SIMPLIFIED PROPOSITIONAL LOGIC AS GAP MANAGEMENT PLAN FOR GRADE 9 TO GRADE 10 OF JUNIOR HIGH SCHOOL

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Abstract : Logic plays an important role in carrying out sound decisions or reasonable judgment. Logic is one of the courses found in high school curriculum in which its principles are earlier taught in the Grade 8 Geometry class according to the K to 12 educational platforms. Its principles are very useful not just in mathematics but also in other fields like computer science and engineering. The importance of tracking learning competencies and assessing learning domains is exhibited in this study. There are several factors that affect the continuity of learning. The need to identify them is a must to prevent gaps. In this study the concepts of propositional logic was used to establish a relationship between the factors affecting the students' learnings in mathematical logic and their learning competencies and learning competencies and learning domains. Item analysis was used to process the data represented by test scores and survey results from Grade 9 to Grade 10 students.

"if-then" logic diagrams were developed to map the relationship between the diminishing factors, the learning competencies and the learning domains. General results show three top factors and a single learning competency that affects the three learning domains at most. Truth tables were created to derive the equations of the relationship.

Results of this study could be further utilized in developing gap management schemes guided by a proposed gap management paradigm.

Keywords: math logic, learning competency, learning domain, item analysis, if-then, truth tables, logic equations

I. THE PROBLEM AND ITS BACKGROUND

1.1 Introduction

Mathematical logic (or math logic) is a set of mathematical disciplines which aims to reduce formal logic to the rules of algebra. Math logic includes Boolean algebra, predicate calculus and preposition calculus. The main objective of math logic is to eliminate the unclearness by virtue of having more than one meaning of a word, a phrase or a sentence.

Mathematical logic is the application of mathematical techniques to logic. Logic has two aspects: formal and informal. Informal logic exists whenever we have a language. Formal logic is structured and associated with its origins in ancient Greece. Math logic has two sides: syntax and semantics. Syntax is how we say things while semantics is what we mean.

The subfields of mathematical logic include set theory, model theory, recursion theory and proof theory. These theories bear close connections to meta-mathematics and theoretical computer science (Hazewinkel, 2001).

1.2 Background of the Study

One area of mathematics that has its roots deep in philosophy is the study of logic. Logic is a way to improve critical thinking skills not just by looking at a problem, but also by studying the problem and implementing strategies to find a solution. It involves both inductive and deductive reasoning. Logic is about being non-contradictory, being rational, and being consistent. It is not related to personal beliefs. It is simply a means to stimulate the mind and apply our critical thinking skills. Logic may even caused a change in opinions and beliefs once rationally thought through a particular proposition.

Logic creates formal languages for reasoning and sets aside the reasoning and proofs to follow the rules of the new language. New derivations of proposed formula and new conclusions may occur, or the rules of logic can lead to derive the truth-value of a formula in some kind of logical arithmetic.

With these definitions, it seems that logic is for higher level learners. But in fact, logic is thought in the junior high-school level and amplified in the senior high-school. It has been a long struggle for pupils to comprehend the "if-then relationship" and the meaning of proof (Smith, 1940).In the past decades, technology has been developing rapidly. Information technology brings about a lot of computer generated applications which are basically operating using the concepts of logic. This rapid change has rendered current high school curricula unable to cope with the students' needs which result to lack of proper skills required in today's modern era (Bouhnik & Giat, 2009).

The cited scenario was in Israel way back 2009. In the Philippine setting, this still holds true as knowledge explosion demands careful discernment from students who are to choose from a variety of information. This requires good reasoning skills that would enable them to filter relevant from irrelevant information. Reasoning is related to students' study habits (Acido, 2010) and this aspect has to be considered in developing critical thinking skills among students through math logic.

Logic is part of the core subjects in general mathematics for senior high school according to the CHED K-12 Transition Program. This is in the latter part which is prone to the chance of not being touched or discussed thoroughly. With this, a problem may arise as students may have difficulties in comprehending the succeeding subjects which require the understanding of logic.

1.3 Conceptual Framework

In order to address this issue, this study has formulated a conceptual framework using the input-process-output-outcome (IPOO) approach as shown in Figure 1.1.

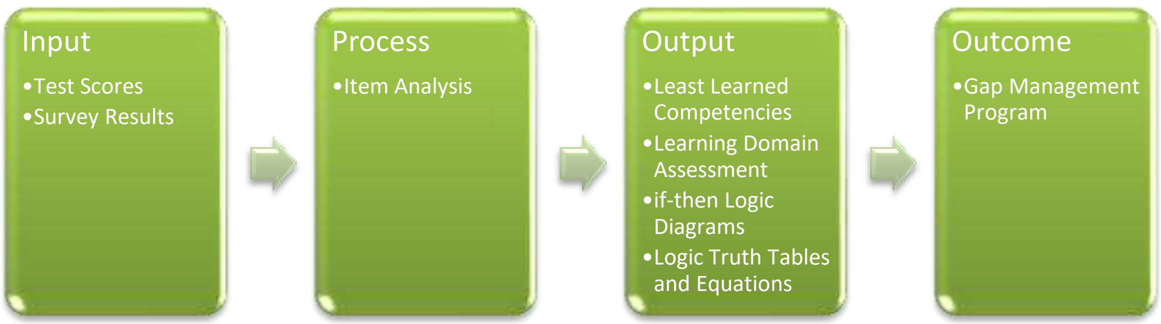

Figure 1.1: Conceptual Paradigm

The input block contains the test scores of the Grade 9 and Grade 10 students given with a test which includes the basic concepts of logic. These concepts are part of the learning competencies in the Grade 8 Geometry subject. Survey results are also part of the input as these will serve as the basis for developing logic diagrams, tables and equations. The process block contains item analysis. Item analysis was used to determine and understand the response of the students as they answer each test item. The output block contains the identified least learned competencies, leaning domain assessment and the if-then logic diagrams, tables and equations. These outputs generalize rules in determining the cause-and-effect behavior towards learning math logic. The results of this study could possibly lead to the development of a proposed gap management program which aims to address the problems identified.

1.4 Objectives of the Study

The general objective of this study is to establish a logical relationship between the factors affecting the students' learnings in mathematical logic and their learning competencies and learning domains.

