Mathematical Model of Face Recognition

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Abstract: Face detection is the process of identifying faces in a photograph using various approaches. Face identification techniques abound, but identifying a face against a complex backdrop, an obscured face, or in a variety of lighting situations may be difficult. Face identification against complicated backdrops and under various lighting conditions will be the focus of this algorithm's development. Surveillance systems that prevent the entry of unfamiliar persons, the identification of criminals in public areas and ATM theft may all benefit from face recognition technology.

Keyword: Face detection, Face identification, Template Matching, Mathematical Model

I. Introduction

Face recognition is a non-intrusive biometric technique that goes back to the 1960s and involves comparing a face to a database of known faces. People have been studying facial characteristics since antiquity in an attempt to figure out which aspects aid us in basic identification tasks like recognising a person, determining their age and gender, and even evaluating their facial expressions and even their attractiveness. When we look at DaVinci's Mona Lisa, we can't help but be taken aback by her grin. Psychologists, social scientists, artists, and most lately, mathematicians and engineers, have all been drawn to this field of study. Some of the mathematical methods employed in the most advanced face-recognition systems are discussed here. When it comes to face-recognition systems, the first rule is that they must be able to handle both external and internal changes. This includes things like the person's location in relation to the camera and the amount of light they're exposed to. Since the majority of commercial applications employ enormous databases of faces, face-recognition algorithms must be very efficient. When faced with all of these constraints, mathematical modelling is anything but straightforward. Simple invariant theories like Lie group invariants are clearly out of the question, and we'll need more specialised equipment to get the job done. What follows are some of the most common methods used to solve these invariant pattern recognition problems. When looking for images with skin or regions that resemble skin, the colour of the skin is a key indicator. The colour of someone's skin may be used to identify their face or hand in both dynamic and static photographs. The colour of a person's skin may also be used to identify photographs of nude persons on the Internet. When it comes to analysing medical photographs, skin colour may also be utilised. Segmenting a picture based on skin colour may be helpful in the detection of skin cancer. Skin is an important characteristic in the field of image detection, notably in face recognition, since

(1) it covers most of the face image area, and

(2) skin of various persons seems to vary over a large range, yet the variance is much less in colour (chromaticity) than in brightness.

Skin region identification in digital images is more practical and straightforward to execute from a computational standpoint. Furthermore, the chromaticity distribution of skin colour among ethnic groups follows a Gaussian pattern. Skin model is studied in three colour spaces (RBG, HSB, and YCCB) and is represented in histograms and skin colour distributions in our study

II. Literature Review

Sawhney et al. (2019), One of the biometric technologies used to enhance this system is face recognition. Facial recognition is widely utilised in a variety of biometric verification applications, such as video surveillance and CCTV footage systems, computer-human interaction, and access control systems in both indoor and network environments. With this architecture, the issue of proxies and students being tagged present even when they are not physically there may be overcome. This research offers a model for the creation of an automated attendance management system for students of a class by utilising Eigenface values, Principal Component Analysis (PCA) and Convolutional Neural Network (CNN) using Eigenface values, PCA and PCA (CNN). After these steps, it should be possible to compare the identified faces to the database of student faces. Student attendance and records may be effectively managed using this methodology.

An & Ruan (2006), In the discussion that follows, a new mathematical model for Enhanced Fisher's Linear Discriminant (EFLD) is presented in this study. Both the within-class and the between-class scatter are taken into consideration as FLD doses in EFLD, which has two key advantages: first, it may adaptively discriminate distinct variables of sample vector according to their scale in statistics. The EFLD characteristics that are extracted have a high degree of classification reliability. EFLD surpasses certain well-known algorithms (PCA, FLD, and ICA) when it comes to dealing with considerable variations in lighting direction, position, and facial expression. Another way EFLD might help classifying algorithms is by replacing FLD with EFLD in algorithms that employ FLD to extract classifiable characteristics. This could lead to new classifying algorithms being presented.

Ushir et al. (2013), In a video frame, an image or a video file, a face recognition system is used to detect human faces without the need for human interaction. An urgent problem in a variety of fields. It has the potential to play a big role in crime management and law enforcement because of the broad spectrum of growth in crime. Eigenfaces approach is utilised in this research for face recognition, and in order to enhance its performance, the key-frame notion is presented via the use of a colour histogram. At this stage, an eigenface for the provided face picture must be generated in order to determine how far apart the pre-stored faces are from each other in terms of Euclidean distance. The individual is most like the eigenface with the shortest Euclidian distance. Using this technology is the first effective face-recognition technique.

