A REVIEW OF F. LONDON'S JUSTIFICATION OF THE LONDON THEORY OF SUPERCONDUCTIVITY

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Abstract :

In this paper is a formal similarity between the ground state of H_{red} and a condensed Bose-Einstein gas, the analogy must be used with care due to the strong overlap of the pair functions. As a result of this overlap the excitation spectrum in real metal exhibits an energy gap rather than a continuous spectrum characteristic of a Bose-gas. If the treatment is extended to include the interactions neglected in H_{red} and one assumes all interactions to be of short range, a continous boson spectrum, starting at zero energy appears in the energy gap corresponding to density fluctuations in the electron system. In real metals these low-lying boson modes ae pushed up to the Plasmon energy $(\sim 10^4 \times 2\nabla_0)$ due to the long range Londons interaction between electrons so that there are no low lying boson modes exact for dressed lattice vibrations (phonons) in coset of physical interval.

Key words : London theory, superconductivity, Plasmon, coherence length, Boltzamann's transport equation

1. Introduction :

F. London pointed out that the equation

$$\bar{J_s} = \frac{n_s e^2}{mc}\,\bar{A} \tag{1}$$

Could be deduced from first principles if one assumed that the many body wave functions Ψ describing the super fluid is valid with respect to perturbations due to a transverse vector potential $(\nabla . \bar{A}=0)$. One can see this as follows. The current density J_{so} in this absence of $\bar{A}$.

$$\overline{J_s}\,(\bar{r}) = -\frac{e\hbar}{2m_i}\sum_{j=1}^{n_s}\int\left(\Psi_s^*\nabla_{\mathrm{j}}\Psi_{\mathrm{s}}-\Psi_{\mathrm{s}}\nabla_{\mathrm{j}}\Psi_s^*\right)\delta\ d^3r_1 \dots d^3r_n \qquad (2)$$

Clearly vanishes. If a weak magnetic field is applied to the system and Ψ_{s} is unaffected to first order by this perturbation, the paramagnetic current continues to vanish, while the diamagnetic current is given by

$$\overline{J_s}\,(\bar{r}) = -\sum_{j=1}^{n_s}\frac{e^2}{mc}\,\bar{A}\,(r)\int\Psi_s^*\Psi_{\mathrm{s}}\delta\left(\mathrm{r_j}-\mathrm{r}\right)\ d^3r_1 \dots d^3r_{ns}\ \frac{-n_s e^2}{mc}\bar{A}\,(r) \qquad (3)$$

in agreement with (1) More accurately, it is assumed that in the long wavelenght limit, the paramagnetic and diamagnetic currents of the normal fluid exactly cancel each other (as they do in the Landau diamagnetism of the normal state) while the paramagnetic current of the superfluied vanishes, leaving the diamagnetic supercurrent. One has suggested that the origin of the London "rigidity" is the energy gap in the excitation spectrum of the sysem. This somewhat imprecise statement is not in conflict with the fact, that insulators also possess an energy gap in their excitation spectra. This follows since interband matrix elements of the magnetic perturbation are large in this case so that the paramagnetic current is nonzero and just cancels the diamagnetic current.

The miscroscopic theory reduces exactly to the form (3) in the limit of fields which very slowly in space.

On the basis of London's quantum interpretation of the London equations, he concluede that the flux ϕ trapped through a hole of a multiply connected superconducting body must be an integral multiple of hc/e$\sim 4\times10^{-7}$gauss cm^2. To understand this result, consider two concentric superconducting cylinders, as shown in figure 2D. Suppose that the thickness of the cylinders is large compared to the penetration depth

λ and that a flux ϕ is trapped within the hole of the inner cylinder. Furthermore. assume that there is no magnetic field in the region between inner and outer cylinders so that the flux through the hols of the outer cylinder is equal to ϕ. The inner cylinder acts only as a shield to ensurethat no magnetic field touches the physically intersting outer cylinder. Let Ψ_o be the wave function for the outer cylinder when there is no flux trapped $\phi = 0$. To determine the wave function Ψ_o, in case $\phi = 0$. One notes that the vector potential in the outer ring is in θ direction and has the value.

$$A_0(r) = \frac{\phi}{2\pi r} = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\phi^{\theta}}{2\pi}\right) = \nabla_{\theta}\left(\frac{\phi^{\theta}}{2\pi}\right) \tag{4}$$

Since $\bar{A}$ is the outer cylinder is the gradient of the scalar $\left(\frac{\phi^{\theta}}{2\pi}\right)$ it follow that Ψ_o and Ψ_o are related by the gauge transformation

$$\Psi_{\phi} = e^{\frac{ie\phi\sum_j Q_j/}{\hbar c\Psi^0}} \tag{5}$$

Where $\theta_j$is the azimuthal coordinate of the Jth electron. If Ψ_{ϕ} and Ψ_0 are the single-valued function of the coordinates θ_j, one must have

$$\mathrm{e}\phi/hc = in\,eger \tag{6}$$

or ϕ is quantized to the London values.

