Study of Boundary Between a Superconductor and an Insulator

Shambhoo Nath Prabhakar
Vill- Andhar, Po- Makhdumpur, Dist.- Gaya (Bihar)
Pin- 824234

Abstract :

The coherence of type I superconductor exceeds their penetration depth ($\xi > \lambda$) so it is not energically favourable for boundaries to from between their normal and superconducting phase. The superconducting element with the exception of Niobium, are all type I. When the penetration depth λ is larger than the coherence length ξ, is becomes energetically favourable for domain walls to form between the superconducting and normal regions. When such a superconductor, called type, II is in a magnetic field, the free energy can be lowered by causing domain of normal material containing trapped flux to form with low energy boundaries created between the normal core and the surrounding superconducting material.

Key words : superconductors boundary condition, insulators critical temperatures, vortex, core.

1. Introduction

We have a differential equation which will give us the variation of Ψwithin any specimen once we know the vector potential A. As always, we must supplement the differential equation with appropriate boundary conditions for Ψ at the surface of the specimen. Ginzburg and Landau argued as follows. The complete expression for the variation δG of G when Ψ^* varies by $\delta\Psi^*$ comparises first the volume ingegral of the lef-hand side of equation (1), multiplied by Ψ , and secondly the surface integral

$$I_s = \int \delta\Psi^* \frac{\partial g}{\partial(\nabla\Psi^*)}. \text{ndS} \qquad (1)$$

where n is the normal to the surface. In the usual formulation of the calculus of variations, this term is neglected because the variation is carried out with the subsidiary condition $\Psi^*= 0$, and therefore $\delta\Psi^* = 0,$ at the surface. For superconductors, however, we cannot simply impose the condition $\Psi^* = 0$ at a surface. Indeed, such a condition would be wrong: for example, it would mean that the critical temperature of a thin film oscillated as a function of its thickness, since a standing-wave condition would be involved. In fact the critical temperature does not oscillate, and is generally independent of film thickness. Ginzburg and Landau therefore took the boundary condition.

$$n.\frac{\partial g}{\partial(\nabla\Psi^*)} = 0 \qquad (2)$$

which of course also makes Is vanish. Written out explicitly, the boundary conditions reads.

$$n. (-ih\nabla - 2eA)\ \Psi = 0. \qquad (3)$$

2. Discussion

The derivation from the microscopic theory shows that this conditions is correct for the boundary between a superconductor and an insulator. If a normal metal adjoins the superconductor; the wave function penetrates some distance into the normal metal, and because of this proximity effect the boundary condition at a superconduting normal interface differs from equation (3). We not finally that equation (3) does give a critical temperature-independent of thickness for a thin film. In the absence of a magnetic field, the condition is simply that the slope of kJ/ is zero at the surface, which means that ill is constant across the film at the critical temperature. Consequently, the critical temperature in unaffected by the thickness.

We must now minimise the free energy G with respect to variations of the vector potential A. The appropriate Euler-Lagrange equation is

$$\frac{\partial g}{\partial A_i} - \sum_i \frac{\partial_j}{\partial x_j} \frac{\partial g}{\partial(\partial A_j/\partial x_j} = 0 \qquad (4)$$

where A_j is the component of A in the ith direction. The second term in this equation gives $(1/\mu_{00})$ curl curl A or $(1/\mu_0)$ curl B, which from Maxwell's equation is J_e. With this replacement. (4) bcomes.

$$J_c = -\frac{ie\hbar}{m}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{4e^2}{m}\Psi^*\Psi^A \qquad (5)$$

that is, the standard expression for a quantum-mechanical current. The fact that we get the standard expression is not surprising: A enters the free energy only in the gradient term and in the field-energy term $B^2/2\mu_0$, and that is exactly where we shuold find it in a quantum-mechancial Hamiltonian. Except that we are now using the mass m rather than 2m, which means only a change in the normalisation of Ψ, equation (5) is therefore identical to our earlier equation, which we wrote down on plausibility grounds from the assumption that a superconductor is characterised by a macroscopic wave function equation (5) is a local expression, of the London type J_c (r) is given by the values v and A at the point r. The fact that we have a local theory implies a severe restriction on the temperature range in which the theory is valid if we are dealing with a pure superconductor. For an alloy, on the other hand, the temperature range need not be restricted by the local nature of the theory.

Note that, as we stated, we should have got the same expression for the current if we had used the Helmholtz density. The two differe by the term H_0 B = H_0. curlA if we put this into the second term of equation (4) we get curl H_o, which is zero inside the superconductor.