Specifically, this study aims to

1. determine the least learned competencies and asses the learning domains of the junior high school students in mathematical logic;
2. develop an if-then logic diagram that maps the factors affecting the learning competencies and learning domains of the junior high school students in learning mathematical logic;
3. build a truth table and derived logic equations that corresponds to the developed if-then logic diagram.

1.5 Significance of the Study

This study finds its significance in the aspect of teaching mathematical logic to high school students. Once a weakness in learning logic is identified between grade levels, gap management plan can be formulated and implemented to amplify the importance learning logic and intensify its application through activities that requires critical thinking skills.

By increasing the awareness about the importance of logic in higher levels of learning and in professional applications in the field of business, science, technology and education, different programs may be developed to further enhance critical thinking skills and analysis leading to inductive or deductive reasoning. Decision making skills are logical in nature. Learning logic is an important aspect to understand the relationship between cause and effect. Leadership or managing positions or offices could benefit if the people placed in them are of sound mind and thinks logically.

1.6 Scope and Limitation of the Study

The outputs of this study will be based only on the available data gathered and presented. Nonetheless, the results are localized only to a specific area of math logic and specific levels in the junior high school. With the broad span of math logic, only the basic principles of logic are considered.

The respondents of this study are the Grade 9 and Grade 10 students of the San Pedro Relocation Center National High School in San Pedro, Laguna. Thirty students from each grade were considered. An examination covering propositional logic was used to determine the least learned competencies of the students. Each question targets a specific learning competency. An item analysis method was used to identify as to which competency has the least score. Learning competencies are defined in the K to 12 Curriculum Guide (K to 12 Mathematics Curriculum Guide, 2013).

The results of the test will be used to form a logic diagram to map the cause-and-effect behavior of the respondents in learning propositional logic. The construction of the diagram is in terms of logical rules basically, "if-then".

The gap management program in this study is a rough proposal and not implemented in the actual scene. M[illegible]r, it is more of a paradigm (see Appendix F). Nonetheless, this study could lead to the development of more gap manageme[illegible] grams to address the needs to increase the awareness in learning logic.

II. REVIEW OF RELATED LITERATURE

Literature plays an important role in establishing the argument of a study. Reviews of past studies are used as bases in proving the existence of unsolved problems and are given the opportunity to be dealt at present or in the near future.

This section discusses the reviews in which this study was founded.

Foreign Literature

2.1 Logic and Mathematical Logic

Logic represents the most general means of reasoning used by people and computers (Detlovs & Podnieks, 2011). This approach deals with reducing human language sentences to variables, constants, functions, predicates and quantifiers that appear to be flexible enough and much more uniform when compared to the variety of constructs used in the natural human languages. They are much easier to use for communication with computers. Mathematical logic comprises mathematical theories which include set theory, number theory and the theory of algebraic structures with the aim of developing tools to examine their consistency, completeness and other similar questions concerning the foundation of these theories (Srivastara, 2013).

The fusion of mathematical logic and the current technology is highlighted in computer science. Logic deals with compound statements formed from given statements by means of the connectives ‘not’, ‘and’, ‘or’, ‘if… then’ and a combination of these (Beardon, 2011). These connectives are undeniably used when building a computer program which is the building block of a computer application used by humans.

2.2 Learning Mathematical Logic

When the word “logic” is mentioned, immediately, what comes-in is the brain and how thoughts, ideas, perceptions, understanding and decisions come to life. The output of the mind may be questioned by another resulting to arguments and further discussions. Engaging in conjecturing and argumentation helps a student to develop an understanding of mathematical proof (Boero, et al., 1996).Attitudinal factors in mathematics are highly important in mathematics education because these variables are amenable to change by educational interventions. Furthermore, the mutual influence between mathematics self-concept and performance has to be addressed. These factors have the potential of being enhanced and modified by new and innovative curricular and instructional approaches to teaching and learning.

To increase mathematics performance, teachers should focus more on enhancing the quality of learning activities and students’ mathematics self-concept than on prioritizing peer or teacher–student relations and liking of mathematics. Students who have confidence and belief in their ability to control their engagement and learning activities achieve more (Winheller, et al., 2013).

Many students seem to concentrate on computations as the essence of mathematics (Kimball & Smith, 2013). Many believe that mathematical activity includes procedures that are divorced from real life, from discovery and from problem solving. The fact that mathematics is usually presented as a body of absolute truths which exists independently of the learners and taught in a hierarchical, linear and prescriptive fashion reinforces the view that mathematics is a difficult subject. There are mathematical concepts that do not involve numbers but are based on truth tables and the factual statements which is hard to comprehend especially if a student is trained to comprehend mathematics by the usual numerical and computational methods.

Many students tend to identify mathematics with arithmetic (Mutodi & Ngirande, 2014). Doing mathematics is normally associated with calculations. It is widely maintained in the literature that negative images and myths of mathematics are widespread among the students. Many students view mathematics as a difficult, cold and abstract subject. It is perceived by many students as an exclusive discipline.

Past experiences affect the perception and beliefs about mathematics (Aguilar, et al., 2016). The negative claims in taking mathematics are widespread among students especially in developed countries (Mtetwa & Garofalo, 1989) (Ernest, 1995) (Gadanidis, 2012). Many students are scared of mathematics and feel powerless when it comes to understanding mathematical ideas (Sam, 2002). With this, it takes time to convince students to learn mathematics and allows them to absorb the principles and concepts that they need to comprehend. This happens in both numerical and non-numerical mathematics.

Local Literature

2.3 Logic in the High School Curriculum

In the Philippine setting, the “K to 12” educational platform is now taking its place and currently affecting curricula and other educational settings. A portion of the K to 12 Curriculum Guide (K to 12 Mathematics Curriculum Guide, 2013) is shown in Appendix A. The concept of logic and reasoning is first introduced in the Grade 8 Geometry. It is scheduled during the latter part of the second quarter. Learning competencies 34 to 35 strongly deal with the concepts of “if-then” statements which are very much useful in computer technology.

In Appendix B, the table shows a part of the K to 12 Basic Education Curriculum for Senior High School – Core Subject (K to 12 Basic Education Curriculum: Senior High School Core Subject, 2013). Logic is then discussed in the 11th Grade. Looking at the 1st performance standard and learning competencies 41 to 45, these areas directly deal with propositions which are part of the scope of this study.