$$\phi_n = n\left(\frac{hc}{e}\right)\ (\mathrm{n} = 0, \pm 1, \pm 2) \tag{7}$$

To complete the argument, suppose that the inner cylinder is made normal to that of the magnetic field fills the entire hole in the outer cylinder. Owing to the meissner effect, the magnetic field will penetrate only a small distance $(\approx 5\times 10^{-6} cm)$ cm into the outer cylinder. Therefore the above argument should continue to hold since this small perturbation should not affect the wavefunct on Ψ appreciably (particularly if London's "rigidity" is effective). On the basis of this argument, London concluded that the flux trapped through any hole in a massive specimen is quantized to multile of hc/e.

In 1953 Onssger[1] suggested that the actual value of the flux quantum might be one-half this value, presumably become of the effective charge of the entites making up the superfluid being 2e. The difficulty in London's argument is that there is another series of low-lying states which are distinct from London's stateΨ_n, and cannot be generated from the ground state Ψ_o, by a gauge transformation. This second series of states leads to the quantized flux values

$$\phi_n = \left(n + \frac{1}{2}\right)\frac{hc}{e} \quad (n = 0, \pm 1, \pm 2 \ldots) \qquad (8)$$

On taking the London Series (1.31) and (1.32) together, one obtains the results suggested by Onsager.

$$\phi_n = n\left(\frac{hc}{e}\right) \qquad (n = 0, \pm 1, \pm 2 \ldots) \qquad (9)$$

In agreement with experiement.

2. Pippard's Nonlocal generalization of the London theory

The basic equation of the London theory are 'local' in the sense that they relate the current densities and the electromagnatic potentials at the same point in space. On the basis of numerous experimental results, Pippard[2] concluded that these local relation must be replaced by nonlocal relation giving the currents at a given point in space as a space average of the field strength taken over a region of extent $\xi_0 = 10^{-4}$ cm of about the point in question. one of the most compelling arguments for this generalization is that the penetration depth λ increases appreciably if a sufficent amount of impurity is introduced into the material. This effects sets in when the mean free path of ℓ electron in the normal state falls below a distance ξ_0 known as Pippard's coherence length ξ_0 is the measure of the size of the pair bound state from which the superfluid have function is constructed. In the miscroscopic theory it is related to the enery gap 22∇ $by\ \xi_0 = h_{\upsilon F}/\pi\Delta$ were $\upsilon_F$is the Fermi velocity. On the other hand in the London theory λ is not expected to be appreciably affected by impurities, particularly neat T = 0. where all of the electrosn are condensed. In choosing a form for the nonlocal relations, Pippard was guided by Chamber's nonocal expression[3] relating the current density and electric field strength in the normal metal

$$J(r) = \frac{3\sigma}{4\pi 1}\int \frac{R[R.E\ (r')]}{R^4} e^{-R/1} d^3r' \qquad R \equiv r - r' \qquad (10)$$

where σ is the long wavelength electrical conductivity. Chamber's

expression is a solution of Boltzmann's transport equation if the seattering mechansim is characterized by a mean free path I. For fields varying slowly over a mean free path I, (10) reduces to Ohm's law J = σE. With Chamber's expression in mind, Pippard assumes that London's equation

$$\bar{J_s}\ (r) = -\frac{1}{C\Lambda(T)} \bar{A}(r) \frac{1}{\Lambda(T)} \equiv \frac{n_s(T)e^2}{m} \qquad (11)$$

Should be replaced by

$$\bar{J_s}\ (r) = -\frac{3}{4\pi\{_n C\Lambda} \int \frac{\bar{R}[\bar{R}.\bar{A}(r')}{R^4} e^{-R/\{} d^3r' \qquad (12)$$

The effective coherence length ξ is given by

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\alpha 1} \qquad (13)$$

Where α is an empirical constant of order unity and ξ_0 is a length characteristic of the material. For a pure material. Pippard's equation reduces to London's equation if A (r) varies slowly over a coherence length. For an impure material, Pippard's equation leads to an extra factor ξ/ξ_0 <1 multiplying $\frac{1}{C\Lambda}$ in London's equation in this long wavelength limit thereby increasing the effective penetration depth. In most cases distances of order $\lambda << \xi$are of importance in penetration phenomena so that the full reductionξ/ξ_0 is not effective. In highly impure specimens λ is of order of grater than ξ and one has $\lambda \sim (\xi_0/1)^{1/2}$

That the effective coherence length ξ, should be bounded by the mean free path I is certainly resonable from a physical point of view. It is a tribute to Pippard's insight into the physics of the pi-oblem that his equation is almost identical to that given by the microscopic theory.

