RELIABILITY ANALYSIS OF A SYSTEM HAVING MAIN UNIT AND HELPING UNIT WITH COLD STANDBY UNIT UNDER PRIORITY BASED REPAIR

Lalit Kumar[1] and A. K. Sharma[2]

1. Department of Mathematics, Kisan PG College, Simbhaoli, Hapur, India

email: abbutyagi@yahoo.com

2. Department of Ag. Statistics, A.S. College, Lakhaoti, Bulandshahr, India

email: drsharmaashok2010@gmail.com

ABSTRACT

This paper deals with the reliability analysis of a system having main unit and helping unit with cold-standby unit under priority based repair. In this system main unit and helping unit are connected in parallel configuration. There is an assumption that main unit can also work when helping units fail but with increasing failure rate. Single repair facility is used for all units. But priority of repair is given to the main unit. The different reliability attributes are obtained by using regenerative point technique. Graphical practices of mean time to system failure and profit function have been studied.

Keywords: Reliability, MTSF, Availability Analysis, Profit Analysis, Regenerative Point Technique.

INTRODUCTION

Various authors [1, 2, 4, 8] have analysed two unit cold and warm standby identical units considering a set of assumptions. Many authors have also analysed the systems of dissimilar units. Agnihotri *et al.* [3] worked with two non-identical operative unit system with repair and inspection. Dhillon and Anude [5] have worked with common case failure analysis of a non-identical units parallel system with arbitrarily distributed repair times. Several authors [6, 7, 9, 10] have also analysed the systems of dissimilar units considering different assumptions as administrative delay in repair, correlated failures and repairs, concept of replacement, common cause failures, human errors etc.

Keeping the idea of dissimilar units we in the present study have analysed a system having main unit and helping unit with cold-standby unit under priority based repair. In this system both the units (main and helping units) are operative and connected in parallel configuration. To improve the effectiveness of the system one helping unit has been taken as cold-standby unit. In the present study it is assumed that even after the failure of all the helping units, main unit will work but with increasing failure rate. Single repair facility is used and priority in repairing is given to the main unit.

By using the regenerative point technique in Markov Renewal Process the following important reliability characteristics of interest are obtained

1. Steady state transition probabilities
2. Mean sojourn time
3. Reliability and mean time to system failure
4. Point-wise and steady state availability of the system
5. Expected busy period of the repairman in time interval (0, t]
6. Profit analysis of the system

MODEL DESCRIPTION AND ASSUMPTIONS

(i) The system consists of non-identical units, main and a helping units are operative and other helping unit is as cold standby.
(ii) Upon failure of an operative helping unit the cold standby helping unit becomes operative instantaneously.
(iii) If the main unit fails, the system becomes down.
(iv) If both the helping units fail then the system will be in up state and in that situation the failure rate of main unit will increase.
(v) Failure rates of main unit and helping unit are constant.
(vi) The repair time distribution of the main unit and helping unit are general.
(vii) Single repairman facility is used.
(viii) If during the repair of helping unit, the main unit fails, then the repair of helping unit is stopped and the priority is given to the main unit. After completing the repair of main unit, the repair of first failed helping unit is re-started and the time already spent in repairing this unit goes to waste.

NOTATIONS AND STATES OF THE SYSTEM

α	Constant failure rate of the helping unit
δ	Constant failure rate of the main unit when helping unit is operative
β	Constant failure rate of the main unit when helping unit are non-operative ($\beta>\delta$)
$f(\cdot)$, $F(\cdot)$	P.d.f. and c.d.f. of time to complete repair of helping unit
$g(\cdot)$, $G(\cdot)$	P.d.f. and c.d.f. of time to complete repair of main unit
$*$	Symbol for Laplace Transform
$\otimes$	Symbol for Convolution
M_o	Main unit as operative
H_o	Helping unit as operative
H_s	Helping unit as cold stand by
H_g	Helping unit is good
M_r	Failed main unit is under repair
H_r	Failed helping unit is under repair
H_R	Repair of helping unit is continued from earlier state
H_{w_1r}	Failed helping unit is under first waiting for repair
H_{w_2r}	Failed helping unit is under second waiting for repair

Using the above notations and assumptions the states of the system are given below:

Up states

$S_0 : (M_o, H_o, H_s)$, $S_1 : (M_o, H_o, H_r)$, $S_2 : (M_o, H_R, H_{w_1r})$, $S_3 : (M_o, H_r, H_{w_1r})$

Down states

$S_4 : (M_r, H_{w_1r}, H_{w_2r})$, $S_5 : (M_r, H_g, H_s)$, $S_6 : (M_r, H_{w_1r}, H_g)$

Transition diagram of the system model is shown in the following figure.