2.4 Gap Management Plans

Gap analysis and management are commonly used in business and industry firms. This is a process in which a company compares its actual performance to its expected performance to determine whether the target expectations are met and the resources are effectively utilized (Investopedia, 2016). Gap basically defines a difference between two similar entities. In terms of educational gap, this may refer to several things with different aspects.

A gap has to be established first before doing the management. A gap is resolved by creating a bridge in-between. A gap management model was proposed by (Heiman, 2010) to bridge the gap for students who are struggling in mathematics. The model is shown in Figure 2.1. Some of the inputs of the model includes math verbal, creating & solving one’s own math problems, mastery learning required for all sub-skills, special structures etc. Each input corresponds to a specific outcome and these outcomes all aims to give a math achievement impact.

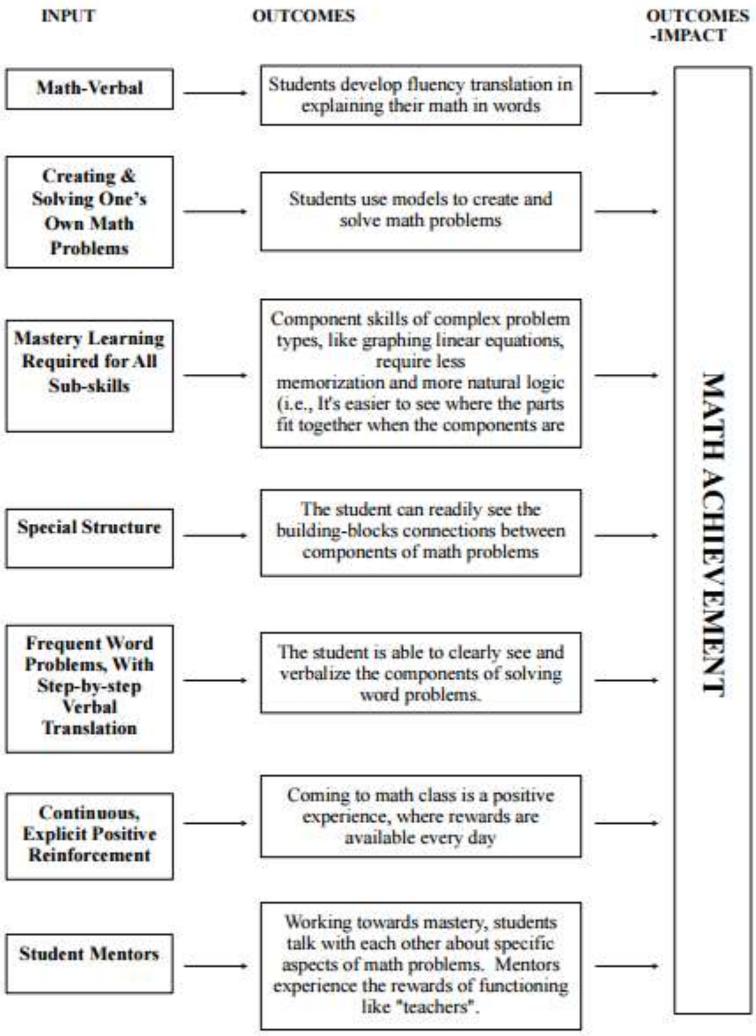

Figure 2.1: Sample Logic Model for Bridging Math Achievement Gap (Heiman, 2010)

Mathematical vulnerability may start in the early stages of learning. In the study made by (Gervasoni, et al., 2010), they explored the impact of bridging the numeracy gap project on the whole-number learning of Prep and Grade 1 students living in a low socio-economic status community. It was recommended that in order to resolve this gap, specialized classroom mathematics programs that caters the diverse range of students' learning needs have to be implemented by a specialist mathematics teacher in partnership with a classroom teacher who will design the individual learning plans.

2.5 Synthesis

Mathematical logic plays a vital role in learning other math subjects and other allied disciplines like computer science. There is an existing negative impression when students take math subjects the reason why this is reflected in their manner of learning math. This results to a gap.

Mathematics is a course that is discussed across majority of the levels in an educational system. Thus, if a gap was made at an early stage, this gap may widen as a student levels-up. Bridging gaps should be dealt level by level in order to not face more problems especially at higher level. The continuous performance and implementation of specialized programs to [illegible]e this concern is of utmost importance to comply with performance standards and achieve target learning competencies.

III. RESEARCH METHODOLOGY

In this chapter, the methods used to achieve the set of objectives were discussed. Furthermore, some of the key areas of this study are explained in more detail.

3.1 Research Design

This study concerns the current status of students' knowledge about mathematical logic between the Grade 8 and Grade 11 high school levels. It aims to define what exists and thus, provide recommendations for further improvements if there are areas that need it.

In relation to this, a descriptive research design suits this purpose. Research design basically helps to provide answers to who, what, when, where and how in relation to a particular research problem.

The data analyzed came from the respondents who were exposed in a completely natural and unchanged environment, in which in this study, refers to a classroom setting. This study may give birth to a more quantitative research given the results derive from the analyzed data.

In the latter part of this study, recommendations can be derived to address the problem identified thus this may serve a useful tool in developing a more focused study.

3.2 Locale and Respondents of the Study

The host academic institution is the San Pedro Relocation Center National High School in San Pedro, Laguna. There were 60 students in the Grade 9 and Grade 10 levels who participated in this study. Each level corresponds to 30 respondents. From a population of 140 students, 30 males and 30 females were asked to take a written test about propositional logic. This test aims to assess their knowledge, process/skills and understanding learning domains.

Since Logic is only discussed in the Grade 8 and Grade 11 levels, the Grade 9 and Grade 10 students were chosen to see if they still have the learnings from Grade 8 and are prepared to take logic in Grade 11.

3.3 Data Gathering Procedures

The data were taken from the test results administered to the students. The sample test questionnaire is shown in Appendix C. The 30-question test is a combination of multiple choice and statement construction type. The students were given an hour to finish the examination. Same set of exams were given to the Grade 9 and Grade 10 students.