A good deal of the qualitative aspects concerning the electro-magnetic properties of superconductors can be understood on the basis of simple energy-gap model. Prior to the BSC theory, Bardeen gave a theoretical derivation of the nonlocal electrodynamics He assumed that the single-particle matrix elements of the magnetic perturbation were unaltered by the condensation and that the single-particle excitation spectrum was altered only by adding a constant to the excitation energy, thereby creating an energy gap. Subsequent to the wrok of BCS, Ferrell, Glover. and Tinkhanm[4] employed the Kramers-Kroning relation to give quire a general discussion of how the electrodynamic behaviour of a superconductor comes about, because of its energy gap.[5]

3. GINSBURG- LANDAU THEORY

In 1950 Ginsburg and Landau proposed an extension of the London theory which takes into account the possibility of the superfluid density n_s varying in space. They phrased the theory in terms of an effective wave function $\Psi(r)$ which one normalize such that the local density of condensed electrons is given by

$$\left|\Psi(r)\right|^2 = \frac{n_s(r)}{n} \qquad (14)$$

where n is the total number of electrons per unit volume_ Roughly speaking $\Psi(r)$corresponds to the center-of mass wave function of the BCS pairs. Ginsburg and Landau treated $\Psi(r)$ as an order parameter which is to be determined at each point in space by minimizing the free-energy functional $F(\Psi, T)$ of the system. The problem is then one of guessing an appropriate form of F.

Suppose that $F(\Psi, T)$ is the difference of free energy per unit volume between S-and N phase when Ψ of is uniform. Then it is natural to include in F the term

$$\int f[\Psi(r), T] d^3r \qquad (15)$$

While $F(\Psi, T)$ is not known a priori, Ginsburg and Landau determined this function for Ψ (which is all that is needed when T is near Tc) by expanding f as a power series in $\left|\Psi\right|^2$ and retating the first two nonvanishing terms, thus

$$F(\Psi, T) \cong a\,(T)\left|\Psi\right|^2 + \frac{1}{2}\, b\,(T)\left|\Psi\right|^4 \qquad (16)$$

For $\left|\Psi\right|^2 << 1$. The equilibrium value $\left|\Psi\right|^2$ is determined by minimizing f.

$$\frac{\partial f}{\partial \left|\Psi\right|^2} = a\,(T) + b\,(T)\left|\Psi\right|^2 \qquad (17)$$

And therefore

$$\left|\Psi\right|^2 = -\frac{a(T)}{b(T)} \qquad (18)$$

From (16) and (18) one finds the (zero-field free-energy difference per unit volume between S- and N- phases is given by

$$f_s\,(T) - f_n(T) \equiv -\frac{1}{2}\frac{a^2(T)}{b\,(T)} = -\frac{H_c^2(T)}{8_\pi} \qquad (19)$$

Where one has used the thermodynamic relation between the critical field and the N-S free- energy difference. If one uses the fact that in the London theory $\lambda^2(T) \sim 1/n_s(T)$, one obtains a second relation between a(T) and b(T).

$$\frac{\lambda^2(0)}{\lambda^2(T)} = \frac{|\Psi_c(T)|^2}{|\Psi_c(0)|^2} = |\Psi_c(T)|^2 = \frac{a(T)}{b(T)} \qquad (20)$$

From (19) and (20) one finds

$$a(T) = - \frac{H_c^2(T)}{4\pi} \frac{\lambda^2(T}{\lambda^2(0)}$$

$$b(T) = - \frac{H_c^2(T)}{4\pi} \frac{\lambda^4(T}{\lambda^4(0)}$$

and therefore f(Ψ, T) given by (16) can be expressed in terms of experimentally measurable quantities.

If Ψ(r) is not uniform in space, Ginsburg and Landau argue that extra terms should be included in F which involve the rate of change of Ψ in space. Presumably these terms would come from (a) the kinetic energy associated with extra wiggles in the many-body wave function describing n_s and/or v_s changing in space and (b) interaction energy density being influenced by the variations of the superfluid density in a region surrounding the point in questions. If $|\Psi|^2$ varies slowly in space it should be sufficient to keep the leading from in grad $|\text{grad}\,\Psi|^2$. On the basis of gauge invariance, one would expect that this term, when combined with the effect of a vector potential A(r) would lead to free-energy contribution of the form

$$\int \frac{n^*}{2m^*} \left| \frac{\hbar}{i} \nabla\Psi(r) + \frac{e^*}{c} A(r)\Psi(r) \right|^2 d^3r \qquad (22)$$

Where e^* is the effective charge of the "entitles" forming the super fluid. (As one shall see, $2n^*$ = n, $e^* = 2e$, and $m^* = 2m$, consistent with the pairing theory)

