Effect of magnetic field on two phase gas particle flow

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Abstract

In this paper, the free jet expansion of a gas-particle mixture is numerically simulated by the piecewise linear method for the gas phase and so called Lagrangian discrete Burtle method to calculate particle motion throughout flow field. The effect of the particle size, the particle loading and the ambient effective pressure on the jet boundary is also presented. In the case, the larger particles, the particle relaxation length is larger than the flow field the shape and structure of the barrel shock and Mach disc almost unaffected by the particles. In case of smaller particles, the barrel shock becomes more dispersed by increasing the loading ratio and the Mach disc is shifted downstream, whereby its strength is reduced.

Keywords: Shock Waves, Magnetic Field, Mach Disc, Gas-dynamics.

1. Introduction:

High speed gas particle free jets and supersonic flow found in space and combustion technology. The interest in the gas-dynamics behavior of gas-particle supersonic grew in the past there due to its application to many engineering problems. Some typical examples are metallized propellants in rocket, jet type dust collectors, blast wave propagation in a dusty atmosphere and the study of drag laws of multiple particle system. In addition mixture of gases heavily laden with particles occurs frequently in industrial process such as particles manufacturing, flour milling, coal-dust converging, powder metallurgy and powdered – food processing.

The numerical simulation particulates two phase flows may be done in two different ways concerning the particles. One is the Eulerian approach where the particles should be rather homogenously distributed throughout the flow field in order to avoid dissipation of particle discontinuities which for examples exist at the particle free jet boundary. The other method is Lagrangian treatment [2-5] of particles, which are followed through the flow field on Lagrangian trajectories. Each computational particles called parcel, represents a number of real particles having the same properties as there are the particle size, the particle velocity and temperature. This method which already was successfully applied to diesel engine spray computation [3-5] has the great advantage that particle boundaries are not diffused. The gas phase equations are solved by piecewise linear method [6], which was modified to account for the interaction with the particles [1-2] and has the advantage of representing shocks and discontinuities rather sharp and having a high resolution.

2. Gas-particle flow-governing equations

The flow governing equations for the gas and the particle flow are formulated by neglecting the volume fraction of the particles and the contribution to the effective pressure of the mixture, which is valid for the considered loading of the flow by solid glan particles. The particle motion is essentially governed by the drag force due to effective pressure gradient in the flow the virtual mean force, the Basselt force, the gravity force and the particle lift force occurring in shear flows are much smaller for the present problem. The heat transfer between the gas and the particles is taken into account with the assumption that the temperature distribution within the particles in homogeneous. The gas phase equations are formulated for a unsteady compressible and inviscid flow in conservation form, where standard form the

$$U_t + F(U)_x + G(U)_r = IG \tag{1}$$

Where

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}, F(U) = \begin{bmatrix} \rho v \\ \rho u^2 + p^* \\ \rho uv \\ u(\rho E + p^*) \end{bmatrix}, \quad G(U) = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p^* \\ v(\rho E + p^*) \end{bmatrix}$$

$$IG = \begin{bmatrix} -j\rho v/r \\ -j\rho uv/r - F_{p_x} \\ -j\rho v^2/r - F_{p_r} \\ -jv(\rho E + p^*)/r - U_p F_{p_x} - v_p F_{p_r} - Q_p \end{bmatrix}$$

For j=0 in two- dimensional case

and j=1 in axis symmetrical case

$$E = \frac{p^*}{\rho(\gamma - 1)} + 0.5(u^2 + v^2) \quad and \qquad p^* = \rho RT$$

Where $p^* = p + \frac{1}{2}H^2$ in which p is fluid pressure and $\frac{H^2}{2}$ is magnetic pressure.

The interaction with the particles is defined in terms of the ship values, the drag coefficient and the Nusselt number and F_{p_x},F_{p_r} and Q_p are obtained by summaries over all parcels be contained in the volume $V_{i,j}$ $(Fig2)$.

$$F_{p_x} = \frac{\pi}{8V_{i,j}} \rho \sum_k \left(\mathrm{D}^2{}_k N_k (u - u_k) |u - u_k| C_{D_k} \right) \tag{2}$$

$$F_{p_r} = \frac{\pi}{8V_{i,j}} \rho \sum_k \left(\mathrm{D}^2{}_k N_k (v - v_k) |v - v_k| C_{D_k} \right) \tag{3}$$

$$Q_p = \frac{\pi}{V_{i,j}} \frac{\mu C_p}{p_r} \sum_k \left(Nu_k N_k \mathrm{D}_k (T - T_k) \right) \tag{4}$$

The velocities and temperature of the gas phase are assumed to be constant within the volume $V_{i,j}$.