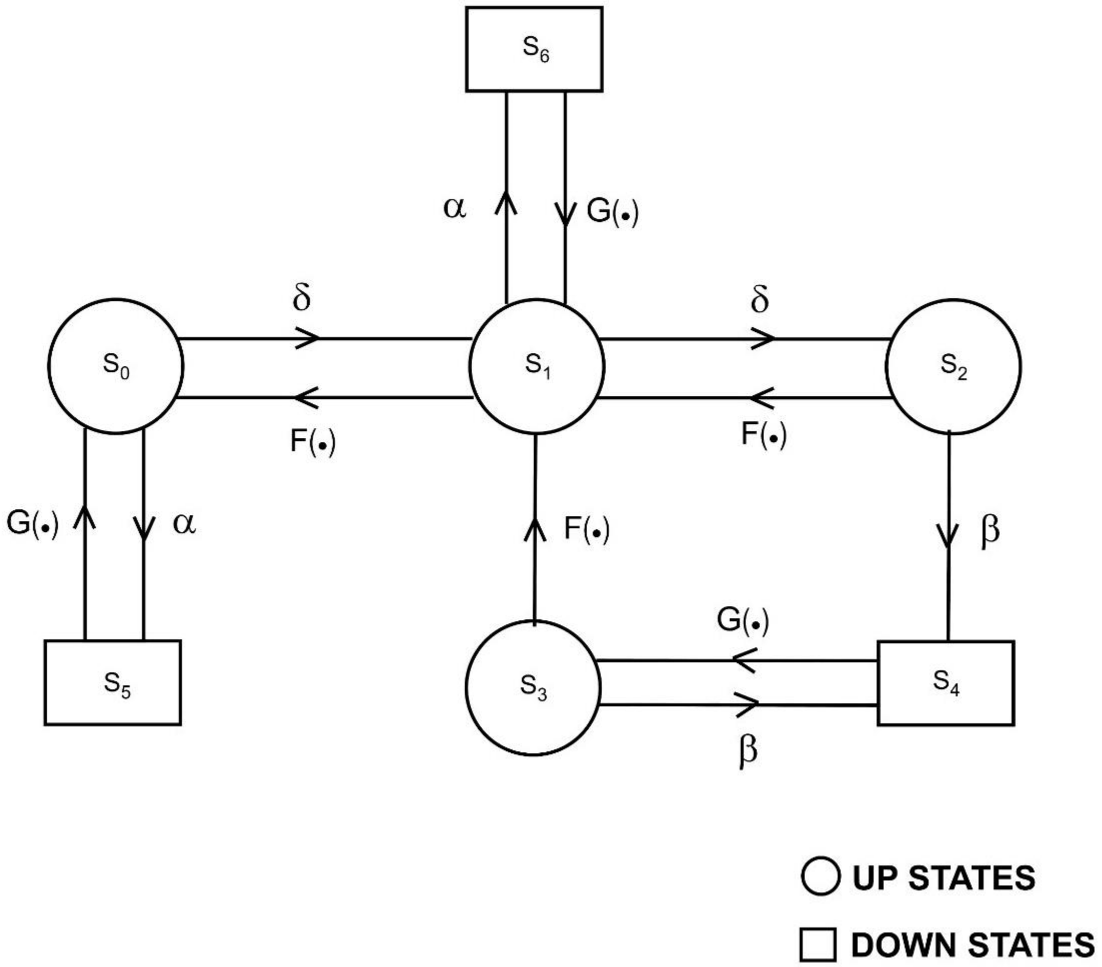

Figure 1

TRANSITION PROBABILITIES

The non zero elements of the transition probability, $P = P_{ij}$ are given below:

$$P_{01} = \frac{\delta}{\alpha+\delta}, \quad P_{05} = \frac{\alpha}{\alpha+\delta}, \quad P_{10} = f^*(\alpha+\delta), \quad P_{12} = \frac{\delta}{\alpha+\delta}[1 - f^*(\alpha+\delta)] = P_{11}^{(2)}$$

$$P_{16} = \frac{\alpha}{\alpha+\delta}[1 - f^*(\alpha+\delta)], \qquad P_{21} = f^*(\beta) = P_{31}, \quad P_{24} = 1 - f^*(\beta) = P_{34}$$

$$P_{43} = 1 = P_{50} = P_{61}$$

The above probabilities satisfies the following relations

$$P_{01} + P_{05} = 1, \qquad P_{10} + P_{12} + P_{16} = 1, \qquad P_{10} + P_{11}^{(2)} + P_{16} = 1$$

$$P_{21} + P_{24} = 1, \qquad P_{31} + P_{34} = 1$$

MEAN SOJOURN TIME

Let ψ_i be the mean sojourn time in state S_i and is defined as the expected time for which the system stays in state S_i before transiting to any other state. Let X_i denotes the sojourn time in state S_i, is given by