By minimizing the total free-energy difference

$$F(\Psi_1 T) = \int \frac{n^*}{2m^*} \left[\frac{\hbar}{i} \nabla\Psi(\bar{r}) + \Psi(\bar{r}) + \frac{e^*}{c^*} A(\bar{r})\Psi(\bar{r}) \right]^{-2} d^3r$$

$$+\int [a(T)\|\Psi(r)\|^2 + \frac{1}{2}\,\mathrm{b(T)}\,|\Psi(r)^4]\,d^3\,r + \int \frac{H(r)^2}{8\pi}\,d^3r \quad (23)$$

With respect to $\Psi\,(r)$, one finds the constitutive equation of the Ginsburg-Landau theory

$$\frac{\hbar^2}{2m^*}\left[\nabla + \frac{ie^*}{\hbar c}\,A(r)\right]^2\Psi(r)$$

$$+\,\frac{H_c^2(T)}{4\pi n^*}\,\frac{\lambda^2(T)}{\lambda^2(0)}\left[1 - \frac{\lambda^2(T)}{\lambda^2(O)}\;\left|\Psi(r)\;\right|^2\right]\Psi(r) = 0 \quad (24)$$

The current density is given by

$$J_s(r) = -\,\frac{n^*\left|\Psi(r)\;\right|^2}{m^*c}\,e^{*2}\,A(r) - \frac{n^*e^*\hbar}{2m^*i}\,\{\Psi^*(\bar{r})\nabla\Psi\,(\bar{r}) - \Psi(\bar{r})\nabla\Psi^*\,(\bar{r})\} \quad (25)$$

With the normalization of Ψ. As the London theory one is to use the gauge $\nabla.\bar{A} = 0$. Therefore, (24) and (25) together with Maxwell's equation $\nabla \times \nabla\bar{A} = 4\pi\bar{J}/c$ lead to two nonlinear differential equations which determine the functions Ψ (r) and A (r)

One notes that if A = 0 and Ψ is uniform in space, (24) reduces to the condition.

$$1-\;\frac{\lambda^2(T)\left|\Psi\;\right|^2}{\lambda^2(O)} = 0 \quad (26)$$

Which states that Ψ is equal to its equilibrium value (20) as required if Ψ is perturbed slightly from its equilibrium value at some point, say r= o, then the linearized Ginsburg- Landau equation.

$$\frac{\hbar^2\nabla^2}{2m^*}\,\widetilde{\Psi}(r) - \frac{H_c^2(T)}{2\pi n^*}\,\frac{\lambda^2(T)}{\lambda^2(O)}\,\widetilde{\Psi} = 0 \quad (27)$$

for the deviation $\widetilde{\Psi}$ (r) leads to

$$\widetilde{\Psi} \sim \frac{e^{-r/d}}{r} \quad (28)$$

Thus the perturbation dies away exponentially, with the characteristic length

$$\mathrm{d}\;=\left[\frac{\pi n^*\hbar^2}{m^*H_e^2\,(T)}\right]^{1/2}\frac{\lambda(0)}{\lambda(T)} \sim \frac{\xi_0}{[1-T/T_c]^{1/2}} \quad (29)$$

Where the last estimate uses the microscopic theory to relate Ho and ξ_0 One see that even though the relation between $\overline{J_s}$ and $\bar{A}$ is approximated by a local expression, the Ginsburg

Landau theory definitely includes nonlocal effects and the co-horence length appears in a natural way.

Gor'kov[6] has given a derivation of the Ginsburg-Landau theory starting from the microscopic theory. He finds the GL wave function Ψ is proportional to the local value of the energy. Gap parameter

The GL theory is particularly useful in calculation whom one cannot treat the magnetic field by perturbation theory. Typical examples of such situations include thin films in strong magnetic fields, N- S phase boundaries, the intermediate state, etc. One can give a simple derivation of flux quantization on the basis of the current equation (25) and one finds the flux quantum to be hc/e*. The experimentally observed value hc/2e leads to the value e* = 2e, as mentioned above. The GL theory has played an important role in explaining the magnetic behavior of So called "hard" superconductors, which are particularly interesting materials, due to their high critical fields ($\sim 10^5\ gauss$). Fundamental theoretical work in this area is due to Abrikosov, who established the vortex picture to account for this magnetic behavior. Each vortex carries one quantum of flux.

Unfortunately, the original Ginsburg-Landau theory is restricted to the temperature range $(T_c-T)/T_c << 1$. Although it has recently been extended to all temperatures suitable conditions by Wertheimer and by Tewordt.

4. Conclusion :

For a large system, whether the physical system has an even or an odd total number of electrons makes no difference in its macroscopic properties; thus the wave functions above apply for any N. The situation is differently different for pairing correlations in atomic nuclei, where these differences lead to the well-known even-odd effects.

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