The particle drag coefficient is calculated by the standard drag function and the Nusselt number for heat convection and conduction is used.

$$C_{D_k} = \frac{24.0}{R_{e_k}} \left(1.0 + \frac{1}{6} R_{e_k}^{1/2} \right) \qquad \text{if } R_e < 1000 \tag{5}$$

$$C_{D_k} = 0.44 \qquad \text{if } R_e > 1000$$

$$N_{u_k} = 2 + 0.6 R_{e_k}^{1/2} - p_r^{1/3} \qquad \left(p_r = 0.75 \right) \tag{6}$$

In case , the ambient effective pressure is rather low, there may exist regions in the flow field where the mean free path of the gas molecules reaches values which are of the order of the particles diameter or even substantially larger. This result is reduced momentum and heat transfer and the relation (5) and(6) which are the valid only in the contium region have to modified accounting for rarefaction effects. The corrections are defined in terms of the particle Knudsen number.

$$K_n = \frac{\Lambda}{D_p} = \sqrt{\frac{\gamma \pi}{2}} \frac{Ma}{R_e}$$

where Λ is the mean free path and D_p is the particle diameter. Throughout the present numerical result the empirical expressions given by the Carlson and Hoagland are used

$$C_D = C_D \left\{ 1 + \frac{M_a}{R_e} \left(3.28 + 1.28 exp \left(-1.25 \frac{R_e}{M_a} \right) \right) \right\}^{-1}$$

$$N_u = N_{uEqn.6} \left(1 + 3.42 \frac{M_a}{R_e p_r} N_{uEqn.6} \right)^{-1}$$

In the case of large particle slip resulting in an increasing particle Mach number, we have to account for compressibility effects whereby the drag is again increased

$$C_D = C_{DEqn.5} \left(1 + e^{\left(-0.427 / {M_a}^{4.63} \right) - \left(3.0 / {R_e}^{0.88} \right)} \right)$$

The Reynolds number and Mach number are defined as following in terms of particles diameter, particle slip velocity and the local speed of sound

$$R_{e_k} = \frac{\rho D_k |\vec{\bar{u}} - \vec{u}_k|}{\mu} \quad , \quad M_{a_k} = \frac{|\vec{\bar{u}} - \vec{u}_k|}{a} \tag{7}$$

Where μ is the function of T (valid for air)

$$\mu = 17.17 \square 10^{-6} (T/273)^{0.77}\ Ns/m^2 \tag{8}$$

For the particles a Lagrangian description of their motion through the flow field is employed. Each computational particle, which we call parcel for convenience, represents a number of real particles and is labeled by subscript k

$$\frac{dx_k}{dt} = u_k, \qquad \frac{dr_k}{dt} = v_k \tag{9}$$

$$\frac{du_k}{dt} = \frac{3}{4}\frac{\rho}{\rho_k D_k} C_{D_k}(u - u_k)|u - u_k| \tag{10}$$

$$\frac{dv_k}{dt} = \frac{3}{4}\frac{\rho}{\rho_k D_k} C_{D_k}(v - v_k)|v - v_k| \tag{11}$$

$$\frac{dT_k}{dt} = 6\mu \frac{C_\rho}{C} \frac{1}{D_k{}^2 \rho_k} \frac{Nu_k}{\rho_r}(T - T_k) \tag{12}$$

3. Numerical Procedure

The structure of a stationary axis-symmetrical free jet expanding through a small arifice into a gliescent region of flow effective pressure is calculated by a time marching method in order to obtain the asymptotic steady solution. A converged solution of a pure gas jet is used as initial condition and the calculation is proceeded by introducing the particles at arifice for the gas phase and the particles. Sonic conditions are assumed to exist $\left(M^* = \frac{u^*}{a^*} = 1\right)$ at the arifice for the gas phase and the particles. Under such condition the gas density and effective pressure are obtained from the stagnation values using isentropic relations.

