CONTRIBUTION TO THE DUALITY THEORY OF TENSOR PRODUCTS OF TWO METRIZABLE LOCALLY CONVEX SPACES IS EQUAL TO PRODUCT OF THEIR TOPOLOGICAL DUAL SPACES.

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Abstract :- The Basic and authentic study of a locally convex space in terms of its dual is the central part of the modern theory of topological vector spaces, for it provides the setting for the deepest and most beautiful results of the subject. Various authors have mentioned In this paper we have proved that the dual space of the ε – tensor product of the two given metrizable locally convex spaces is equal to the ε – tensor product of their topological dual spaces . Duality theory of ε – tensor product of two metrizable locally convex spaces considered about topological dual space of tensor products. objective of this paper is For all our purposes, topological vector spaces are locally convex, in the sense of having a basis at consisting of convex opens. We prove below that a separating family of seminorms produces a locally convex topology. Conversely, every locally convex topology is given by separating families of semi-norms: the seminorms are functionals associated to a local basis of balanced, convex opens. Giving the topology on a locally convex V by a family of seminorms exhibits V as a dense subspace of a projective limit of Banach spaces, with the subspace topology. This chapter presents the most basic results on topological vector spaces. With the exception of the last section, the scalar field over which vector spaces are defined can be an arbitrary.

(**Keywords** :- ε – tensor product, Metrizable, Locally, convex spaces, Topological dual spaces)

Introduction :- In the same chapter I presents the most basic results on topological vector spaces. With the exception of the scalar field over which vector spaces are defined can be an arbitrary with the uniformity derived from its absolute value. The purpose of this generality is to clearly identify those properties of the commonly used real and complex number that are essential for these basic results. The description of vector space topologies in terms of neighborhood bases of a the uniformity associated with such a topology. Constructing new topological vector spaces from given ones.The standard tools used in working with spaces of finite dimension are collected, which is followed by a brief discussion of affine sub spaces the extremely important notion of boundedness. Metrizability is treated although not overly important for the general theory, deserves special attention for several reasons among them are its connection with category, role in applications in analysis, and its role in the history of the subject. By the Herms [1] , Kelly [2] , Komura [3] , Kaplan [4] , Loventz [5] and Nakaro [6] duaity theory of dual space of the ε – tensor product of the given metrizable locally convex spaces is equal to the ε – tensor product of their topological dual spaces. We have proved about duality theory of ε – tensor product of two metrizable locally convex spaces. In this Connection we have considered about topological dual space of tensor products, polar of sets, field of scalars, subsets of tensor product spaces with topological dual spaces.

NOTATION :-

Let P and Q be locally convex spaces. Then by P' and Q' we denote the topological duals of P and Q respectively. Let U be a set in P. Then by U^0 we denote the polar of U with $U^0 \subset P'$. by $P \otimes \varepsilon Q$ we denote the ε -tensor product of P and Q. We denote by K the filled of scalars. Let U be a set in P and let V be a set in Q. Then by $(U^0 \times V)^{00}$ we denote a set in $P \otimes Q = B(P'\sigma\ Q'\sigma)$ We denote by $(U^0 \times V^0)^{00}$ a set in$(P \otimes Q)'$. We denote by $(U^{00} \times V^{00})^0$ a set in $P' \otimes Q'$..

DEFINITION : - I

Let P and Q be two locally convex spaces and let U, V be two finite sets in P and Q such that U^0, V^0 are equicontinuous subsets of P' and Q' respectivly. Then the ε -topology on $P \otimes Q$ is the topology of uniform convergence defined on the product $U^0 \times V^0$ of P' × Q' such that the space foremd in this way is $P \otimes \varepsilon Q$ called the ε -tensor product of P and Q.

DEFINITION :-II

Let P be a locally convex space such that P has a countable fundamental system of zero neighbourhoods in P. Then P is said to be metrizable.

DEFINTION :- III

Let P be a metric locally convex space with x^m , $x^n \in P$. Let there exist a finite positive number no(ε) for each ε > 0 such that m, n ≥ no(ε) = x^m , $x^n < \varepsilon$

Thus, { x^n } n =1, 2 .... is a cauchy sequence.

DEFINITION :- IV

Let the exist each cauchy sequence in a metric locally covex sapce P such that this sequence has a limit in the space of real numbers. Thus P is said to be complete.

