FIXED POINT THEOREMS FOR SEQUENCE OF MAPPINGS IN COMPLEX VALUED METRIC SPACE

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Abstract. The aim of this paper is to establish fixed point results for the collection of sequence of mappings by using the concept of weakly compatibility and continuity in complex valued metric space. Our results generalize the results proved earlier by [4].

1. Introduction

In 2011, Azam et al. [1] introduce the notion of of new space called complex valued metric space and establishes existence of fixed point theorems under the contraction condition.

Theorem 1.1. *([1]). Let (X,d) be a complete complex valued metric space and S,T : X → X, If S and T satisfy*

$$d(Sx,Ty) \preceq \lambda d(x,y) + \frac{\mu d(x,Sx)d(y,Ty)}{1+d(x,y)} \qquad (1.1)$$

for all x,y ∈ X, where λ,μ are nonnegative reals with λ + μ < 1. Then S and T have a common fixed point.

In 2012, Rouzkard and Imdad [8] established some common fixed point theorems satisfying certain rational expressions in Complex valued metric space.

Theorem 1.2. *([8]). If S and T are self mappings defined on a complex valued metric space (X,d) satisfying the condition*

$$d(Sx,Ty) \preceq \lambda d(x,y) + \frac{\mu d(x,Sx)d(y,Ty) + \gamma d(y,Sx)d(x,Ty)}{1+d(x,y)} \qquad (1.2)$$

for all x,y ∈ X, where λ,μ,γ are nonnegative reals with λ + μ + γ < 1. Then S and T have a unique common fixed point.

Later on Sintunavarat W. and Kumam P. [9] extend and improve the condition of contraction of theorem (1.1) from the constant of contraction to some control functions and establish the common fixed point theorems which are more general than the result of [1] and also give the results for weakly compatible mappings in complex valued metric spaces.

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Theorem 1.3. *([1]). Let (X,d) be a complete complex valued metric space and $S,T : X \to X$, if there exists a mappings $\Lambda,\Xi : X \to [0,1)$*

(1) $\Lambda(Sx) \le \Lambda(x)$ *and* $\Xi(Sx) \le \Xi(x)$;
(2) $\Lambda(Tx) \le \Lambda(x)$ *and* $\Xi(Tx) \le \Xi(x)$;
(3) $(\Lambda + \Xi)(x) < 1$;

$$d(Sx,Ty) \preceq \Lambda(x)d(x,y) + \frac{\Xi(x)d(x,Sx)d(y,Ty)}{1+d(x,y)} \tag{1.3}$$

for all $x,y \in X$, where λ,μ are nonnegative reals with $\lambda + \mu < 1$. Then S and T have a common fixed point.

In 2014 Hakwadiya et.al. [4], Proved common fixed point theorems for six self mappings as follows

Theorem 1.4. *Let (X,d) be a complex valued metric space and A,B,D,M,S and T be six self mappings in X satisfying the condition:*

(1) $S(X) \subset BD(X)$ *and* $T(X) \subset AM(X)$;

(2) (AM,S) *and* (BD,T) *are commuting pairs;*

(3) *The pair* (AM,S) *and* (BD,T) *are weakly compatible;*

(4) *For each $x,y \in X$ and x 6= y,;*

(i): *If* $d(Ty,AMx)+d(Sx,BDy)+d(BDy,AMx) = 0$. *then* $d(Sx,Ty) = 0$;

(ii): *If* $d(Ty,AMx) + d(Sx,BDy) + d(BDy,AMx)$ 6= 0 *than following identity holds;*

$$\begin{aligned} d(Sx,Ty) &\preceq \alpha\left(\frac{d(AMx,Sx)+d(BDy,Ty)d(AMx,Sx)}{1+d(Sx,Ty)}\right) \\ &+ \beta max\{d(AMx,BDy)), d(AMx,Sx), d(BDy,Sx)\} \\ &+ \gamma\{d(BDy,Ty)+d(Ty,AMx)+d(Sx,BDy)\} \\ &+ \eta\left(\frac{d(Ty,BDy)d(Sx,AMx)}{d(Ty,AMx)+d(Sx,BDy)+d(BDy,AMx)}\right) \end{aligned}$$

Where $\alpha+\beta+2\gamma+\eta < 1$. Then AM,BD,S and T have a unique common fixed point.

2. preliminaries

We recall some basic concept, notion and definition in complex valued metric spaces.

Let C be the set of complex numbers and $z_1,z_2 \in$ C. We define a partial order $'\prec'$ on $\mathbb{C}$ as follows:
(A): Two complex number z_1,z_2 such that $z_1 \preceq z_2 \iff Re(z_1) \le Re(z_2)$ and $Im(z_1) \le Im(z_2)$.

Thus $z_1 \preceq z_2$ if one of the following holds:

(C1): $Re(z_1) = Re(z_2)$ *and* $Im(z_1) = Im(z_2)$;

(C2): $Re(z_1) < Re(z_2)$ *and* $Im(z_1) = Im(z_2)$;

(C3): $Re(z_1) = Re(z_2)$ *and* $Im(z_1) < Im(z_2)$; **(C4):** $Re(z_1) < Re(z_2)$ *and*

$Im(z_1) < Im(z_2)$.

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of $(C2)$, $(C3)$, and $(C4)$ is satisfied and we will write $z_1 \prec z_2$ if only $(C4)$ is satisfied.

(B): It follows that,

(1) $0 \preceq z_1 \precnsim z_2$ $implies |z_1| < |z_2|$;

(2) $z_1 \preceq z_2$ and $z_2 \prec z_3$ imply $z_1 \prec z_3$; (3) $0 \preceq z_1 \preceq z_2$ implies $|z_1| \leq |z_2|$;

(4) if $a,b \in \mathbb{R}$, $0 \leq a \leq b$ and $z_1 \preceq z_2$, then $az_1 \preceq bz_2$ for all $z_1,z_2 \in \mathbb{C}$.

Definition 2.1. [1]. Let X be a nonempty set. A mapping $d : X \times X \to \mathbb{C}$ is called a complex valued metric on X if the following conditions satisfied :

1. $0 \leq d(x,y)$, *for all* $x,y \in X$ *and* $d(x,y) = 0$ *if and only if* $x = y$;
2. $d(x,y) = d(y,x)$ *for all* $x,y \in X$; (3) $d(x,y) \leq d(x,z) + (z,y)$, *for all* $x,y,z \in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Example 2.2. Let $X = \mathbb{C}$, Define the mapping $d : X \times X \to \mathbb{C}$ by

$$d(z_1,z_2) = e^{-il}|z_1 - z_2|,$$

where $z_1,z_2 \in \mathbb{C}$. and $l \in \mathbb{R}$ Then (X,d) is a complex valued metric space.