The particle flow rate is given according to the loading ratio $\eta = \frac{m_p}{m_g}$. The number of inflowing particles at the arifice additionally depends on the gas velocity, the time step size and the area of the grid. Since the total number of parcels injected per time step and the number of parcels in one grid is fixed, the number of particles represented by one parcel is found from

$$N_k = \frac{6}{\pi} \frac{\eta \rho^* u^* \Delta A_j}{N_{p_j} \rho_k D_k^3} \Delta t \tag{13}$$

The starting locations of the parcels are statistically sampled using random number generators. At the boundaries (2) *and* (5) we apply reflection conditions, at (3) ambient conditions $\left(u = 0, v = 0, p = p^*_{\infty,} \rho = \rho_\infty\right)$ and at (4) outflow conditions. A particle crossing the lower boundary (5) is reflected elastically.

The gas-phase equations are solved using the piecewise-linear method proposed by Colella which was modified to take into account the in homogeneous terms describing the momentums and energy exchange with the particles[12]. These terms (Equations $2-4$) are obtained by summarizing over all the parcels contained in the volume $V_{i,j}$. The final steady state solution is obtained by employing the operator splitting technique expressible in the form

$$W_r = \{L_x(\Delta t/2), L_r(\Delta t), \dots\dots\dots\dots\dots\dots, L_r(\Delta t), L_x(\Delta t/2)\ \}W_0 \tag{14}$$

The particle equations are solved simultaneously after each time step consisting of $L_x(\Delta t)$, $L_r(\Delta t)$. Prior to the particle calculation new parcels are added at the arifice. After introducing and at any later stage the parcels are moved according to their respective velocities and new parcels positions are found according to:

$$x_k^{n+1} = x_k^n + \Delta t\, u_k^n, \qquad r_k^{n+2} = r_k^n + \Delta t\, v_k^n \tag{15}$$

Parcels leaving the computational domain are removed from the calculation and the parcel array is replaced.

The new parcel velocities and temperature are calculated.

$$u_k^{n+1} = u_k^n + \Delta t \frac{3}{4}\frac{\rho^n}{\rho_k D_k} C_{D_k}^n (u^n - u_k^n)|u^n - u_k^n| \tag{16}$$

$$v_k^{n+1} = v_k^n + \Delta t \frac{3}{4}\frac{\rho^n}{\rho_k D_k} C_{D_k}^n (v^n - v_k^n)|v^n - v_k^n| \tag{17}$$

$$T_k^{n+1} = T_k^n + \Delta t\ 6\mu^n \frac{C_p}{C}\frac{1}{\rho_k D_k^2}\frac{Nu_k^n}{p_r}(T^n - T_k^n) \tag{18}$$

The gas phase properties ρ^n, u^n, v^n and $T^n$are linearly interpolated from the grid point values associated with the parcel location.

4. Numerical Results and Discussion

The most important properties influencing the particle motion in magnetic field and the gas flow of the gas particle free jet are the particle diameter, the particle material density, the particle mass loading and the ambient effective pressure. The effect of three properties on the free jet structure and configuration of embedded shock waves and the shape of free jet are discussed with the help of same numerical results. If no other values and parameters are given the correction for rarefaction and compressibility effect are included in the calculation and the effective pressure ratio is $\rho_0^*/\rho_{00}^* = 20$, the stagnation effective pressure $\rho_0^* = 20\square 10^5 \rho_a$, the particle material density $\rho_\rho = 2500\ kg/m^3$ and the ratio of specific heat δ=0.763.

From the conservation equation for the gas phase it may be seen, that an increasing particle loading results in a strong coupling between particles and gas and hence the gas flow will be influenced more and more. How the flow field will be changed largely depends on the particle size.

In the case larger particles are added to the flow the particle jet will remains almost straight since the particle relaxation length is much larger as the considered flow field. Therefore the particles only may influence the structure and shape of the Mach disc which becomes only straight more airved compared to the pure gas case. The location of the Mach disc at jet axis remains nearly constant with increasing particle loading given in Figure 1,2.