Juneja & Gill (2015), Facial ROI Extraction is critical to the accuracy and speed of face identification. Identifying the face image in a surveillance system with many photos of both objects and people gets increasingly difficult. This study proposes a precise face localisation in real-time photos and videos. The low-level strategies used in this study have been used to create a hybrid model. In order to identify the face area more precisely, the window-based mathematical filter model is combined with a colour model transformation. Group photos and films are used for the experimenting. Facial regions may be accurately extracted from the data.

Suvorov & Shleymovich (2020), Biometric features may be used to automatically identify a person's identification based on their unique traits or characteristics. The process of biometric identification involves creating reference templates and comparing them to fresh input data. Algorithms for iris pattern recognition have a high degree of accuracy and a low percentage of incorrect identifications in actual use. Many other biometric characteristics can't compete with the great degree of freedom (almost 249), density, and consistency of the iris pattern. A high degree of recognition reliability is critical for searching large datasets. In contrast to the one-to-one check mode, which can only be used for modest calculation counts, the one-to-many identification mode is applicable to all calculations. Recognition accuracy, false acceptance rate, and false rejection rate are used to describe the qualitative properties of biometric identification systems. Due to these qualities, it is possible to compare various techniques of identifying people and evaluate the overall effectiveness of the system. The mathematical model of iris pattern biometric identification is explained in this article, along with its features. There are also studied comparison data between the model and the actual recognition procedure. Such a study was carried out by reviewing current iris pattern recognition algorithms based on unique features vectors in order to produce such an assessment Detailed information on the Python-based software package is provided herein. It produces probabilistic distributions and generates big data sets for testing. Such data sets may also be utilised to train the neural network that makes the identification decision. To further improve the system's quality, it was recommended that multiple different iris pattern detection algorithms be combined into a single algorithm.

Spreeuwers (2017), This approach is known as de-duplication, and it's used to remove or connect several copies of the same data. Specific de-duplication applications, such as those for a new passport and a check to see if the person already has one under another name, are taken into account. For this purpose, an image of the subject's face is compared to every passport photo in the country. Using state-of-the-art face recognition, they find that for a big database, around two out of three duplicates may be detected, while just a few or no false duplicates are discovered. This suggests that face-recognition-based de-duplication in practise is possible. It is further shown that by employing a variable, subject-specific false match rate, a Poisson distribution accurately describes the likelihood that k false duplicates are returned during de-duplication. Using a vast database of genuine passport photos, they provide experimental findings and conclude that our model accurately predicts the outcomes.

Rassomakhin et al. (2018), For this project, a mathematical model for the probability distribution of minutiae in biometric fingerprint photographs is being developed. Using heuristic analysis of fingerprint scanning data, the proposed model takes into consideration the nature of the inaccuracies.

Miao et al. (2020), One more complex visual processing challenge is tackled using the partial differential equation learning model: face recognition. Partial differential equation learning model-based feature selection is suggested in this paper. Rotation and translation are invariant to the retrieved features, whereas illumination changes are less likely to affect the results. This article initially employs a face identification method in face recognition technology to identify the face and intercept expression data, and then determines the rising rate. Facial expressions are then detected and weighted according to the better model of concentration analysis and assessment developed by Chinese college students. In order to arrive at a final concentration score, we multiply the head-up rate by the expression score. Teachers may use the outcomes of the experiments and the analysis of the data they gathered in the classroom to draw their own conclusions and make their own instructional decisions. It is possible to combine the locality of the -nearest neighbour approach with the resilience of sparse representation by selecting a large neighbourhood set for each face and then extracting sparse representations of the sample points in the neighbourhood. Nonnegative block alignment uses sparse preserving nonnegative reconstruction coefficients to describe local geometry and weighted distance to explain class separability in constructing a discriminant partial optimization model. In a variety of actual and simulated occlusion scenarios, both algorithms provide excellent grouping and recognition results, demonstrating the durability and efficacy of the methods. An in-class practise exam, instructor questions, and interviews with students and teachers were all used in this research to confirm the model's dependability. There is great accuracy and reliability in the suggested combined assessment approach based on expression and head-up rate.