Definition 2.3. [9] Let (X,d) be a complete complex valued metric space, $\{x_n\}$ be a sequence in X,

1. *A point* $x \in X$ *is called interior point of a set* $A \subseteq X$ *whenever there exists* $0 < r \in C$ *such that* $B(x,r) := \{y \in X | d(x,y) < r\} \subseteq A$.
2. *A point* $x \in X$ *is called a limit point of* Z *whenever for every* $0 < r \in \mathbb{C}, \{B(x,r) \cap (A - X)\} 6= \varphi$
3. *A subset* $A \subseteq X$ *is called open whenever each element of* A *is an interior point of* A
4. *A subset* $A \subseteq X$ *is called closed whenever each limit point of* A *belongs to* A.
5. *A sub − basis for a Hausdorff topology* τ *on* X *is a family*

Definition 2.4. [9]. Let (X,d) be a complex valued metric space, $\{x_n\}$ be a sequence in X and $x \in X$.

1. If for every c C , with $0 < c$ there is $N \in \mathbb{N}$ such that for all $n > N$. $d(x_m,x) < c$, then $\{x_n\}$ is said to *convergent*, $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. we denote this by $lim_{n\to\infty}x_n = x$.
2. If for every $c \in C$, with $0 < c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n,x_{n+m}) < c$, where $m \in \mathbb{N}$, then $\{x_n\}$ is said to be Cauchy sequence.
3. If every Cauchy sequence in X is convergent, then (X,d) is said to be a complete valued metric space.

Remark 2.5. (1) If A^0 is the set of limit points of 'A' and there exist 'u' such that $0 < u < z$ for each $z \in A^0$ then $u = 0$,

(2) If $z \leq \lambda z$ and $0 \leq \lambda < 1$, then $z = 0$.

Lemma 2.6. [1] *Let* (X,d) *be a complex valued metric space and* $\{x_n\}$ *be a sequence in X. Then* $\{x_n\}$ *converges to x if and only if* $|d(x_n),x| \to 0$ *as* $n \to \infty$.

Lemma 2.7. [1] *Let* (X,d) *be a complex valued metric space and let* $\{x_n\}$ *be a sequence in X. Then* $\{x_n\}$ *is a cauchy sequence if and only if* $|d(x_n,x_{n+m})| \to 0$ *as* $n \to \infty$, *where* $m,n \in \mathbb{N}$

Definition 2.8. [1] Let s and T be self mappings of a nonempty set X

(1) A point $x \in X$ is said to be a fixed point of T if $Tx = x$.
(2) A point $x \in X$ is said to be a coincidence point of S an T if $Sx = Tx$ and we shall called $w = Sx = Tx$ that a point of coincidence of S and T.
(3) A point $x \in X$ is said to be a common fixed point of S and T if $x = Sx = Tx$.

In 1976, Jungck [5] introduced concept of common mappings as follows:

Definition 2.9. Let X be a non-empty set. The mappings S and T are commuting if $TSx = STx$, for all $x \in X$.

In 1986, [5] introduced the more generalized commuting mappings in metric spaces, called compatible mappings., which also are more general than the concept of weakly commuting mappings as follows:

Definition 2.10. Let S and T be mappings from a metric space (X,d) into itself.
The mappings S and T are said to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Remark 2.11. In general, commuting mappings are weakly commuting and weakly commuting mappings are compatible, but the converse are not necessarily true and some examples can be found in [5].

Example 2.12. Let $X = [0,2]$ with usual metric d where $d(x,y) = |x - y|$ for all x and y in X. We define $T(x)$ and $S(x)$ as follows

$$Tx = \begin{cases} x, & \text{if } x \in [0,1) \\ 2, & \text{if } x \in [1,2] \end{cases}$$

$$Sx = \begin{cases} 2 - x, & \text{if } x \in [0,1) \\ 2, & \text{if } x \in [1,2] \end{cases}$$

by choosing $x_n = 1 - \frac{1}{n}$ then $Tx_n = 1 - \frac{1}{n}$ and $Sx_n = 1 + \frac{1}{n}$ one can easily show the commuting mappings are weakly commuting and weakly commuting mappings are compatible, but the converse are not necessarily true.

In 1996, Jungck introduced the concept of weakly compatible mappings as follows:

Definition 2.13. Let S and T be self mappings of a non empty set X. The mappings S and T are weakly compatible if $STx = TSx$ whenever $Sx = Tx$.

We can see an example to show that there exist weakly compatible mappings which are not compatible mappings in Djoudi and Nisse.
The following lemma proved by Haghi et al. is useful for our main results:

Example 2.14. Let $X = [2,20]$ with usual metric d where $d(x,y) = |x - y|$ for all x and y in X.We define $T(x)$ and $S(x)$ as follows

$$Tx = \begin{cases} 2, & \text{if } x = [0,1) \text{ if } 2 \geq x \leq 5 \\ 13 + x, & \\ x - 3, & \text{if } x > 5 \end{cases}$$

$$Sx = \begin{cases} 2, & \text{if } x \in 2 \cup (5,20] \\ 8, & \text{if } x \in (2,5] \end{cases}$$

by choosing $x_n = 5 + \frac{1}{n}$ for all $n \geq 1$. The map S and T are compatible maps.

Lemma 2.15. *Let X be a nonempty set and $T : X \to X$ be a function. Then there exists a subset $E \subseteq X$ such that $T(E) = T(X)$ and $T : E \to X$ is one to one.*

Definition 2.16. Let $f^n : X \to X$, where $n = 1,2,3...$ be a sequence of mapping in topological space X. A point $x \in X$ is said to be a fixed point of the sequence $\{f^n\}_{n=1}^{\infty}$, if f^n converges to x as $n \to \infty$.

If $f^n = f$ for every n, for some fixed f, then a point is a fixed point of the sequence $\{f^n\}_{n=1}^{\infty}$, when and only when it is a fixed point of the mapping f. Also if f^n converges to f point wise, then a point is a fixed point of the sequence $\{f^n\}_{n=1}^{\infty}$, when and only when it is a fixed point of the mapping f.