The shock strength, however, will be considerably reduced shown in Figure.1,3. Smaller particles where the relaxation length is in the order of the jet size, will also pass through the barrel shock and influence its structure and shape. In this case the barrel shock will become more and more dispersed by increasing the particle loading. The Mach disc is shifted downstream in Figure 1,2, decreases in size and finally disappear where by the barrel shock interact with the jet axis. The shock strength at the jet axis is reduced to smaller values than in the case of larger particle (Figure 3).

If we continuously decreases the particle size the Mach disc moves from the location for the pure gas case further downstream away from the arifice. Even for the smaller particles introduced in the computations an equilibrium location was not reached because the particle kundich number increases resulting in a reduced drag (Figure 4). The particle jet for very large particles to alter in a given by the gas stream line.

When lower stagnation effective pressure are employed Kurdien effects are more pronounced and due to reduced drag the particle jet expansion angle only slightly increases with decreasing particle size (Figure 5) and the location of the Mach disc does not change considerably.

5. Conclusion

The numerical computational of particulate two-phase supersonic free jet and the structure and configuration of the embedded shock waves.The influence of variation in the particle mass loading, magnetic field intensity, the particle diameter and the ambient effective pressure was discussed in detail.

The numerical method used for the computation of the particle motion throughout the flow field, called Lagrangian discrete particle method, was found to be a powerful method, as soon as particle discontinuities present in the flow field. Furthermore this method has the advantage, that additional force influencing the particle motion e.g. the interaction of the particle with the solid walls, particle-particle collisions and particle left forces may be easily traced. Since individual particles are followed throughout the flow field a poly disperse particle phase represented by this method without additional effort.