Zhang et al. (2020), Mathematical models are a common and efficient way to learn about the world around us. The mathematical depiction of the human face is particularly difficult because of the face's complex physiological systems and dynamic activities. In this article, a multi-Gaussian function-based facial image representation model named GmFace is suggested. Two-dimensional Gaussian function gives a symmetric bell surface with a shape that may be modified by parameters in this model. If you want to change the GmFace parameter solution issue into a network optimization problem, you use Gaussian functions as neurons and assign each parameter's value to one of the GmNet's parameters. Steps in face modelling include:

(1) initialization of GmNet;

(2) feeding GmNet face images;

(3) training and converging the face model GmFace, and

(4) pulling GmFace's parameters out (as if they were face model parameters).

In addition, a number of mathematically based facial picture transformations are possible with GmFace.

III. Mathematical model

A mathematical model is a representation of a system in terms of mathematical ideas and terminology. In order to understand a system, a model may be used to examine the interactions between its many components and forecast the system's behaviour. Our system's mathematical modelling looks like this:

S= {∑, F, δ, C} S = Face Recognition. ∑ = set of input symbols = {Video File, image, character information} F = set of output symbol = {Match Found then notification to user, Not Found} δ = 1.

1. Start
2. Read training set of images N*N images
3. Resize image dimensions to N2*1
4. Select training set of N2*M Dimensions, M: number of sample images
5. Find average face, subtract from the faces in the training set, create matrix A

$$\Psi = \frac{1}{M} \sum_{i=1}^{M} \Gamma i$$

Where, Ψ= average image, M= number of images, and Γi= image vector. Φi = Γi – Ψ Where, i = 1, 2, 3, …, M. A = [Φ1, Φ2, Φ3…ΦM]

6. Calculate covariance matrix: AA'

$C=A^{T}*A$

7. Calculate the c covariance matrix's eigenvectors.

8. In order to find the eigenfaces, divide the entire number of eigenvectors by the total number of training pictures.

9. The chosen eigenvectors are multiplied by the A matrix in order to construct a reduced eigenface space.

10. Determine the eigenface of the picture.

11. Calculate the eigenfaces' Euclidian distances from the picture.

12. Calculate the shortest distance between two points using the Euclidian formula.

13. Image with the smallest Euclidian distance or an image that cannot be recognised are the output options. Eigenfaces will create the grayscale pictures, and the method will only execute on key frames if C is true.

IV. Face Recognition Methods

There are a variety of ways to recognise a person's face, including the following:

a. Geometric Based / Template Based

Algorithms for facial recognition may be categorised as either geometry-based or template-based. Tools like SVM, PCA, LDA, LDA-like statistical approaches, kernel methods, and trace transforms may be used to build template-based methods, as can other statistical methods like SVM. The geometric feature-based approaches examine the geometric connection between local face characteristics. Additionally, it's called feature-based.

b. Piecemeal / Wholistic

In an effort to identify the most important qualities, numerous academics used this method to examine the relationship between the parts of a face and its functions. The eyes, a combination of traits, and so on, have been used in many ways to identify someone. Hidden Markov Models and facial recognition feature processing come within this area.

c. Appearance-Based / Model-Based

Images of a face are shown using an appearance-based technique. As a high-dimensional vector, images are often taken into account. A feature space may be derived from an image divide using this method. The training set compared to the sample picture. The model-based method, on the other hand, aims to model a face. The model's parameters were utilised to recognise the picture and the fresh sample it was fed into the model.

There are two ways to categorise the appearance-based technique. In the direct method, we employ Ex-PCA, LDA, and IDA; in the nonlinear approach, we use Kernel PCA. As an alternative, the model-based technique is divided into two categories: either 2D or 3D. Matching of an Elastic Bunch Graph from the past is used.

V. Template / Statistical / Neural Networks Based

5.1. Template Matching

Samples, models, pixels, textures, and so on are all used in template matching to represent the patterns to be matched. In most cases, a correlation or distance measure is used as the recognition function.

5.2. Statistical Approach

When using the statistical technique, features are the ways in which the patterns are represented. The discriminant function's recognition function. For each characteristic, a picture was created. We want to make sure we choose and use the best statistical tool for the job.

VI. Conclusion

It has been shown that mathematical skin colour modelling has a theoretical foundation and may be used in the field of face identification. As a result, this paper explained the theoretical foundations of skin colour modelling and how it might be used to face identification. Face detection is the technique of detecting individuals in an image by using a variety of methods. Despite the availability of several face recognition methods, distinguishing a face against a complex background, an occluded face, or in a range of lighting settings may be challenging to do. The development of this algorithm will be centred on the recognition of faces against complex backgrounds and in a variety of lighting situations, among other things. Face recognition technology may be useful in a variety of applications, including surveillance systems that block the admission of unknown people, the detection of criminals in public places, and ATM robbery.

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