$$\psi_i = \int P[X_i > t]dt$$

so that

$$\psi_0 = \frac{1}{\alpha+\delta}, \qquad \psi_1 = \frac{1 - f^*(\alpha+\delta)}{\alpha+\delta}, \qquad \psi_2 = \psi_3 = \frac{1 - f^*(\beta)}{\beta}$$

$$\psi_4 = \psi_5 = \psi_6 = \int \bar{G}(u)du$$

RELIABILITY AND MEAN TIME TO SYSTEM

Let random variable T_i denotes the time to system failure when the system initially starts from states $S_i \in E$, then the reliability of the system is given by

$$R_i(t) = P[T_i > t]$$

To determining $R_i(t)$, we assume the failed states (S_4 to S_6) of the system as observing. By using the simple probability arguments, we observe that $R_0(t)$ is the sum of the following mutually exclusive contingencies.

(i) The system remains up in state S_0 upto time t, the probability of this contingency is given as $Z_0(t) = e^{-(\alpha+\delta)t}$

(ii) System transits from state S_0 to S_1 during time $(u,\ u + du); u \le t$ and then starting from S_1 it remains up continuously during remaining time $(t - u)$. the probability of this contingency is $\int_0^t q_{01}(u)R_1(t-u)du = q_{01}(t)\otimes R_1(t)$

Therefore

$$R_0(t) = Z_0 + q_{01}(t)\otimes R_1(t)$$

Similarly

$$R_1(t) = Z_1(t) + q_{10}(t)\otimes R_0(t) + q_{12}(t)\otimes R_2(t)$$
$$R_2(t) = Z_2(t) + q_{21}(t)\otimes R_1(t) \tag{1-3}$$

where

$$Z_1 = e^{-(\alpha+\delta)t}\bar{F}(t), \qquad Z_2 = e^{-\beta t}\bar{F}(t)$$

by taking the Laplace transform of the relation (1-3), we have

$$R_0^*(s) = Z_0^*(s) + q_{01}^*(s)R_1^*(s)$$
$$R_1^*(s) = Z_1^*(s) + q_{10}^*(s)R_0^*(s) + q_{12}^*(s)R_2^*(s)$$
$$R_2^*(s) = Z_2^*(s) + q_{21}^*(s)R_1^*(s) \tag{4-6}$$

After solving the relations (4-6), we have

$$R_0^*(s) = \frac{\left(1 - q_{12}^*(s)q_{21}^*(s)\right)Z_0^*(s) + q_{01}^*(s)Z_1^*(s) + q_{01}^*(s)q_{12}^*(s)Z_2^*(s)}{1 - q_{01}^*(s)q_{10}^*(s) - q_{12}^*(s)q_{21}^*(s)} \tag{7}$$

Taking the inverse Laplace Transform of the equation (7), we have the reliability of the system starting from state S_0 for known values of parameters. The MTSF is given by

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = \frac{(1 - P_{12}P_{21})\psi_0 + P_{01}\psi_1 + P_{01}P_{12}\psi_2}{1 - P_{01}P_{10} - P_{12}P_{21}} \tag{8}$$

as $Z_i^*(0) = \psi_i$ and $q_{ij}^*(0) = P_{ij}$

AVAILABILITY ANALYSIS

By using the probabilistic argument and defining $A_i(t)$ as the probability that the system is in up state at instant t, given that the system entered in regenerative state S_i at t=0, we get the following recursive relations:

$$A_0(t) = Z_0(t) + q_{01}(t)\otimes A_1(t) + q_{05}(t)\otimes A_5(t)$$
$$A_1(t) = Z_1(t) + q_{10}(t)\otimes A_0(t) + q_{11}^{(2)}(t)\otimes A_1(t) + q_{16}(t)\otimes A_6(t)$$
$$A_2(t) = Z_2(t) + q_{21}(t)\otimes A_1(t) + q_{24}(t)\otimes A_4(t)$$
$$A_3(t) = Z_3(t) + q_{31}(t)\otimes A_1(t) + q_{34}(t)\otimes A_4(t)$$
$$A_4(t) = q_{43}(t)\otimes A_3(t)$$
$$A_5(t) = q_{50}(t)\otimes A_0(t)$$
$$A_6(t) = q_{61}(t)\otimes A_1(t)$$

(9-15)

Taking the Laplace Transform of the relation (9-15) and solving them for $A_0^*(s)$ and them steady state availability of the system is given as