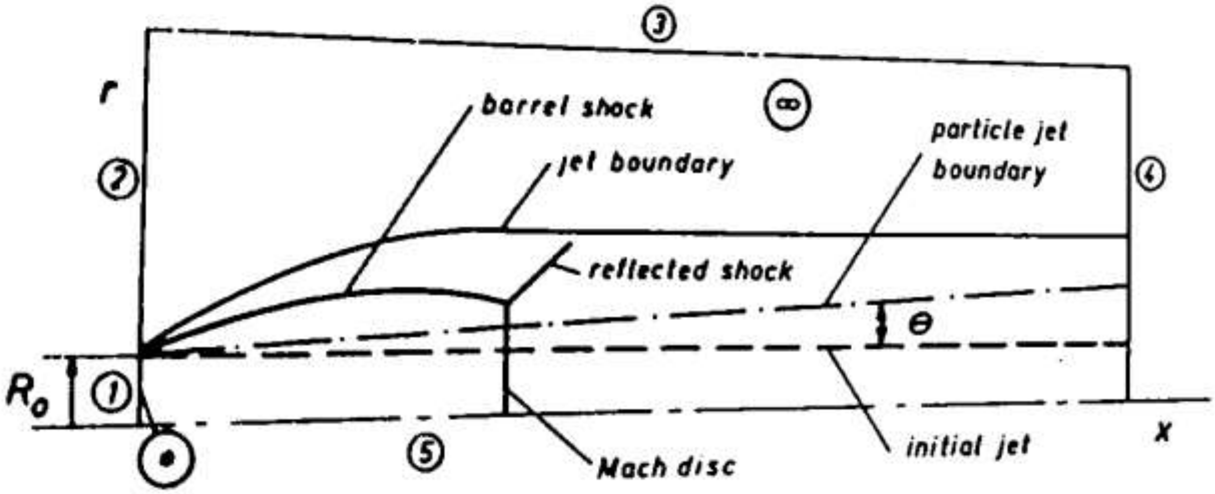


Fig. 1: Free Jet Boundary conditions

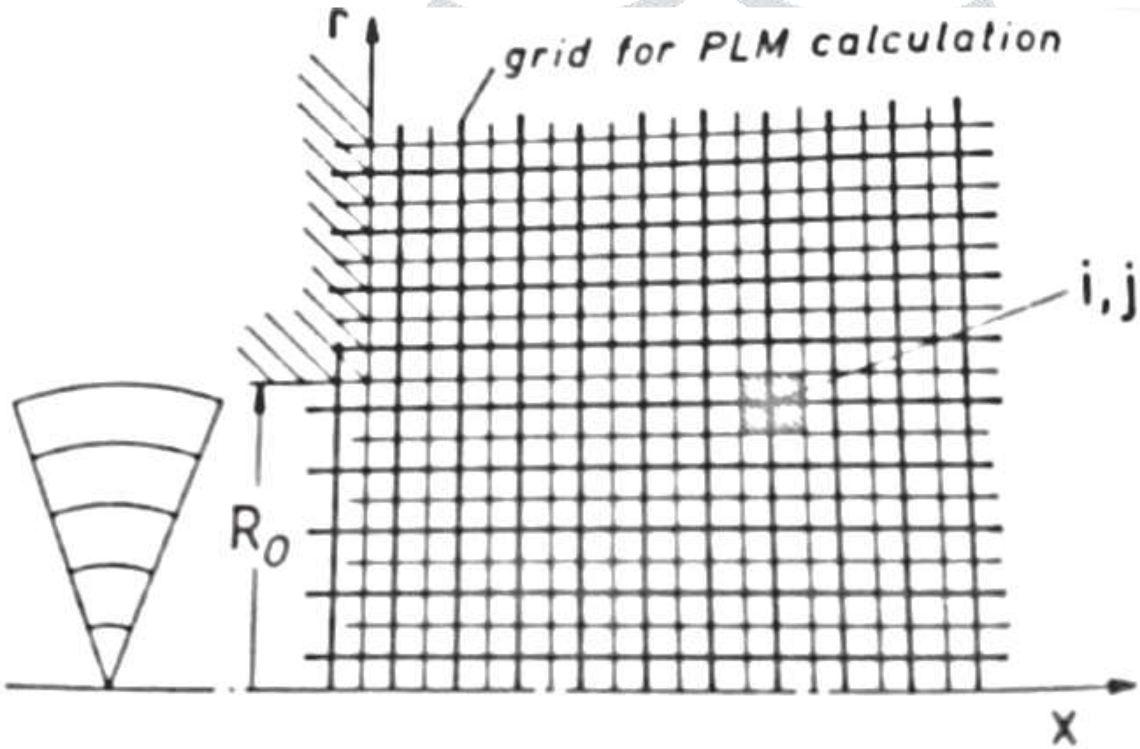
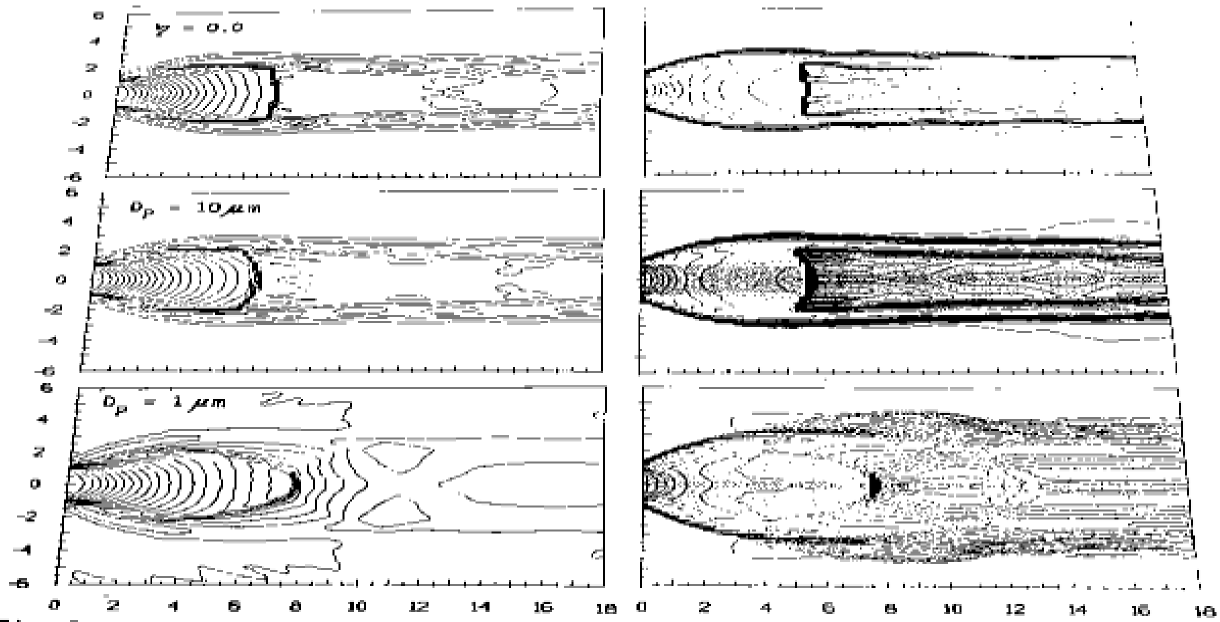