We have now developed the full framework of the GL theory: the Gibbs free energy, Ψ, with the boundary condition of equation (3); and finally (5) for the current. The temperature dependence of the parameters α and β remains that of the Landau theory

$$\alpha = -A(T_c - T) \qquad (6)$$

$$\beta = \text{constants} \qquad (7)$$

Our first task is to derive expressions for the penetration depth, and for the critical field H_{cb} of a bulk type I superconductor. If the dimensions of the specimen are much greater than λ , then we have B=0 inside the specimen. We then have Ψ varied the gradient term would mean that the free energy increased. The constant value of Ψ is given from literature.

$$\left|\Psi\right|^2 = \frac{\alpha}{\beta} = \left|\alpha\right|/\beta \qquad (8)$$

(recall that α is negative), which of course is the ordinary Landau theory Since $\nabla_\Psi = 0$ (2.23) the current (5) is simply given by the London equation

$$Jc = -\frac{4e^2}{m}\frac{\left|\alpha\right|}{\beta}\text{A}. \qquad (9)$$

this gives for the penetration depth

$$\lambda\left(\frac{m\beta}{4e^2\mu_0\left|\alpha\right|}\right)^{1/2} \qquad (10)$$

Equation (6) and (7) lead to an explicit form for the temperature dependence of λ.

$$\lambda \infty\ (1-t)^{-1/2} \qquad (11)$$

We saw earlier that the Gorter-Casimir temperature dependence $\lambda \infty\ (1-t)^{-1/2}$ fits experimental data at all temperatures. Near to Tc, which is where the Landau theory holds, the two forms of temperature dependence are in agreement. In fact, $(1\text{-}t^4)^{-1/2} = (1\text{+}t^2)^{-1.2}\ (1+t)^{-1.2}\ (1-t)^{-1.2}$ and for t near 1 the first two terms are slowly varying, so that the dependence on t is dominated by the singularity given by the last term.

H_{cb} is the field at the which the Gibbs free energies Gs and Gn in the superconducting and normal phase are equal. B is zero in the superconducting phase, and Ψ, is zero in the normal phase. With

$|\Psi^2| = |\alpha|/\beta,$ the equation (15) therefore gives for the energies at applied field H_0.

Gs = $V\left(f_n - |\alpha|^2/2\beta + \frac{1}{1}\mu_0 H_0^2\right)$ (12)

$G_n = Vf_n$ (13)

Where V is the volume of the specimen. The critical field is given by $G_s = G_n$

$H_{cb}^2 = |\alpha|^2/\mu_0\beta$ (14)

which is of coures the usual result that at the critical field the flux-exclusion energy is equal to the condensation energy. Equation (14) gives the temperature dependence $H_{cb}\alpha(1-t)$, which again for t near l is in agreement with the Gorter-Casimir form1-t^2, and therefore in agreement with the experimental results. As Ginzburg and Landau remarked, this is strong support for the postulated temperature dependences of α and β, equations (6) and (7).

The phase change at H_{cb} is first order, that is, there is a latent heat associated with the phase change. Within the Landau theory, the nature of a given phase change is connected with the behaviour of the order parameter Ψ at the phase boundary. In this present case, Ψ retains the constant value $(|\alpha|/\beta)^{1/2}$ upto H_{cb}, and then drops discontinuously to zero. The discontinuous change of Ψ means that there is a finite difference in the degree of order between the two phases. This implies an entropy different $\nabla\sum and$ an associated latent heat $T\nabla\sum$. Since the superconducting phase is more ordered, it has the lower entropy, so if the experiments is done isothermally heat is absorbed from the surroundings in the passage from the superconducting to the normal phase. An expression for the latent heat can be obtained from equation

3. Conclusion :

From the above investigation on type II superconductivity we can draw the following conclusion.

I. Tinkham hingh-Kappa approximation works very well in the evaluation of flux in the vortex core.

II. Ginzburg- Landau Phenomenological theory works quite well in explaining the various properties of type II superconductors.

References

1. V. C. Ginzburg and L.D. Landau zh. Eksp. Teor. Fiz. 20, 1064 (1950).
2. L.D. Landau, Zh. Eksp. Teor. Fiz 34, 1240 (1958).
3. P. G. de Gennes Superconductivity of Metals and alloys (New York: Benjamin 1966).
4. T. E. Faber, Prov. R. Soc. A. 214, 392 (1952).
5. T.E. Faber, Prov. R. Soc. A 231, 353 (1955).
6. T.E. Fabber, Proc. R. Soc. A241, 531 (1957).
7. J. Faber and D. S. Melachlen Phys. Rev. 177, 763 (1969).
8. D. Saint-James and P G de Gennes, Phys. Lett. 7, 306 (1963).
9. J. Feber and D.S. Malachlean, Phys. Rev 140, A1638 (1965).
10. L.J. Campbell, J. Low Temp. Phys. JLTP 8, 105 (1972).