3. Main results

Theorem 3.1. *Let (X,d) be a complex valued metric space and S,T be the self mappings and P^n and R^n be the collection of self mappings in X, such that for all $x,y \in X$.*

(1) *$S(X) \subset R^n(X)$ and $T(X) \subset P^n(X)$;*

(2) *(P^n,S) and (R^n,T) are commuting pairs;*

(3) *The pair (P^n,S) and (R^n,T) are compatible;*

(4) *For each x,y X and x 6= y,;*

(i): *If $d(Ty,P^nx)+d(Sx,R^ny)+d(R^ny,P^nx) = 0$. then $d(Sx,Ty) = 0$;*

(ii): *If $d(Ty,P^nx)+d(Sx,R^ny)+d(R^ny,P^nx)$ 6= 0 than following identity holds;*

$$
\begin{aligned}
d(Sx,Ty) \quad &\preceq \quad \alpha\left(\frac{d(P^nx,Sx)+d(R^ny,Ty)d(P^nx,Sx)}{1+d(Sx,Ty)}\right)\\
&+ \quad \beta max\{d(P^nx,R^ny)),d(P^nx,Sx),d(R^ny,Sx)\}\\
&+ \quad \gamma\{d(R^ny,Ty)+d(Ty,P^nx)+d(Sx,R^ny)\}\\
&+ \quad \eta\left(\frac{d(Ty,R^ny)d(Sx,P^nx)}{d(Ty,P^nx)+d(Sx,R^ny)+d(R^ny,P^nx)}\right)\\
&+ \quad \theta max\{d(P^nx,Sx),d(R^ny,Tx),d(P^nx,R^ny),d(Sx,Tx)\} \quad (3.1)
\end{aligned}
$$

Where $\alpha + \beta + 2\gamma + \eta + \theta < 1$. Then P^n,R^n,S and T have a unique common fixed point.

Proof. Let $x_0 \in X$ be arbitrary, Since $S(X) \subset R^n(X)$ and $T(X) \subset P^n(X)$ define for each $n \geq 0$ the sequence $\{y_n\}$ in X by $y_{2n+1} = Sx_{2n} = R^nx_{2n+1}$ and $y_{2n+2} = Tx_{2n+1} = P^nx_{2n+2}$ where $n = 0,1,2...$ then,

$$
\begin{aligned}
\implies d(y_{2n+1}, y_{2n+2}) &= d(Sx_{2n}, Tx_{2n+1}) \\
&\preceq \alpha\left(\frac{d(P^n x_{2n}, Sx_{2n}) + d(R^n x_{2n+1}, Tx_{2n+1}) d(P^n x_{2n}, Sx_{2n})}{1 + d(Sx_{2n}, Tx_{2n+1})}\right) \\
&+ \beta max\{d(P^n x_{2n}, R^n x_{2n+1}), d(P^n x_{2n}, Sx_{2n}), d(R^n x_{2n+1}, Sx_{2n})\} \\
&+ \gamma\{d(R^n x_{2n+1}, Tx_{2n+1}) + d(Tx_{2n+1}, P^n x_{2n}) + d(Sx_{2n}, R^n x_{2n+1})\} \\
&+ \eta\left(\frac{d(Tx_{2n+1}, R^n x_{2n+1}) d(Sx_{2n}, P^n x_{2n})}{d(Tx_{2n+1}, P^n x_{2n}) + d(Sx_{2n}, R^n x_{2n+1}) + d(R^n x_{2n+1}, P^n x_{2n})}\right) \\
&+ \theta max\{d(P^n x_{2n}, Sx_{2n}), d(R^n x_{2n+1}, Tx_{2n}), d(P^n x_{2n}, R^n x_{2n+1}) \\
&, \ d(Sx_{2n}, Tx_{2n})\} \\
\implies d(y_{2n+1}, y_{2n+2}) &\preceq \alpha d(y_{2n}, y_{2n+1}) + \beta d(y_{2n}, y_{2n+1}) + \gamma d(y_{2n}, y_{2n+1}) \\
&\quad + \eta d(y_{2n}, y_{2n+1}) + \theta d(y_{2n}, y_{2n+1}) \\
&\preceq (\alpha + \beta + \gamma + \eta + \theta) d(y_{2n}, y_{2n+1}) \\
Now, \ |d(y_{2n+1}, y_{2n+2})| &\preceq (\alpha + \beta + \gamma + \eta + \theta) d(y_{2n}, y_{2n+1})
\end{aligned}
$$

Similarly,

$$\begin{aligned}
d(y_{2n+3}, y_{2n+4}) &= d(Sx_{2n+2}, Tx_{2n+3}) \\
&\preceq \alpha\left(\frac{d(P^n x_{2n+2}, Sx_{2n+2}) + d(R^n x_{2n+3}, Tx_{2n+3})d(P^n x_{2n+2}, Sx_{2n+2})}{1 + d(Sx_{2n+2}, Tx_{2n+3})}\right) \\
&+ \beta max\{d(P^n x_{2n+2}, R^n x_{2n+3}), d(P^n x_{2n+2}, Sx_{2n+2}), d(R^n x_{2n+3}, Sx_{2n+2})\} \\
&+ \gamma\{d(R^n x_{2n+3}, Tx_{2n+3}) + d(Tx_{2n+3}, P^n x_{2n+2}) + d(Sx_{2n+2}, R^n x_{2n+3})\} \\
&+ \eta\left(\frac{d(Tx_{2n+3}, R^n x_{2n+3})d(Sx_{2n+2}, P^n x_{2n+2})}{d(Tx_{2n+3}, P^n x_{2n+2}) + d(Sx_{2n+2}, R^n x_{2n+3}) + d(R^n x_{2n+3}, P^n x_{2n+2})}\right) \\
&+ \theta max\{d(P^n x_{2n+2}, Sx_{2n+2}), d(R^n x_{2n+3}, Tx_{2n+2}), d(P^n x_{2n+2}, R^n x_{2n+3}) \\
&, \quad d(Sx_{2n+2}, Tx_{2n+2})\}
\end{aligned}$$

$$\begin{aligned}
\Longrightarrow d(y_{2n+3}, y_{2n+4}) &\preceq \alpha d(y_{2n+2}, y_{2n+3}) + \beta d(y_{2n+2}, y_{2n+3}) + \gamma d(y_{2n+2}, y_{2n+3}) \\
&\quad + \eta d(y_{2n+2}, y_{2n+3}) + \theta d(y_{2n+2}, y_{2n+3}) \\
&\preceq (\alpha + \beta + \gamma + \eta + \theta) d(y_{2n+2}, y_{2n+3})
\end{aligned}$$

i.e.

$$|d(y_{2n+3}, y_{2n+4})| \leq (\alpha + \beta + \gamma + \eta + \theta)|d(y_{2n+2}, y_{2n+3})|$$