Fig. 2 : Grid Configuration



<u>Fig. 3</u>: Free jet structure (left column: gas density contour lines: right column gas velocity contour lines and parcel location, represented by small dots.

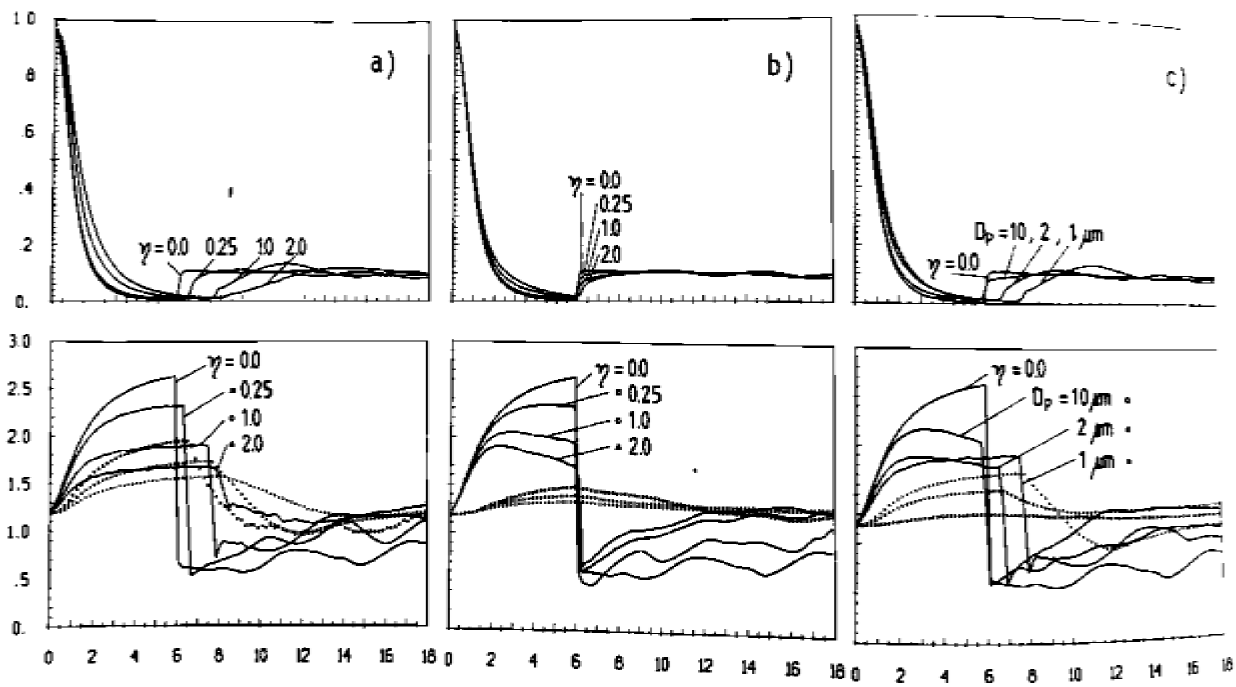
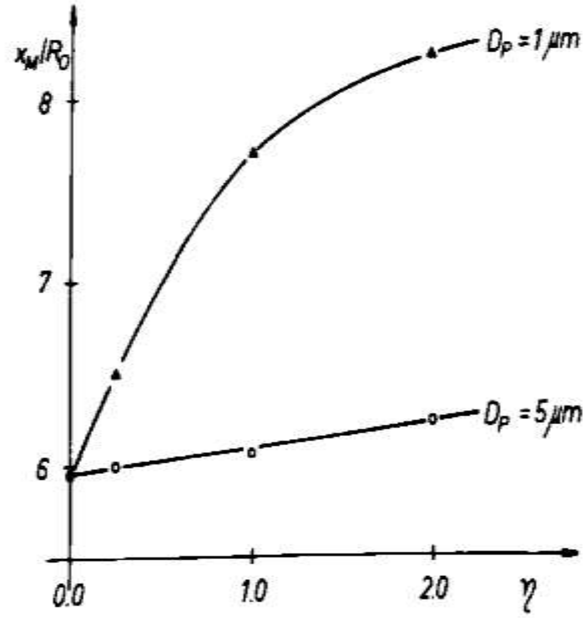