Each item of the test targets a pre-determined learning competency under Geometry for Grade 8 students (see Appendix A). These is to measure the post-learnings of the students. Table 3.1 shows the test items which target a specific learning competency.

Table 3.1: Learning Competencies for each Test Item

Learning Competencies (code)	Item Nos.
1. Determines the relationship between the hypothesis and the conclusion of an if-then statement (M8GE-IIf-1)	2, 3, 9
2. Transforms a statement into an equivalent if-then statement (M8GE-IIf-2)	4, 5, 8
3. Determine the inverse, converse, and contrapositive of an if-then statement (M8GE-IIg-1)	6-12, 17-30
4. Illustrates the equivalence of (a) the statement and its contrapositive; and (b) the converse and inverse of a statement (M8GE-IIg-2)	10, 13
5. Uses inductive or deductive reasoning in an argument (M8GE-IIh-1)	1
6. Write both direct and indirect proof (M8GE-IIj-1)	14, 15, 16

Each item of the test also corresponds to assessment points targeting three different learning domains which is either knowledge, process/skills or understanding. Table 3.2 shows their correspondence.

Table 3.2: Learning Domains for each Test Item

Learning Domains	Item Nos.
1. Knowledge	1-9, 12-13, 17-20
2. Process / Skills	21 – 30
3. Understanding	10, 11, 14-16

Items 1-9, 12-13, 17-20 deal with knowledge. Items 21 – 30 deal with process/skills. Items 10, 11, 14-16 deal with understanding. To summarize, there are 15 items to assess knowledge, 10 items to assess process/skill and 5 items to asses understanding.

3.4 Data Analysis

Item analysis was used to assess the students' performance. A learning competency is assumed to be least learned when at most 10 students got the correct answer to its corresponding item. All the instances of at most 10 students got the correct answer for each learning competency was tallied and the one with the highest number of instances was considered to be the least. This was done for both grade levels.

In addition, it was also tested if there is a significant difference between the performance of the Grade 9 and Grade 10 students. This was done using a two-sample t-test assuming equal variances with 95% confidence level. The t-test equation is defined in Equation 1.

$$t = \frac{\bar{x}_1{}^2 - \bar{x}_2{}^2}{\sqrt{\left(\left(\frac{ss_1 - ss_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} - \frac{1}{n_2}\right)\right)}} \quad \text{(Eq.1)}$$

where t – t-stat value
$\bar{x}_1$- mean of group 1
$\bar{x}_2$- mean of group 2
ss_1 – sum of squares of group 1
ss_2 – sum of squares of group 2
n_1-number of observation in group 1
n_2-number of observation in group 2

To assess the learning domains, the number of students who got the correct answer for each item was divided by the total number of takers. The mean percentage was calculated for each learning domain. The two-sample t-test assuming equal variances was also used to determine if there is a significant difference between the learning domains of the Grade 9 and Grade 10 students. The confidence level is also 95%.

The weights for knowledge, process/skills and understanding are 60%, 30% and 10%, respectively. A 75% cut-off value was assumed to be the passing mark as this is considered to be a standard passing rate in the high school level. Ratings below the passing rate were assessed to be low level learning domains.

A survey was conducted to determine the possible reasons why the students encountered difficulty in taking the test. The survey checklist is shown in Appendix E. Item analysis was also used in order to determine the least learning competencies and the least learning domains. The results were tallied and served as the basis for the construction of the if-then logic diagram.

3.5 “if - then” Logic Diagram Development

In general, a mathematical statement consists of two parts: the hypothesis or assumptions, and the conclusion. Most mathematical statements are in the form “if A, then B” or “A implies B”. The assumptions are the conditions that make up “A” and the conclusion makes up “B” (Tourlakis, 2008). “If-then” statements can be visualized in the diagram as shown in Figure 3.1.

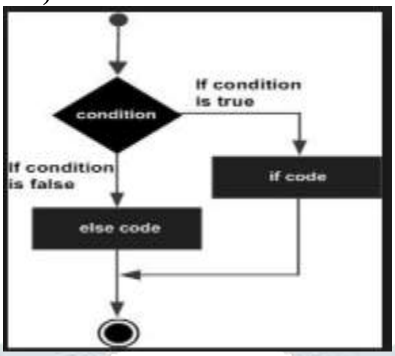

Figure 3.1: if-then block diagram (PilotLogic, 2016)

The “if” condition appears in the diamond block. It has two possible outputs, that is, when the statement is satisfied (yes or Y) and when the condition not satisfied (no or N).

There are 15 factors considered as possible causes of difficulty among the students. From the survey results, the top 3 factors and the top 5 test items with the highest hits or scores were considered. These were used to create a logical relationship in the form “if <factor> and <learning competency> then <learning domain>”.

3.6 Truth Tables and Logic Equations

The truth table was derived from the form and the conditions found in the logic diagram. Truth tables are formed by one or more inputs but only a single output. The number of input combinations can be determined using 2^n, where n is the number of inputs.

Inputs and outputs are commonly represented in terms of 1’s and 0’, True or False, and T or F. Several information about this is found in Appendix D.

Logic equations are similar to normal equations except for the negations represented by bars or quotes (c ne). A negation of true means false and the negation of false means true. If a variable A represents a true, then it is represent A. If A represents a false, then it is represented as $\bar{A}$ or A'.

IV. PRESENTATION, ANALYSIS AND INTERPRETATION OF DATA

The data were obtained from the results of a 30-item test taken by Grade 9 and Grade 10 students. Survey data were also obtained from them. This chapter presents the results and how these data were used to achieve the objectives of this study.

4.1 Least Learned Competencies and Learning Domain Assessment

Item analysis was used to determine the number of students who got the correct answer on each test question which corresponds to one of the learning competencies and one of the learning domains. The number of students who got the correct answer for each item was counted and the results are shown in Table 4.1.The learning competencies are expressed using their respective codes as shown in Table 3.1.