So, $d(y_n, y_{n+1}) \preceq (\alpha + \beta + \gamma + \eta + \theta)d(y_{n-1}, y_n)$, if $\delta = \alpha + \beta + \gamma + \eta + \theta <1$ then it can be concluded that

$$\begin{aligned}
d(y_n, y_{n+1}) &\preceq \delta d(y_{n-1}, y_n) \\
\Longrightarrow d(y_n, y_{n+1}) &\preceq \delta^2 d(y_{n-2}, y_{n-1}) \preceq \ldots \preceq \delta^n d(y_0), y_1)
\end{aligned}$$

now for all $m > n$, we have,

$$\begin{aligned}
d(y_m, y_n) &\preceq \delta^n d(y_0, y_1) + \delta^{n-1} d(y_0, y_1) + \delta^{n-2} d(y_0, y_1) + \ldots + \delta^{m-1} d(y_0, y_1) \\
\Longrightarrow, d(y_m, y_n) &\preceq (\delta^n + \delta^{-1} + \delta^{n-2} + \ldots + \delta^{m-1}) d(y_0, y_1) \\
\Longrightarrow, |d(y_m, y_n)| &\leq \frac{\delta^n}{1-\delta}|d(y_0, y_1)|
\end{aligned}$$

Hence,

$$|d(y_m, y_n)| \leq \tfrac{\delta^n}{1-\delta}|d(y_0, y_1)| \to 0 \text{ as } m, n \to \infty$$

i.e.

$$lim_{n\to\infty}|d(y_m,y_n)| = 0.$$

Hence $\{y_n\}$ is a cauchy sequence. Since X is complete so sequence $\{y_n\}$ is converges to some point z those sub sequence $\{Sx_{2n}\}$,$\{R^n x_{2n+1}\}$,$\{Tx_{2n+1}\}$ and $\{P_n x_{2n+2}\}$ also converges to z. That is $lim_{n\to\infty} y_n = lim_{n\to\infty} Sx_{2n} = lim_{n\to\infty} R_n x_{2n+1} = lim_{n\to\infty} Tx_{2n+1} = lim_{n\to\infty} P^n x_{2n+2} = z$, there exist some $u \in X$ such that $y_n \to u$ as $n \to \infty$,$Su = P^n u = R^n u = Tu = z$ and also the pair (P^n,S) and (R^n,T) are weakly compatible, then they commute at their coincidence point.

Hence $Sz = S(P^n u) = (P^n Su) = (P^n z)$ and $R^n z = R^n Tu = T(R^n u) = Tz.$

Case I:

Now we shall show that $Tz = Sz$, put $x = z$ and $y = x_{2n+1}$ in equation 3.1. We have,

$$\begin{aligned} d(Sz,Tx_{2n+1}) &\preceq \alpha\left(\frac{d(P^n z,Sz)+d(R^n x_{2n+1},Tx_{2n+1})d(P^n z,Sz)}{1+d(Sz,Tx_{2n+1})}\right) \\ &+ \beta max\{d(P^n z,R^n x_{2n+1})),d(P^n z,Sz),d(R^n x_{2n+1},Sz)\} \\ &+ \gamma\{d(R^n x_{2n+1},Tx_{2n+1})+d(Tx_{2n+1},P^n z)+d(Sz,R^n x_{2n+1})\} \\ &+ \eta\left(\frac{d(Tx_{2n+1},R^n x_{2n+1})d(Sz,P^n z)}{d(Tx_{2n+1},P^n z)+d(Sz,R^n x_{2n+1})+d(R^n x_{2n+1},P^n z)}\right) \end{aligned}$$

$$+\theta max\{d(P^n z,Sz),d(R^n x_{2n+1},Tz),d(P^n z,R^n x_{2n+1}),d(Sz,Tz)\} \tag{3.2}$$

$$\begin{aligned} \Longrightarrow d(Sz,y_{2n+2}) &\preceq \alpha\left(\frac{d(Sz,z)+d(y_{2n+1},y_{2n+2})d(Sz,Sz)}{1+d(Sz,y_{2n+2})}\right) \\ &+ \beta max\{d(Sz,y_{2n+1})),d(Sz,Sz),d(y_{2n+1},Sz)\} \\ &+ \gamma\{d(y_{2n+1},y_{2n+2})+d(y_{2n+2},Sz)+d(Sz,y_{2n+1})\} \\ &+ \eta\left(\frac{d(y_{2n+2},y_{2n+1})d(Sz,Sz)}{d(y_{2n+2},Sz)+d(Sz,y_{2n+1})+d(y_{2n+1},Sz)}\right) \\ &+\theta \ max \ \{d(Sz,Sz),d(y_{2n+1},y_{2n+1}),d(Sz,y_{2n+1}),d(Sz,y_{2n+1})\} \end{aligned}$$

If let $n \to \infty$, we get

$$\begin{aligned} d(Sz,z) &\preceq \alpha\left(\frac{d(Sz,Sz)+d(z,z)d(Sz,Sz)}{1+d(Sz,z)}\right)+\beta max\{d(Sz,z),d(Sz,Sz),d(z,Sz)\} \\ &+ \gamma\{d(z,z)+d(z,Sz)+d(Sz,z)\}+\eta\left(\frac{d(z,z)d(Sz,Sz)}{d(z,Sz)+d(Sz,z)+d(z,Sz)}\right) \\ &+ \theta max\{d(Sz,Sz),d(z,z),d(Sz,z),d(Sz,z)\} \end{aligned}$$

$$\begin{aligned} \Longrightarrow d(Sz,z) &\preceq \beta d(Sz,z)+2\gamma d(Sz,z)+\theta d(Sz,z) \\ \Longrightarrow d(Sz,z) &\preceq (\beta+2\gamma+\theta)d(Sz,z) \\ \Longrightarrow |d(Sz,z)| &\leq (\beta+2\gamma+\theta)|d(Sz,z)| \end{aligned}$$

which is the contradiction that $(\beta+2\gamma+\theta) < 1$ therefore $Sz = z$, since $Sz = z$,then $P^n z = Sz \Longrightarrow P^n z = z$.