DEFINITION :- V

Let P ans Q be two metric locally convex spaces. Let Un and Vn be finite sets in P and Q respectively such that and are equicontinuous subsets of P' and Q' respectively. Then $(U^0_n \times V^0_n)^{00}$ are subsets of P⊗ ε Q and $(U^{00}_n \times V^{00}_n)^0$ are subsets of P' ⊗ ε Q'

We utilize the above notation and definitions along with theorems, propositions, corollaries, hypothesis etc. from some books given under references to solve the following unsolved problem as theorem.

Theorem :- Let there exists a relation (P⊗ ε Q)' = P⊗ ε Q' Then the locally convex spaces P and Q are complete metric spaces.

Proof :- It is given that (P⊗ ε Q)' = P⊗ ε Q' 1

P is a locally convex space Such that P has a fundamental system U *f* (P) of zero neighbourhoods Gn in P 2

Q is a locally convex space such that Q has a fundamental system of U *f* (Q) zero neighbourhoods Mn in Q. 3

From the concept of the ε -topology it is obvious that Gn are finite subsets such that Gn^0 are equicontinuous subsets of P' 4

In the sdame way Mn are finite subsets of M^0n are equicontinuous subsets of Q 5

It is obvious from the concept of (1) that G^{00}_n, M^{00}_n are equicontinuous subsets of P" and Q" respectively 6

From the concepts of (2) and (4) it follows that G^0_n are bounded subsets Of P'such that P' has countable fundamental system Bf(P') of bounded subsets 7

From the concepts of (3) and (5) it is obvious that M^0_n are bounded subsets of Q' such that Q' has a countable fundamental system B *f* (Q') of bounded subsets M^0_n 8

All Gn, Mn are finite such that G^{00}_n and M^{00}_n are countable 9

from the cocept of (6) and (9) it follows by hypothesis that each countable subset of P" and each Countable subset of Q" are equicontinuous. 10

From the concept of (10) it is obvious by hypothesis thaw P' and Q' are brrelled sapces. 11

From the concepts of (7) and (11) it is clear that P' is a dual metric locally convex space. 12

From the concepts of (8) and (11) it follows that Q' is a dual metric locally convex space 13

By hypothesis from (12) it clear that the locally convex space P is a metric space. 14

By hypothesis from (13) it is obvious that the locally convex space Q is a metric space. 15

On the basis of the concept of (2) there can exist a convergent sequence which converges to a point X_0 in P such that X_0 is a limit of this sequence with $[X_m, X_n] < \in$ Where there exists a number n(ε) for each ε >0 with m, n≥ n(ε) showing that every Cauchy sequence {Xn} in P has a limit X_0 in P 16

Similarly on the basis of the concept of (3) there can exist a convergent sequence{Yn} which converges to a point Y_0 inQ such that Y0 is a limit of this sequence with $[y_m, y_n] < \varepsilon$ Where there exists a positive finite number n(ε1) for each ε1>0 with m, n≥ n(ε1) showing that each cauchy sequence{Xn}has a limit $\{Y_0\}$ in Q. 17

From the concepts of (14) and (16) it follows by hypothesis that the locally convex space P is a complete metric space. 18

In the same way fromt the concepts of (15) and (17) it is obvious by hypothesis that the locally convex sapce Q is a complete metric space. 19

From (18) and (19) the theorem is completly proved.

. Corollary

subspace in (⊗n s (Πn j=1Ej), τs).

Indeed, the maps JE1,...,En and QE1,...,En are continuous

E1,...,En ◦ JE1,...,En = Id.

In the next theorem we prove that when one restricts an n-tensor topology τ to the space of symmetric tensors and th:- For every locally convex spaces E1, . . . , En, (⊗n j=1Ej , τes) is a complemented en extends it with the procedure given in the Theorem obtains an n-tensor topology finer than τ . Both topologies are the same when τ is symmetric

In our work We propose to consider problems of following types

1. The equality of ε– tensor product of two metrizable locally convex spaces can be equal to π – tensor product of said two dual metric locally convex spaces.
2. Two given locally convex spaces can be normable if the duality of ε – tensor products of the given first dual space and the second given locally convex space is equal to the π – tensor products of the first given bidual space and the second given dual locally convex space. The duality of ε – tensor product of two given dual nuclear dual metric spaces can be equal to the π – tensor product of the first given dual space and second given bidual space

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