Table 4.1. Student Count who got the Correct Answer for each Test Item

Item No.	**Learning Competency Code**	**GRADE 9**	**GRADE 10**
1	M8GE-IIh-1	18	9
2	M8GE-IIf-1	22	13
3	M8GE-IIf-1	21	14
4	M8GE-IIf-2	18	15
5	M8GE-IIf-2	20	18
6	M8GE-IIg-1	16	14
7	M8GE-IIg-1	5	7
8	M8GE-IIf-2, M8GE-IIg-1	8	21
9	M8GE-IIf-1, M8GE-IIg-1	23	15
10	M8GE-IIg-1, M8GE-IIg-2	8	3
11	M8GE-IIg-1	13	8
12	M8GE-IIg-1	5	9
13	M8GE-IIg-2	22	12
14	M8GE-IIj-1	14	6
15	M8GE-IIj-1	15	12

16	M8GE-IIj-1	15	10
17	M8GE-IIg-1	18	15
18	M8GE-IIg-1	8	6
19	M8GE-IIg-1	7	8
20	M8GE-IIg-1	11	10
21	M8GE-IIg-1	23	16
22	M8GE-IIg-1	22	12
23	M8GE-IIg-1	7	3
24	M8GE-IIg-1	6	4
25	M8GE-IIg-1	15	10
26	M8GE-IIg-1	16	9
27	M8GE-IIg-1	9	4
28	M8GE-IIg-1	7	0
29	M8GE-IIg-1	5	0
30	M8GE-IIg-1	2	0

Looking at the gray shaded cells, results show that there are items in which both grade levels exhibits low performance indicative of low competency levels. There are also items in which only the Grade 9 students are low while the Grade 10 students are high or vice-versa. To highlight the least learned competencies, Table 4.2 was presented.

Table 4.2: Instances of Low Learning Competencies

Learning Competencies (code)	**Instances for Grade 9 Students**	**Instances for Grade 10 Students**
1. Determines the relationship between the hypothesis and the conclusion of an if-then statement (M8GE-IIf-1)		
2. Transforms a statement into an equivalent if-then statement (M8GE-IIf-2)	1	
3. Determine the inverse, converse, and contrapositive of an if-then statement (M8GE-IIg-1)	12	15
4. Illustrates the equivalence of (a) the statement and its contrapositive; and (b) the converse and inverse of a statement (M8GE-IIg-2)	1	1
5. Uses inductive or deductive reasoning in an argument (M8GE-IIh-1)		1
6. Write both direct and indirect proof (M8GE-IIj-1)		2

Table 4.2 shows that there are 2 occurrences in M8GE-IIj-1 and a single occurrence in M8GE-IIh-1 and M8GE-IIg-2 under the Grade 10 level. For the Grade 9 level, there is 1 occurrence in M8GE-IIf-2 and M8GE-IIg-2. For both grade levels, high occurrences were observed in M8GE-IIg-1 which corresponds to the learning competency; determine the inverse, converse, and contrapositive of an if-then statement.

The performance of the two grade levels was compared statistically. The two-sample t-test assuming equal variances was used to accomplish this task. Results show a t-value of 2.4973. Tabular values are 1.6716 for one-tail and 2.0017 for two-tail. It can be concluded that there is a significant difference between the performances of the two grade levels since the computed t-value is greater than the tabular values.

The learning domains were also assessed using item analysis. Table 4.3 to 4.5 shows the percentage of those who correctly answered each item that pertains to knowledge, process/skill or understanding for both Grade 9 and Grade 10 students. The percentages for each item were averaged and this value was used to assess the learning domains.

For the knowledge learning domain, the Grade 9 class got 49.40% while the Grade 10 class got 40.47% (refer to Table 4.3). For the process/skills learning domain, the Grade 9 class got 37.30 % while the Grade 10 class got 21.20% (refer to Table 4.4). And for the understanding learning domain, the Grade 9 class got 31.40 % while the Grade 10 class got only 20.00% (refer to Table 4.5). The higher percentage of the Grade 9 class could be possibly due to their closeness to the Grade 8 level. They have just finished Grade 8 and the knowledge, the skills and their understanding about logic still remains in their mind. The Grade 10 class could have possibly forgotten the principles they have learned in Grade 8 since it has been 2 years since then.

Table 4.3: Knowledge Learning Domain Assessment

Item No.	Grade 9		Grade 10	
	Number of Correct Responses	Percentage (%)	Number of Correct Responses	Percentage (%)
1	18	60	9	30
2	22	73	13	43
3	21	70	14	47
4	18	60	15	50
5	20	67	18	60
6	16	53	14	47
7	5	17	7	23
8	8	27	21	70
9	23	77	15	50
12	5	17	9	30
13	22	73	8	27
17	18	60	15	50
18	8	27	6	20
19	7	23	8	27
20	11	37	10	33
		49.40		**40.47**

Table 4.4: Process / Skills Learning Domain Assessment

Item No.	Grade 9		Grade 10	
	Number of Correct Responses	Percentage (%)	Number of Correct Responses	Percentage (%)
21	23	77	10	33
22	22	73	16	53
23	7	23	12	40
24	6	20	3	10
25	15	50	10	33
26	16	53	9	30
27	9	30	4	13
28	7	23	0	0
29	5	17	0	0
30	2	7	0	0
		37.30		**21.20**

Table 4.5: Understanding Learning Domain Assessment

Item No.	Grade 9		Grade 10	
	Number of Correct Responses	Percentage (%)	Number of Correct Responses	Percentage (%)
10	8	27	3	10
11	13	43	8	27
14	9	30	6	20
15	6	20	6	20
16	11	37	7	23
		31.40		**20.00**

The learning domains assessment of the two classes was statistically verified if they are same or not. Using the two sample t-test assuming equal variances, results show that there is no significant difference between the knowledge and process/skills learning domains of the Grade 9 and Grade 10 students. For the knowledge learning domain, the t-stat value is 1.3015 which is less than the critical t-values of 1.7011 and 2.0484 for one-tail and two-tail tests, respectively. For the process / skills learning domain, the t-stat value is 1.6459 which is less than the critical t-values of 1.7341 and 2.1009 for one-tail and two-tail tests, respectively.

In the understanding learning domain, results show a significant difference. The t-stat value is 2.3387 which is greater than the critical t-values of 1.8595 and 2.3060 for one-tail and two-tail tests, respectively.