Now we will prove that $Tz = z$, putting x=y=z in equation 3.1. We have,

$$\begin{aligned} d(Sz,Tz) &\preceq \alpha\left(\frac{d(P^n z,Sz)+d(R^n z,Tz)d(P^n z,Sz)}{1+d(Sz,Tz)}\right) \\ &+ \beta max\{d(P^n z,R^n z)),d(P^n z,Sz),d(R^n z,Sz)\} \\ &+ \gamma\{d(R^n z,Tz)+d(Tz,P^n z)+d(Sz,R^n z)\} \\ &+ \eta\left(\frac{d(Tz,R^n z)d(Sz,P^n z)}{d(Tz,P^n z)+d(Sz,R^n z)+d(R^n z,P^n z)}\right) \\ &+ \theta max\{d(P^n z,Sz),d(R^n z,Tz),d(P^n z,R^n z),d(Sz,Tz)\} \\ \Longrightarrow d(z,Tz) &\preceq \alpha\left(\frac{d(z,z)+d(Tz,Tz)d(z,z)}{1+d(z,Tz)}\right)+\beta max\{d(z,Tz)),d(z,z),d(Tz,z)\} \\ &+ \gamma\{d(Tz,Tz)+d(Tz,z)+d(z,Tz)\} \\ &+ \eta\left(\frac{d(Tz,Tz)d(z,z)}{d(Tz,z)+d(z,Tz)+d(Tz,z)}\right) \\ &+ \theta max\{d(z,z),d(Tz,Tz),d(z,Tz),d(z,Tz)\} \end{aligned}$$

$$\Longrightarrow d(z,Tz) \preceq \beta d(z,Tz)+2\gamma d(z,Tz)+\theta d(z,Tz)$$
$$\Longrightarrow d(z,Tz) \preceq (\beta+2\gamma+\theta)d(z,Tz)$$

Then

$$\Longrightarrow |d(z,Tz)| \leq (\beta + 2\gamma + \theta)|d(z,Tz)|$$

which is again a contradiction. Therefore $Tz = z$ and this implies $R^n z = Tz \Longrightarrow R^n z = z,$ now we will prove that $Pz = z.$ Putting $x = pz$ and $y = z$ in equation 3.1, we get,

$$\begin{aligned}
d(S(pz),Tz) &\preceq \alpha\left(\frac{d(P^n pz, Spz) + d(R^n z, Tz)d(P^n pz, Spz)}{1 + d(Spz, Tz)}\right)\\
&+ \beta max\{d(P^n pz, R^n z)), d(P^n pz, Spz), d(R^n z, Spz)\}\\
&+ \gamma\{d(R^n z, Tz) + d(Tz, P^n pz) + d(Spz, R^n z)\}\\
&+ \eta\left(\frac{d(Tz, R^n z)d(Spz, P^n pz)}{d(Tz, P^n pz) + d(Spz, R^n z) + d(R^n z, P^n pz)}\right)\\
&+ \theta max\{d(P^n pz, Spz), d(R^n z, Tpz), d(P^n pz, R^n z), d(Spz, Tpz)\}
\end{aligned}$$

$$\begin{aligned}
d(pz,z) &\preceq \alpha\left(\frac{d(pz,pz) + d(z,z)d(pz,pz)}{1 + d(pz,z)}\right)\\
&+ \beta max\{d(pz,z), d(pz,pz), d(z,pz)\}\\
&+ \gamma\{d(z,z) + d(z,pz) + d(pz,z)\}\\
&+ \eta\left(\frac{d(z,z)d(pz,pz)}{d(z,pz) + d(pz,z) + d(z,pz)}\right)\\
&+ \theta max\{d(pz,pz), d(z,z), d(pz,z), d(pz,z)\}
\end{aligned}$$

$$\begin{aligned}
d(pz,z) &\preceq \beta d(pz,z) + 2\gamma d(z,pz) + \theta d(pz,z)\\
\Longrightarrow d(pz,z) &\preceq (\beta + 2\gamma + \theta)d(pz,z)
\end{aligned}$$

Now,

$$|d(pz,z)| \leq (\beta + 2\gamma + \theta)|d(pz,z)|$$

Which is a contradiction, since $(\beta + 2\gamma + \theta) < 1,$ therefore $pz = z : P^n z = z \Longrightarrow P^{n-1}z = z.$ Now we will prove that $Rz = z$ in equation 3.1 we may put $x = z$ and $y = Rz$

$$\begin{aligned}
d(Sz,T(Rz)) &\preceq \alpha\left(\frac{d(P^n z, Sz) + d(R^n(Rz), T(Rz))d(P^n z, Sz)}{1 + d(Sz, T(Rz))}\right)\\
&+ \beta max\{d(P^n z, R^n z)), d(P^n z, Sz), d(R^n(Pz), Sz)\}\\
&+ \gamma\{d(R^n(Rz), T(Rz)) + d(T(Rz), P^n z) + d(Sz, R^n(Rz))\}\\
&+ \eta\left(\frac{d(T(Rz), R^n(Rz))d(Sz, P^n z)}{d(T(Rz), P^n z) + d(Sz, R^n(Rz)) + d(R^n(Rz), P^n z)}\right)
\end{aligned}$$

$$+\theta max\{d(P^n z, Sz), d(R^n(Rz), T(Rz)), d(P^n z, R^n(Rz)), d(Sz, T(Rz))\}$$

$$\begin{aligned} d(z,Rz) &\preceq \alpha\left(\frac{d(z,z)+d(Rz,Rz)d(z,z)}{1+d(z,Rz)}\right) \\ &+ \beta max\{d(z,Rz)),d(z,z),d(Rz,z)\} \\ &+ \gamma\{d(Rz,Rz)+d(Rz,z)+d(z,Rz)\} \\ &+ \eta\left(\frac{d(Rz,Rz)d(z,z)}{d(Rz,z)+d(z,Rz)+d(Rz,z)}\right) \\ &+ \theta max\{d(z,z),d(Rz,Rz),d(z,Rz),d(z,Rz)\} \end{aligned}$$

$$\begin{aligned} \Longrightarrow d(z,Rz) &\preceq \beta d(z,Rz)+2\gamma d(Rz,z)+\theta d(z,Rz) \\ \Longrightarrow d(z,Rz) &\preceq (\beta+2\gamma+\theta)d(z,Rz) \end{aligned}$$

now,

$$|d(z,Rz)| \leq (\beta + 2\gamma + \theta)|d(z,Rz)|$$

but, $(\beta + 2\gamma + \theta) < 1$ which is a contradiction, therefore $Rz = z$.

