2,3,4,………..} .

When under the attribute $\widetilde{x'}_{ij}$ the values of all alternatives got bigger and it indicates variations, as the attribute plays an significant role in alternatives graded. The better one, from the previous review the degree of variance of the variable will be greater than the weight. Therefore the value of the weight vector attribute will be optimize the total variance on all attributes class of the feasible alternatives. The confusion and the ambiguity of the aims and the vagueness of the individual remember, partial information is provided on the weights of the attributes. The attribute weight of the given limited facts. Weight $W_j \in [a,b]$,
$0 \le a_j \le b_j \le 1$, a linear programming model is manufactured as follows:

$$P: \max D(x) = max \sum_{j=1}^{n} D_j\, w_j$$

$$\text{s. t} \sum_{j=1}^{n} w_j = 1, w_j \in [a_j, b_j], w_j \ge 0, j \in N \ldots\ldots(5)$$

4. Ranking of the alternatives

Depending on the degree of the possibility updated in this section, the related matrix approach for rating is proposed alternate grade. In Carlsson and Filler (2001), the explanation of interval description of the potential mean value of the fuzzy number shall be given as follows:

4.1 Definition

Let A = $(x_{ij})_{n\times n}$be a matrix if $x_{ij} + x_{ji}$= 1, the matrix is considered as a supplemental decision matrix. For a given fuzzy performance matrix, $P = (p_{ij})_{m\times n}$ known as a potential matrix by the rule the P is supplemental decision matrix, using the ranking formula Xu (2001), we have

$$v_i = \frac{1}{m(m-1)} \sum_{j=1}^{m} \left(p_{ij} + \frac{m}{2} - 1\right), i \in M \text{..........(6)}$$

This is, the position vector v =($v_1, v_2, v_3 \ldots\ldots v_m$) for the classification that can be determined with the potential matrix P. The alternatives, instead will be ranked according to the scale of the specific vector components.

A five stages of hybrid technique based on the earlier discussion to MADM is the following proposal:

Step 1:

Arrangement of the GFN based on decision matrix and normalized graph of decision –making.

Step 2:

Establishing on linear programming model maximizing degree of deviation and calculating the weight factor.

Step 3:

To calculate the average weighted decision matrix vector.

Step 4:

To calculate matrix for pair wise comparisons based on probability degree.

Step 5:

List on the basis of alternatives ranking formula (6).

5. Numerical example

A numerical example is considered in this section. Assume that a bike showroom needs to buy some automobile parts, and there are four auto mobile parts suppliers A_1,A_2,A_3andA_4. Six attribute C_1(Rate of the spare), C_2(State of environmental protection), C_3 (Trait of the product), C_4(State of suppliers service), C_5(Time of the reaction), C_6

(Maintenance), are taken into the thought. During the decision making process the public surrounding is somewhat intricate and the opinion of decision maker are usually not reliable, vague, and ambiguous; people are usually unwilling are unable to appoint exact values in the evaluation process. They prefer to provide their evaluations in linguistic process.

The corresponding relations between linguistic variable and positive trapezoidal fuzzy number are given in Table 5.

Using Eqn. (2) and (3), we can obtain the normalized decision matrix $\tilde{R}^1$ of the expert P_1 according to Table 1 and 4.

Table 1

Decision information given by the decision maker P_1

Bike Showroom	Attributes					
	C_1(mach)	C_2(mile)	C_3(Ib)	$C_4(\$ \times 10_6)$	C_5	C_6
A_1	2	1,500	20,000	5.5	Medium	Very high
A_2	2.5	2,700	18,000	6.5	Low	Medium
A_3	1.8	2,000	21,000	4.5	High	High
A_4	2.2	1,800	20,000	5.0	Medium	Medium

Table 2

Decision information given by the decision maker P_2

Bike Showroom	Attributes					
	C_1(mach)	C_2(mile)	C_3(Ib)	$C_4(\$ \times 10_6)$	C_5	C_6
A_1	2	1,500	20,000	5.5	High	High
A_2	2.5	2,700	18,000	6.5	Low	Medium
A_3	1.8	2,000	21,000	4.5	Medium	Very high
A_4	2.2	1,800	20,000	5.0	Medium	Medium

Table 3
Decision information given by the decision maker P_3

Bike Showroom	Attributes C_1(mach)	C_2(mile)	C_3(Ib)	$C_4(\$ \times 10_6)$	C_5	C_6
A_1	2	1,500	20,000	5.5	Medium	Very high
A_2	2.5	2,700	18,000	6.5	Low	High
A_3	1.8	2,000	21,000	4.5	Medium	Medium
A_4	2.2	1,800	20,000	5.0	High	Medium

Table 4
Decision information given by the decision maker P_4

Bike Showroom	Attributes C_1(mach)	C_2(mile)	C_3(Ib)	$C_4(\$ \times 10_6)$	C_5	C_6
A_1	2	1,500	20,000	5.5	Medium	High
A_2	2.5	2,700	18,000	6.5	Medium	Medium
A_3	1.8	2,000	21,000	4.5	High	High
A_4	2.2	1,800	20,000	5.0	Low	Medium

Table 5
The relations between linguistic and trapezoidal fuzzy number

Linguistic Variable	Trapezoidal fuzzy number
Very high	(0.7,0.8,0.9,1)
High	(0.6,0.7,0.8,0.9)
Medium	(0.3,0.4,0.5,0.6)
Low	(0.2,0.3,0.4,0.5)
Very low	(0,0.1,0.2,0.3)

Table 6
Borda's scores of all bike showroom with respect to every experts

Bike showroom	Experts P_1	P_2	P_3	P_4	Borda's score
A_1	2	2	3	2	9
A_2	0	0	1	1	2
A_3	3	3	0	0	6
A_4	1	2	2	3	8

Comparing these distance, the ranking orders of the four bike showroom for the four exports P_p (P = 1,2,3,4) are generated respectively as follows: $A_3> A_1> A_4> A_2> A_3> A_1> A_4> A_2> A_1> A_4> A_2> A_3 >A_4>A_1> A_2>A_3$. From Table 6, the ranking order of the four bike showroom for the auto mobile purchase group is generate as follows

$$A_1>A_4>A_3>A_2$$

Therefore, the best selection is the bike showroom A_1.

6.Conclusions

In this paper, A new fuzzy decision-making method towards decision-making problems also finds reasonable decision-making alternatives. The multi attribute decision making problems contain both quantitative and qualitative attribute that are many times assessed using imprecise data and human judgment. The attribute weights are determined of the construction a linear programming model, and we calculate the possibility degree for the FMADM problem. Linguistic variable as well as crisp numerical values are used to assess qualitative and quantitative attributes. Trapezoidal fuzzy number are used to asses alternatives with respect to qualitative attributes. Also linguistic terms and GFN_S are used in the calculating process. The membership function of the GFN has been calculated. The normalization constraints on weights are imposed, which ensures that the weights generated are not zero. The developed methods is established using an bike showroom problem. Especially, in situations where multiple decision makers are involved and the weights of the attributes are not provided a priori. Hence, simulation is a better away to show the robustness of MADM.

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