The overall assessment of the learning domains is shown in Table 4.6. The Grade 9 class has 49.40% with knowledge, 37.30% with process/skills and 31.40% with understanding. The overall assessment is 43.97%. For the Grade 10 class, they got 40.47% with knowledge, 21.20% with process/skills and 20.00% with understanding. Overall, they got an assessment of 32.64%.

Table 4.6: Overall Assessment of the Learning Domains

Grade Level	Learning Domains			Overall Assesment
	Knowledge (60%)	Process/Skills (30%)	Understanding (10%)	
9	49.40%	37.30%	31.40%	43.97%
10	40.47%	21.20%	20.00%	32.64%

It has been set that the acceptable passing rate is 75%. Overall, both grade levels failed to satisfy the acceptable passing rate. The learning domain percentages of the two grade levels were statistically compared and results show that there is no significant difference between them. The obtained t-stat is 1.4308, lower than the t-critical values of 2.1318 and 2.7764, for one-tail and two-tail tests, respectively.

This low learning domain percentage proves the need for a bridging program for learning logic between the Grade 8 and Grade 11 levels.

4.2 Survey Results and the if-then Logic Diagram

A survey was conducted to determine what factor/s could have possibly affected the students the reason why they answered incorrectly. There are 15 factors considered and each test item was also analyzed.

Table 4.7 shows the survey results for the Grade 9 students. There are 3 factors which are very significant in terms of total hits. In descending order, ‘poor in math skills’, ‘little knowledge on the concept’ and ‘incomplete mastery of math basics’. The top 5 items with the highest hits were considered. Analyzing each item, item 19 got 22 hits, items 12 and 30 got 21 hits, item 29 got 19 hits, item s 7, 11, and 18 got 18 hits and items 10, 24 and 28 got 16 hits. Each item corresponds to specific learning competencies and learning domains. Table 4.8 shows this correspondence.

Table 4.7: Survey Results for Grade 9 Students

Factors Affecting the Students Performance in Mathematics	Why is your answer wrong in item number																														TOTAL HITS
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1. little knowledge on the concept	4	3	2	1	1	1	4	4	1	5	3	5		1	3	2	1	2	4	2		2	2	2			1	1		1	**58**
2. teaching strategies							1																								1
3. Inattentiveness		1	1	1							1						1														5
4. Incomplete mastery of math basics		1	1	1	2	2	6	5		1	1	2	1	1		2	2	3	4	1			1	1	1	1	1	1	1	1	**44**
5. short span of memory	2		2	3	1	2	2	1			1		2	1	1			3	4	3				3	2	1			1	1	36
6. poor in math skills	1	1	2	2	3	4	4	4	3	8	8	9	3	8	7	7	5	9	9	8	4	5	9	9	6	7	12	13	15	16	**201**
7. lack of time											1					1	1											1	2	2	8
8. manner of questioning						1	1	1		1	3	5		1									1								14
9. poor health																															0
10. classroom environment																															0
11. poverty																															0
12. family problem																															0
13. peer influences									1	1								1	1		1	1	1	1							8
14. absenteeism														1									1								2
15. poor comprehension																															0
TOTAL	7	6	8	8	7	10	**18**	15	5	**16**	**18**	**21**	6	13	11	12	10	**18**	**22**	14	5	8	15	**16**	9	9	14	**16**	**19**	**21**	377

Majority of the least learned competency is M8GE-IIg-1 (Determine the inverse, converse, and contrapositive of an if-then statement). Minority is M8GE-IIg-2 (Illustrates the equivalence of (a) the statement and its contrapositive; and (b) the converse and inverse of a statement). The Grade 9 students exhibited difficulties in the 3 learning domains.

Table 4.8: Survey Results Item Analysis for Grade 9 Students

Hits	Item No.	Learning Competency Code	Learning Domain
22	19	M8GE-IIg-1	Knowledge
21	12	M8GE-IIg-1	Knowledge
	30	M8GE-IIg-1	Process/Skills
19	29	M8GE-IIg-1	Process/Skills
18	7	M8GE-IIg-1	Knowledge
	11	M8GE-IIg-1	Understanding
	18	M8GE-IIg-1	Knowledge
16	10	M8GE-IIg-1, M8GE-IIg-2	Understanding
	24	M8GE-IIg-1	Process/Skills
	28	M8GE-IIg-1	Process/Skills

Table 4.9 shows the survey results for the Grade 10 students. Similar to the Grade 9 class, there are also 3 factors which are very significant in terms of total hits. In descending order, ‘poor in math skills’, ‘short span of memory’ and ‘little knowledge on the concept’.

The top 5 items with the highest hits were also considered for the Grade 10 class. Analyzing each item, item 23 got 24 hits, item 30 got 23 hits, items 28 and 24 got 22 hits, item 12 got 21 hits and items 18 and 29 got 20 hits.

Table 4.9: Survey Results for Grade 10 Students

Factors Affecting the Students Performance in Mathematics	Why is your answer wrong in item number																														TOTAL HITS
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
1. little knowledge on the concept	1	2		2		3	5	2	5	1	2	3	5	3	3	4	3	9	2	2	2	2	4	3	2	2	4	4	3	2	**85**
2. teaching strategies										1	1															2	1	1	1	1	8
3. Inattentiveness				1																			1	1							3
4. Incomplete mastery of math basics		1		1	1							1			1	1					1	1	1	2	2	1	1	1	2	1	19
5. short span of memory	5	6	3	4	5	3	7	4	4	5	6	7	2	5	2	2	4	4	5	5	4	5	6	5	5	3	5	6	6	6	**139**
6. poor in math skills	6	5	6	3	3	6	6	3	8	10	9	10	10	8	7	5	4	6	8	5	4	4	8	7	6	7	7	8	8	7	**194**
7. lack of time			1																		1	1	1	1				2		2	9
8. manner of questioning						1																								2	3
9. poor health																									1	1	1			2	5
10. classroom environment	1	1	1		1									1	1	1	1	1	1	1	1	1	1	1							15
11. poverty		1					1															1			1						4
12. family problem																															0
13. peer influences																						1	1	1	1						4
14. absenteeism																							1	1	1						3
15. poor comprehension																															0
TOTAL	13	16	11	11	10	13	19	9	17	17	18	**21**	17	17	14	13	12	**20**	16	13	13	16	**24**	**22**	19	16	19	**22**	**20**	**23**	491

Table 4.10 shows the corresponding learning competencies and learning domains for each item. Majority of the least learned competency is M8GE-IIg-1 (Determine the inverse, converse, and contrapositive of an if-then statement). The Grade 10 students exhibited difficulties in the knowledge and process/skills learning domains.