Uniqueness:

Let u be another fixed point of S,T,R^n and P^n then we have

$$\begin{aligned} d(Sz,Tu) &\preceq \alpha\left(\frac{d(P^nz,Sz)+d(R^nu,Tu)d(P^nz,Sz)}{1+d(Sz,Tu)}\right) \\ &+ \beta\ max\{d(P^nz,R^nu)d(P^nz,Sz)d(R^nu,Sz)\} \\ &+ \gamma\{d(R^nu,Tu)+d(Tu,P^nz)+d(Sz,R^nu)\} \\ &+ \eta\left(\frac{d(Tu,R^nu)d(Sz,P^nz)}{d(Tu,P^nz)+d(Sz,R^nu)+d(R^nu,P^nz)}\right) \\ &+ \theta\ max\{d(P^nz,Sz),d(R^nu,Tu),d(P^nz,R^nu),d(Sz,Tu)\} \end{aligned}$$

$$\begin{aligned} d(z,u) &\preceq \alpha\left(\frac{d(z,z)+d(u,u)d(z,z)}{1+d(z,u)}\right)+\beta\ max\{d(z,u)d(z,z)d(u,z)\} \\ &+ \gamma\{d(u,u)+d(u,z)+d(z,u)\}+\eta\left(\frac{d(u,u)d(z,z)}{d(u,z)+d(z,u)+d(u,z)}\right) \\ &+ \theta\ max\{d(z,z),d(u,u),d(z,z),d(z,u)\} \end{aligned}$$

$$\begin{aligned} d(z,u) &\preceq \beta d(z,u)+2\gamma d(z,u)+\theta d(z,u) \\ \Longrightarrow d(z,u) &\preceq (\beta+2\gamma+\theta d(z,u) \\ Now,\ |d(z,u)| &\preceq (\beta+2\gamma+\theta)|d(z,u)| \end{aligned}$$

which is again a contradiction since $(\beta + 2\gamma + \theta) < 1$ than $z = u$ is a unique common fixed point of S,T,R^n and P^n. **Case II:**

We consider the case $d(Tx_{2n+1},P^nx_{2n})+d(Sx_{2n},R^nx_{2n+1})+d(R^nx_{2n+1},P^nx_{2n}) = 0$ for any ‘n_0. $\Longrightarrow$ $d(Sx_{2n},Tx_{2n+1}) = 0$. So that $y_{2n} = Sx_{2n} = y_{2n+1} = R_nx_{2n+1} =$

$Tx_{2n+1} = P_nx_{2n+2} = y_{2n+2}$ thus we have $y_{2n+1} = Sx_{2n} = P_nx_{2n} = y_{2n}$ then there exist n_1 and m_1 such that $n_1 = Sm_1 = P^nm_1 = m_1$.

Similarly $y_{2n+2} = Tx_{2n+1} = R^nx_{2n+1} = y_{2n+1}$ then there exist n_2 and m_2 such that $n_2 = Tm_2 = R^nm_2 = m_2$. As $d(Tm_2,P^nm_1)+d(Sm_1,R^nm_1)+d(R_2^n,P^nm_1)=0 \Longrightarrow$ that $d(Sm_1,Tm_2) =$ 0, so that $n_1 = Sm_1 = P^nm_1 = Tm_2 = R^nm_2 = n_2$. which in turn yields that $n_1 = Sn_1 = P^nm_1 = P^nn_1$. Similarly one can also have $n_2 = Tn_2 = R^nn_2$. As $n_1 = n_2$, *implies* $n_1 = ST_1 = Tn_1 = R^nn_1$, therefore $n_1 = Sn_1 = Pn_1 = Rn_1 = Tn_1$ for all P^n and R^n. Hence $n_1 = n_2$ is common fixed point.

Uniqueness:

If v is an another fixed point of S,T,R^n and P^n then we have $v = Sv = Pv = Rv = Tv$ for even p^n and R^n therefore $d(Tv_1,P^n n_1) + d(Sn_1,R^n v_1) + d(R^n v,P^n n_1)) = 0$ so that $d(n_1,v_1) = 0$.

Hence $d(Sn_1,Tv) = 0 \Longrightarrow n_1 = v_1$. Hence n_1 is unique fixed point of S,T,R^n and P^n.

Corollary 3.2. *Let (X,d) be a complex valued metric space and S,T,R and P be four mappings in X satisfies the condition*

(1) *$S(X) \subset R(X)$ and $T(X) \subset P(X)$;*

(2) *The Pair (P,S) and (R,T) are weakly compatible;*

(3) *For each $x,y \in X$ and $x 6= y$,;*

(i): *If $d(Ty,Px) + d(Sx,Ry) + d(Ry,Px) = 0$. then $d(Sx,Ty) = 0$;*

(ii): *If $d(Ty,Px) + d(Sx,Ry) + d(Ry,Px)$ 6= 0 than following identity holds;*

$$\begin{aligned} d(Sx,Ty) &\preceq \alpha\left(\frac{d(Px,Sx)+d(y,Ty)d(Px,Sx)}{1+d(Sx,Ty)}\right) \\ &+ \beta max\{d(Px,Ry)), d(Px,Sx), d(Ry,Sx)\} \\ &+ \gamma\{d(Ry,Ty)+d(Ty,Px)+d(Sx,Ry)\} \\ &+ \eta\left(\frac{d(Ty,Ry)d(Sx,Px)}{d(Ty,Px)+d(Sx,Ry)+d(Ry,Px)}\right) \\ &+ \theta max\{d(Px,Sx), d(Ry,Tx), d(Px,Ry), d(Sx,Tx)\} \quad (3.3) \end{aligned}$$

Where $\alpha + \beta + 2\gamma + \eta + \theta < 1$. Then P,R,S and T have a unique common fixed point.

Theorem 3.3. *Let (X,d) be a complex valued metric space and S,T be the self mappings and P^n and R^n be the collection of self mappings in X, such that for all $x,y \in X$.*

(1) *$S(X) \subset R^n(X)$ and $T(X) \subset P^n(X)$;*

(2) *(P^n,S) are compatible, P^n or S is continuous and (R^n,T) are weakly compatible;*

(3) *(R^n,T) are compatible, R^n or T is continuous and (P^n,S) are weakly compatible;*

(4) *For each $x,y \in X$ and $x 6= y$,;*

$$\begin{aligned} d(Sx,Ty) &\preceq \alpha\left(\frac{d(P^nx,Sx)+d(R^ny,Ty)d(P^nx,Sx)}{1+d(Sx,Ty)}\right) \\ &+ \beta max\{d(P^nx,R^ny)), d(P^nx,Sx), d(R^ny,Sx)\} \\ &+ \gamma\{d(R^ny,Ty)+d(Ty,P^nx)+d(Sx,R^ny)\} \\ &+ \eta\left(\frac{d(Ty,R^ny)d(Sx,P^nx)}{d(Ty,P^nx)+d(Sx,R^ny)+d(R^ny,P^nx)}\right) \\ &+ \theta max\{d(P^nx,Sx), d(R^ny,Ty), d(P^nx,R^ny), d(Sx,Tx)\} \, (3.4) \end{aligned}$$

holds where α, β, γ, η and θ are non-negative real number with $\alpha+\beta+2\gamma+\eta+\theta < 1$. Then P^n,R^n,S and T have a unique common fixed point.