Fig. 4: Pressure and velocity distribution along the jet axis a) $D_p = 1$ μm b) $D_p = 5$ μm c) y=1.0



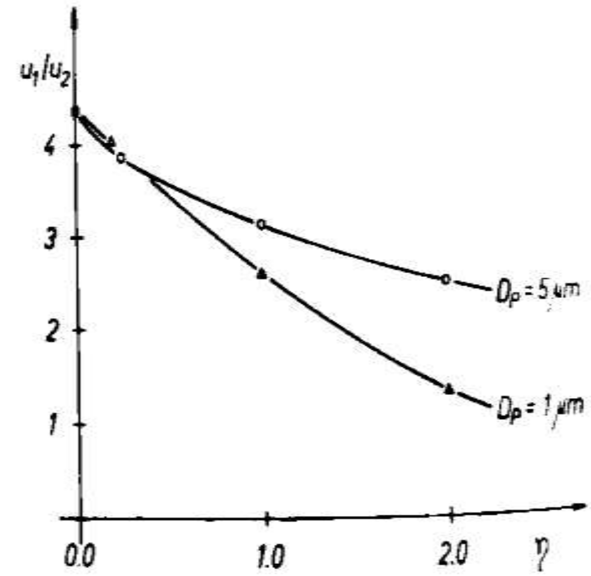
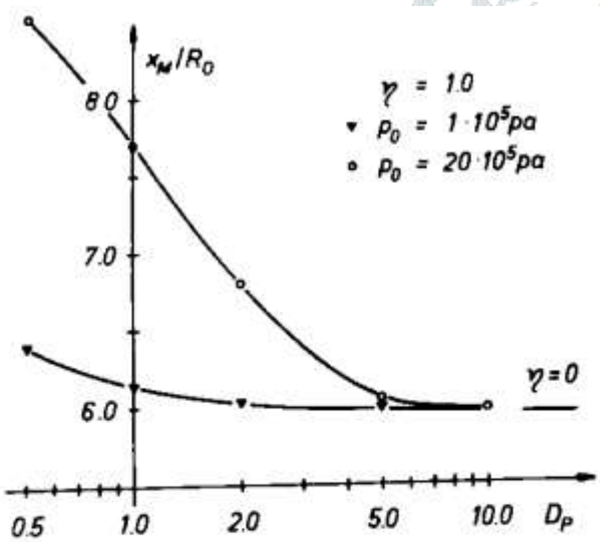


Fig. 5 : **Location of the Mach disc at the jet axis**

Fig. 6: Change of gas velocity across the Mach disc



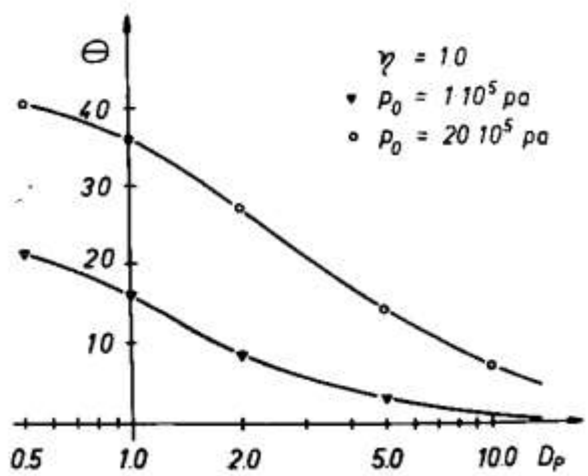


Fig. 7: Location of the Mach disc

Fig. 8: Particle jet angle

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