Table 4.10: Survey Results Item Analysis for Grade 10 Students

Hits	Item No.	Learning Competency Code	Learning Domain
24	23	M8GE-IIg-1	Process/Skills
23	30	M8GE-IIg-1	Process/Skills
22	28	M8GE-IIg-1	Process/Skills
	24	M8GE-IIg-1	Process/Skills
21	12	M8GE-IIg-1	Knowledge
20	18	M8GE-IIg-1	Knowledge
	29	M8GE-IIg-1	Process/Skills

The results of Tables 4.7 to 4.10 were used to build the if-then logic diagrams as shown in Figures 4.1 and 4.2. The diagrams were based on the statement, *"if <factor> and <learning competency> then <learning domain>"*. The top 3 factors and the top 5 test items with the highest hits or scores were considered. Each diagram starts and ends with "start" and "end". It is assumed that the factors and the learning competencies are inputs and the learning domain is the output with reference to the students.

For Figure 4.1, this diagram can be interpreted as "**if** the factor is *poor in math skils* **and** *little knowledge on the concept* **and** *incomplete mastery of math basics* **and** the learning competency is *M8GE-IIg-1* **then** the *knowledge, process/skills, understanding* learning domains are affected; else **if** the learning competency is *M8GE-IIg-2* **then** the *understanding* learning domain is affected."

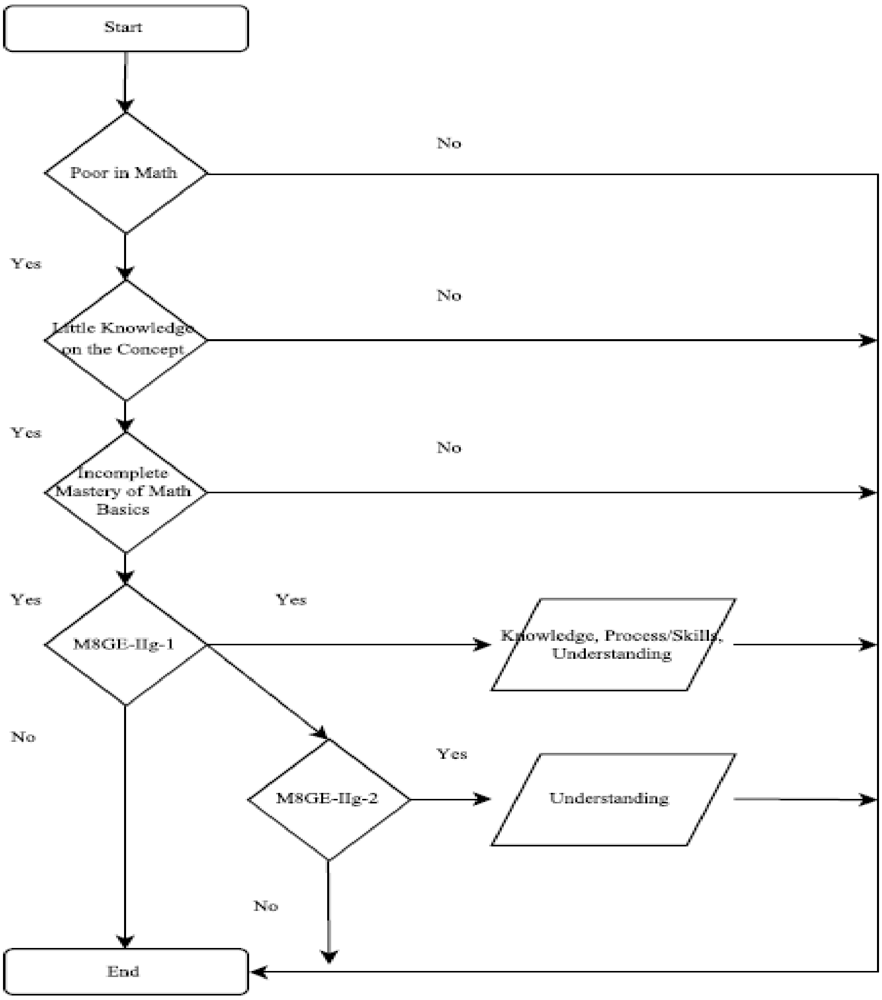

Figure 4.1: if-then block diagram for Grade 9 Survey Results

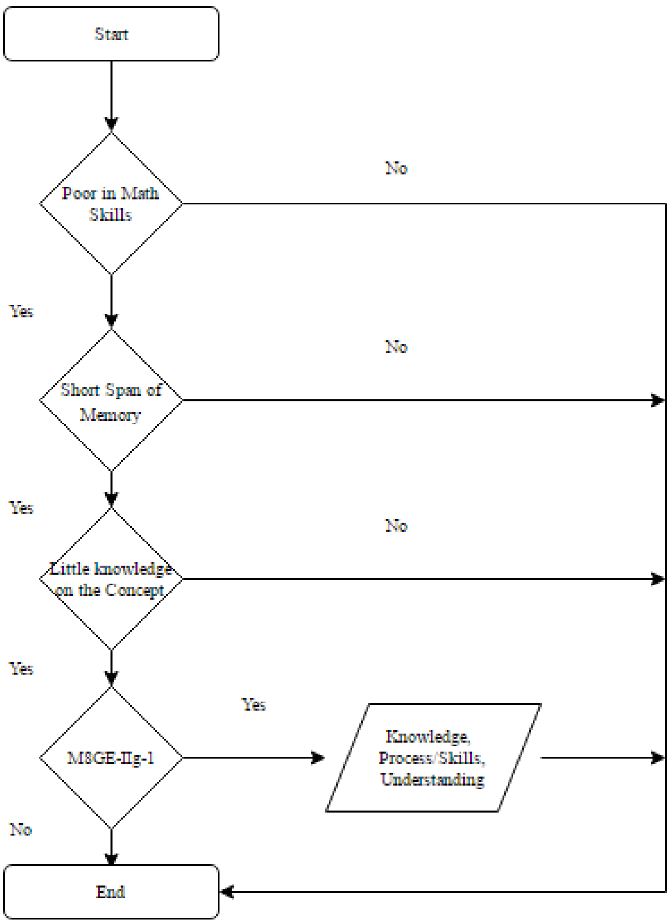

Figure 4.2: if-then block diagram for Grade 10Survey Results

For Figure 4.2, this diagram can be interpreted as "**if** the factor is *poor in math skills* **and** *short span of memory* **and** *little knowledge on the concept* **and** the learning competency is *M8GE-IIg-1***then** the *knowledge, process/skills, understanding* learning domains are affected." It is important to take note that the negative effects, as in low assessment levels, were assumed when it comes to the learning domains.