Proof. By Theorem [3.1] $\{y_n\}$ is a Cauchy sequence. Since X is complete, that means $\{y_n\}$ must converge to some point z (say).Thus subsequence $\{Sx_{2n}\}$, $\{R^n x_{2n+1}\}$,$\{Tx_{2n+1}\}$, and $\{P^n x_{2n+2}\}$, also converges to z. that is

$$lim_{n\to\infty} y_n = lim_{n\to\infty} Sx_{2n} = lim_{n\to\infty} R_n x_{2n+1} = lim_{n\to\infty} P_n x_{2n+2}$$

$$= lim_{n\to\infty} Tx_{2n+1} = z \tag{3.5}$$

Assume that S is Continuous,since (P^n,S) are compatible, we have

$$lim_{n\to\infty} P^n(Sx_{2n+2}) = lim_{n\to\infty} S(P^n x_{2n+2}) = Sz \tag{3.6}$$

Now putting $x = x_{2n+2}$, $y = x_{2n+1}$ then we have

$$\begin{aligned}
&d(P^n(Sx_{2n+2}), Tx_{2n+1})\\
&\preceq \alpha\left(\frac{d(P^n x_{2n+2}, Sx_{2n+2}) + d(R^n x_{2n+1}, Tx_{2n+1})d(P^n x_{2n+2}, Sx_{2n+2})}{1 + d(Sx_{2n+2}), Tx_{2n+1}))}\right)\\
&+\beta max\{d(P^n x_{2n+2}, R^n x_{2n+1}), d(P^n x_{2n+2}, Sx_{2n+2}), d(R^n x_{2n+1}), Sx_{2n+2})\}\\
&+\gamma\{d(R^n x_{2n+1}, Tx_{2n+1}) + d(Tx_{2n+1}, P^n x_{2n+2}) + d(Sx_{2n+2}, R^n x_{2n+1})\}\\
&+\eta\left(\frac{d(Tx_{2n+1}), R^n x_{2n+1})d(Sx_{2n+2}, P^n x_{2n+2})}{d(Tx_{2n+1}, P^n x_{2n+2}) + d(Sx_{2n+2}, R^n x_{2n+1}) + d(R^n x_{2n+1}, P^n x_{2n+2})}\right)\\
&+\theta max\{d(P^n x_{2n+2}, Sx_{2n+2}), d(R^n x_{2n+1}, Tx_{2n+1}), d(P^n x_{2n+2}, R^n x_{2n+1})\\
&\quad (Sx_{2n+2}, Tx_{2n+2})\}
\end{aligned}$$

,*d*

Let $n \to \infty$, in the above inequality and using 3.5 and 3.6, we get

$$\begin{aligned}
d(Sz, z) \quad &\preceq \alpha\left(\frac{d(z,z) + d(z,z), d(z,z)}{1 + d(z,z)}\right) + \beta max\{d(z,z), d(z,z), d(z,z)\}\\
&+\gamma\{d(z,z) + d(z,z) + d(z,z)\} + \eta\left(\frac{d(z,z), d(z,z)}{d(z,z) + d(z,z) + d(z,z)}\right)\\
&+\theta max\{d(z,z), d(z,z)), d(z,z), d(z,z)\}
\end{aligned}$$

$$\implies d(Sz,z) \preceq 0 \ that \ is \ |d(Sz,z) \leq 0| \ hence \ Sz = z.$$

Now putting $x = z$, and $y = x_{2n+1}$ then we have in equation 3.8

$$\begin{aligned}
d(Sz, Tx_{2n+1}) \ \preceq \ & \alpha\left(\frac{d(P^n z, Sz) + d(R^n x_{2n+1}, Tx_{2n+1})d(P^n z, Sz)}{1 + d(Sz, Tx_{2n+1})}\right)\\
+ \ & \beta max\{d(P^n z, R^n x_{2n+1}), d(P^n z, Sz), d(R^n x_{2n+1}, Sz)\}\\
+ \ & \gamma\{d(R^n x_{2n+1}, Tx_{2n+1}) + d(Tx_{2n+1}, P^n z) + d(Sz, R^n x_{2n+1})\}\\
+ \ & \eta\left(\frac{d(Tx_{2n+1}, R^n x_{2n+1})d(Sz, P^n z)}{d(Tx_{2n+1}, P^n z) + d(Sz, R^n x_{2n+1}) + d(R^n x_{2n+1}, P^n z)}\right)\\
+ \ & \theta max\{d(P^n z, Sz), d(R^n x_{2n+1}, Tx_{2n+1}), d(P^n z, R^n x_{2n+1}), d(Sz, Tz)\}
\end{aligned}$$

Let $n \to \infty$, in the above inequality and using 3.5 and 3.6, we get

$$\begin{aligned}
d(z,z) \ \preceq \ & \alpha\left(\frac{d(P^n z, z) + d(z,z)d(P^n z, Sz)}{1 + d(z,z)}\right)\\
+ \ & \beta max\{d(P^n z, z), d(P^n z, z), d(z,z)\}\\
+ \ & \gamma\{d(z,z) + d(z, P^n z) + d(z,z)\}\\
+ \ & \eta\left(\frac{d(z,z)d(z, P^n z)}{d(z, P^n z) + d(z,z) + d(z, P^n z)}\right)\\
+ \ & \theta max\{d(P^n z, z), d(z,z), d(P^n z, z), d(z,z)\}
\end{aligned}$$

i.e.

$$0 \preceq \{\alpha + \beta + \gamma + \theta\}|d(P^n z, z)|$$

Then, $|d(P^n z,z)| \geq 0$, hence $P^n z = z$. Since $S(x) \subset R^n x$, there exist a point $w \in X$ such that $Sz = R^n w$. Suppose that $R^n w$ 6= Tw. Now to prove that $R^n w = Tw$ and given that $Sz = z = R^n w$. From 3.8 putting $x = z$ and $y = w$,
we obtain

$$\begin{aligned} d(Sz,Tw) &\preceq \alpha\left(\frac{d(P^n z,Sz)+d(R^n w,Tw)d(P^n z,Sz)}{1+d(Sz,Tw)}\right) \\ &+ \beta max\{d(P^n z,R^n w)),d(P^n z,Sz),d(R^n w,Sz)\} \\ &+ \gamma\{d(R^n w,Tw)+d(Tw,P^n z)+d(Sz,R^n w)\} \\ &+ \eta\left(\frac{d(Tw,R^n w)d(Sz,P^n z)}{d(Tw,P^n z)+d(Sz,R^n w)+d(R^n w,P^n z)}\right) \\ &+ \theta max\{d(P^n z,Sz),d(R^n w,Tw),d(P^n z,R^n w),d(Sz,Tz)\} \quad (3.7) \end{aligned}$$