4.3 Truth Tables and Logic Relationships

The logic diagrams shown in the previous section can be converted into truth tables to determine the possible outcome for a given input combination. In Appendix D, a short lecture about propositional logic was presented. It discusses the different truth tables for the OR and the AND logic operation. In arithmetic, OR is synonymous to addition while AND is to multiplication. The truth table that corresponds to Figure 4.1 is shown in Table 4.11. The three input factors (A, B and C) have 8 possible combinations since the T (true) and F (false) represents the base, which is2, and the 3 factors is the exponent, resulting to $2^3 = 8$. The learning competencies X and Y, has 4 possible combinations which is obtained from $2^2 = 4$ using the aforementioned calculations.

Table 4.11: Truth Table for Figure 4.1

A	B	C	X	Y	O	A	B	C	X	Y	O
F	F	F	F	F	F	T	F	F	F	F	F
			F	T	F				F	T	F
			T	F	F				T	F	F
			T	T	F				T	T	F
F	F	T	F	F	F	T	F	T	F	F	F
			F	T	F				F	T	F
			T	F	F				T	F	F
			T	T	F				T	T	F
F	T	F	F	F	F	T	T	F	F	F	F
			F	T	F				F	T	F
			T	F	F				T	F	F
			T	T	F				T	T	F
F	T	T	F	F	F	T	T	T	F	F	F
			F	T	F				F	T	F
			T	F	F				T	F	T
			T	T	F				T	T	T

Legend: T - True; F - False

A - Poor Math Skills
B - Little Knowledge on the Concept
C - Incomplete Mastery of Math Basics
X - M8GE-IIg-1
Y - M8GE-IIg-2
O - Output (Learning Domain/s)

Each ABC combinations has to pair with each XY combinations forming Table 4.11. A true or T output (O) will only be produced when A = T, B = T, C = T, X = T and Y = F, or A = T, B = T, C = T, X = T and Y = T. Hence, the output can be defined as

$$O_1 = ABCX\bar{Y} \quad \text{(Eq.2)}$$

or

$$O_2 = ABCXY \quad \text{(Eq.3)}$$

Two subscripts, 1 and 2, were used because the learning domains for each output is different. A true O_1 is consists of the three learning domains while O_2 is just the understanding learning domain.

For Figure 4.2, its truth table is shown in Table 4.12. The three input factors (A, B and C) still have 8 combinations however; the learning competency X only has 2 combinations following $2^1 = 2$.

Table 4.12: Truth Table for Figure 4.2

A	B	C	X	O	A	B	C	X	O
F	F	F	F	F	T	F	F	F	F
			T	F				T	F
F	F	T	F	F	T	F	T	F	F
			T	F				T	F
F	T	F	F	F	T	T	F	F	F
			T	F				T	F
F	T	T	F	F	T	T	T	F	F
			T	F				T	T

Legend: T - True; F - False

A - Poor Math Skills
B - Short Span of Memory
C - Little Knowledge on the Concept
X - M8GE-IIg-1
Y - M8GE-IIg-2
O - Output (Learning Domain/s)

Each ABC combinations has to pair with each X combinations. A true or T output will be produced only when A = T, B = T, C = T, and X = T. Hence, the output can be defined as

$$O = ABCX \quad \text{(Eq.4)}$$

For Equations 2, 3 and 4, the operation is AND corresponding to the multiplication process in basic arithmetic.

V. SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

This study was able to establish a logical relationship between the factors affecting the students' learnings in mathematical logic and their learning competencies and learning domains.

5.1 Findings

The least learning competencies and the assessment of the learning domains were determined using item analysis. The test results were analyzed per item which corresponds to a one or more learning competency/ies or learning domain.

For both grade levels, it was found out that their least learning competency in logic is in determining the inverse, converse and contrapositive of an if-then statement (M8GE-IIg-1).Though their test results are significantly different, they have a common least learned competency.

For a set learning domain passing rate of 75%, both grade levels failed. Grade 9 got 43.97% while Grade 10 got 32.64%. There is no significant difference between the learning domain assessments of the two grade levels.

Hence, this low learning domain percentage proves the need for a bridging program for learning logic between the Grade 8 and Grade 11 levels.

5.2 Conclusions

Three major factors were identified that causes the students to not correctly answer the test. For the Grade 9 class, these are 'poor in math skills', 'little knowledge on the concept' and 'incomplete mastery of math basics'. For the Grade 10 class, these are 'poor in math skills', 'short span of memory' and 'little knowledge on the concept'. The Grade 9 class exhibited difficulties in the three learning domains while the Grade 10 class exhibited difficulty in the knowledge and process/skills learning domains.

To apply the concepts of mathematical logic, the factors affecting the students and their learning competencies and learning domains were logically mapped using if-then logic diagrams. The diagrams come with a truth table resulting to a logical equation. For the Grade 9 class, the relationship can be defined by the equations $O_1 = ABC\bar{X}Y$ and $O_2 = ABCX\bar{Y}$. Two subscripts, 1 and 2, were used because the learning domains for each output is different. O_1 is consists of the three learning domains while O_2 is just the understanding learning domain. For the Grade 10 class, the relationship can be defined by the equation, $O = ABCX$.

5.3 Recommendations

The study is a descriptive research in which the results call for a more quantitative research. The gap in learning math logic between the Grade 8 and Grade 11 levels was established and given substantial evidence to require a gap management plan or a bridging module to be implemented. This study also recommends the implementation of the gap management paradigm as shown in Appendix F.

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