$$\begin{aligned} d(z,Tw) &\preceq \alpha\left(\frac{d(z,z)+d(z,Tw)d(z,z)}{1+d(z,Tw)}\right)+\beta max\{d(z,z)),d(z,z),d(z,z)\} \\ &+ \gamma\{d(z,Tw)+d(Tw,z)+d(z,z)\}+\eta\left(\frac{d(Tw,z)d(z,z)}{d(Tw,z)+d(z,z)+d(z,z)}\right) \\ &+ \theta max\{d(z,z),d(w,Tw),d(z,z),d(z,z)\} \end{aligned}$$

$$\begin{aligned} d(z,Tw) &\preceq +2\gamma d(z,Tw)+\theta d(z,Tw) \\ &\preceq (2\gamma+\theta)d(z,Tw) \end{aligned}$$

$$\implies |d(z,Tw)| \leq (2\gamma+\theta)|d(z,Tw)| \implies Tw = z$$

Therefore $Tw = z$ hence $R^n w = z = Tw$. Thus $R^n w = Tw$. since $R^n w$ and T are weakly compatible then $R^n z = R^n(Tw) = Tz$. Thus z is a coincidence point of
R^n and T, now to prove $Tz = z$, putting $x = z$ and $y = z$ in 3.8

$$\begin{aligned} d(Sz,Tz) &\preceq \alpha\left(\frac{d(P^n z,Sz)+d(R^n z,Tz)d(P^n z,Sz)}{1+d(Sz,Tz)}\right) \\ &+ \beta max\{d(P^n z,R^n z)),d(P^n z,Sz),d(R^n z,Sz)\} \\ &+ \gamma\{d(R^n z,Tz)+d(Tz,P^n z)+d(Sz,R^n z)\} \\ &+ \eta\left(\frac{d(Tz,R^n z)d(Sz,P^n z)}{d(Tz,P^n z)+d(Sz,R^n z)+d(R^n z,P^n z)}\right) \\ &+ \theta max\{d(P^n z,Sz),d(R^n z,Tz),d(P^n z,R^n z),d(Sz,Tz)\} \end{aligned}$$

$$\begin{aligned} d(z,Tz) &\preceq \alpha\left(\frac{d(z,z)+d(z,Tz)d(z,z)}{1+d(z,Tz)}\right)+\beta max\{d(z,z)),d(z,z),d(z,z)\} \\ &+ \gamma\{d(z,Tz)+d(Tz,z)+d(z,z)\}+\eta\left(\frac{d(Tz,z)d(z,z)}{d(Tz,z)+d(z,z)+d(z,z)}\right) \\ &+ \theta max\{d(z,z),d(z,Tz),d(z,z),d(z,Tz)\} \end{aligned}$$

$$\begin{aligned} d(z,Tz) &\preceq 2\gamma d(z,Tz)+\theta d(z,Tz) \\ d(z,Tz) &\preceq (2\gamma+\theta)d(z,Tz) \end{aligned}$$

$\Longrightarrow |d(z,Tz)| \leq (2\gamma + \theta)|d(z,Tz)| \Longrightarrow Tz = z$

Uniqueness Let u be an another common fixed point of P^n, R^n, S *and* T, then we have from 3.8

$$
\begin{aligned}
d(Sz,Tu) &\preceq \alpha\left(\frac{d(P^n z,Sz)+d(R^n u,Tu)d(P^n z,Sz)}{1+d(Sz,Tu)}\right)\\
&+ \beta max\{d(P^n z,R^n u)),d(P^n z,Sz),d(R^n u,Sz)\}\\
&+ \gamma\{d(R^n u,Tu)+d(Tu,P^n z)+d(Sz,R^n u)\}\\
&+ \eta\left(\frac{d(Tu,R^n u)d(Sz,P^n z)}{d(Tu,P^n z)+d(Sz,R^n u)+d(R^n u,P^n z)}\right)\\
&+ \theta max\{d(P^n z,Sz),d(R^n u,Tu),d(P^n z,R^n u),d(Sz,Tz)\}
\end{aligned}
$$

$$
\begin{aligned}
d(z,u) &\preceq \alpha\left(\frac{d(z,z)+d(u,u)d(z,z)}{1+d(z,u)}\right)+\beta max\{d(z,u),d(z,z),d(u,z)\}\\
&+ \gamma\{d(u,u)+d(u,z)+d(z,u)\}+\eta\left(\frac{d(u,u)d(z,z)}{d(u,z)+d(z,u)+d(u,z)}\right)\\
&+ \theta max\{d(z,z),d(u,u),d(z,u),d(z,z)\}
\end{aligned}
$$

$$
\begin{aligned}
d(z,u) &\preceq \beta d(z,u)+2\gamma d(z,u)+\theta d(z,u)\\
d(z,u) &\preceq (\beta+2\gamma+\theta)d(z,u)\\
\Longrightarrow |d(z,u)| &\leq (\beta+2\gamma+\theta)|d(z,u)|
\end{aligned}
$$

i.e.

$$|d(z,u)| \leq (\beta + 2\gamma + \theta)|d(z,u)|$$

Which is a contradiction since $(\beta+2\gamma+\theta) < 1$. therefore $d(z,u) = 0 \Longrightarrow z = u$.
Hence z is a common fixed point of P^n, R^n, S *and* T.

Corollary 3.4. *Let* (X,d) *be a complex valued metric space and* R,P,S *and* T *be reciprocally continuous self mappings in* X, *such that for all* $x,y \in X$.

(1) $S(X) \subset R(X)$ *and* $T(X) \subset P(X)$;

(2) (P,S) *are compatible,* P *or* S *is continuous and* (R,T) *are weakly compatible;*

(3) (R,T) *are compatible,* R *or* T *is continuous and* (P,S) *are weakly compatible;*

(4) *For each* $x,y \in X$ *and* x 6= y,;

$$
\begin{aligned}
d(Sx,Ty) &\preceq \alpha\left(\frac{d(Px,Sx)+d(Ry,Ty)d(Px,Sx)}{1+d(Sx,Ty)}\right)\\
&+ \beta max\{d(Px,Ry)),d(Px,Sx),d(Ry,Sx)\}\\
&+ \gamma\{d(Ry,Ty)+d(Ty,Px)+d(Sx,Ry)\}\\
&+ \eta\left(\frac{d(Ty,Ry)d(Sx,Px)}{d(Ty,Px)+d(Sx,Ry)+d(Ry,Px)}\right)\\
&+ \theta max\{d(Px,Sx),d(Ry,Ty),d(Px,Ry),d(Sx,Tx)\} \qquad (3.8)
\end{aligned}
$$

holds where α, β, γ, η *and* θ *are non-negative real number with* $\alpha+\beta+2\gamma+\eta+\theta < 1$. *Then* P,R,S *and* T *have a unique common fixed point.*

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