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1'} \tag{16}$$

where

$$N_1 = P_{10}P_{31}\psi_0 + P_{01}P_{31}\psi_1$$

and

$$D_1' = P_{10}P_{31}\psi_0 + P_{01}P_{31}\psi_1 + P_{05}P_{10}P_{31}\psi_5 + P_{01}P_{16}P_{31}\psi_6$$

using

$$q_{ij}'^{*} = -\int tq_{ij}^*(t) \text{ and } \psi_i = \sum_j m_{ij}$$

The expected up time of the system during time interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du \tag{17}$$

so that

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s} \tag{18}$$

BUSY PERIOD ANALYSIS

Let $B_i(t)$ is defined as the probability that the repairman is busy at epoch t starting from $S_i \in E$. By the elementary probabilistic arguments, we get

$$B_0(t) = q_{01}(t)\otimes B_1(t) + q_{05}(t)\otimes B_5(t)$$
$$B_1(t) = W_1(t) + q_{10}(t)\otimes B_0(t) + q_{11}^{(2)}(t)\otimes B_1(t) + q_{16}(t)\otimes B_6(t)$$
$$B_2(t) = W_2(t) + q_{21}(t)\otimes B_1(t) + q_{24}(t)\otimes B_4(t)$$
$$B_3(t) = W_3(t) + q_{31}(t)\otimes B_1(t) + q_{34}(t)\otimes B_4(t)$$
$$B_4(t) = W_4(t) + q_{43}(t)\otimes B_3(t)$$
$$B_5(t) = W_5(t) + q_{50}(t)\otimes B_0(t)$$
$$B_6(t) = W_6(t) + q_{61}(t)\otimes B_1(t)$$

(19-25)

where

$$W_1 = e^{-(\alpha+\delta)t}\bar{F}(t), \qquad W_2 = W_3 = e^{-\beta t}\bar{F}(t), \qquad W_4 = W_5 = W_6 = \bar{G}(t)$$

Taking the Laplace Transform of the relations (19-25) and then solving them for $B_0^*(s)$. Omitting the arguments ‘s’ for brevity, we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)} \tag{26}$$

The steady busy period, when the system starts from S_0, is obtained as

$$B_0 = \lim_{s\to 0} s\, B_0^*(s) = \frac{N_2}{D_1'} \tag{27}$$

where

$$N_2 = P_{01}P_{31}(\psi_1 + P_{16}\psi_6) + P_{05}P_{10}P_{31}\psi_5$$

using $W_i^*(0) = \psi_i$ and $q_{ij}^* = P_{ij}$

and D_1' is same as defined in availability analysis.

$$\mu_b(t) = \int_0^t B_0(u)du \tag{28}$$

so that

$$\mu_b^* = \frac{B_0^*(s)}{s} \tag{29}$$

PROFIT ANALYSIS

The expected profit incurred by the system during time interval (0, t] is given by

$$P(t) = \text{Expected total revenue in } (0,t] - \text{Expected total repair cost in } (0,t] = C_0\mu_{up}(t) - C_1\mu_b(t)$$

where,

C_0= the revenue per unit up time by the system

C_1= the cost per unit time for which the repairman is busy

The expected profit per unit time in steady state is

$$\begin{aligned} P(t) &= C_0 \lim_{t\to 0} \frac{\mu_{up}(t)}{t} - C_1 \lim_{t\to 0} \frac{\mu_b(t)}{t} \\ &= C_0 \lim_{s\to 0} sA_0^*(s) - C_1 \lim_{s\to 0} sB_0^*(s) \\ &= C_0A_0 - C_1B_0 \end{aligned} \tag{30}$$

PARTICULAR CASE

Let all the repair time distributions are also follow the exponential distribution:

$$F(t) = 1 - e^{-\phi t} \text{ and } G(t) = 1 - e^{-\gamma t}$$

then

$$P_{10} = \frac{\phi}{\alpha+\delta+\phi}, \quad P_{11}^{(2)} = \frac{\delta}{\alpha+\delta+\phi}, \quad P_{16} = \frac{\alpha}{\alpha+\delta+\phi}$$

$$P_{21} = \frac{\phi}{\beta+\phi} = P_{31}, \quad P_{24} = \frac{\beta}{\beta+\phi} = P_{34}, \quad P_{43} = P_{50} = P_{61} = 1$$

$$\psi_1 = \frac{1}{\alpha+\delta+\phi}, \quad \psi_2 = \psi_3 = \frac{1}{\beta+\phi}, \quad \psi_4 = \psi_5 = \psi_6 = \frac{1}{\gamma}$$

GRAPHICAL INTERPRETATION

For more clear understanding of the system characteristic with respect to failure rates and time, graphs of MTSF and profit function are given in fig. 2 and fig. 3 respectively.

Fig. 2 shows the variation in MTSF with respect to δ for different values of the failure rate of helping unit α=0.03, 0.04, 0.05 when the other parameters kept fixed as β=0.20 and ϕ=0.03. It can be interpreted from this graph that as failure rate (δ) is increased; MTSF goes down, this concludes that the reliability of the system decreases with an increase in the failure rate.

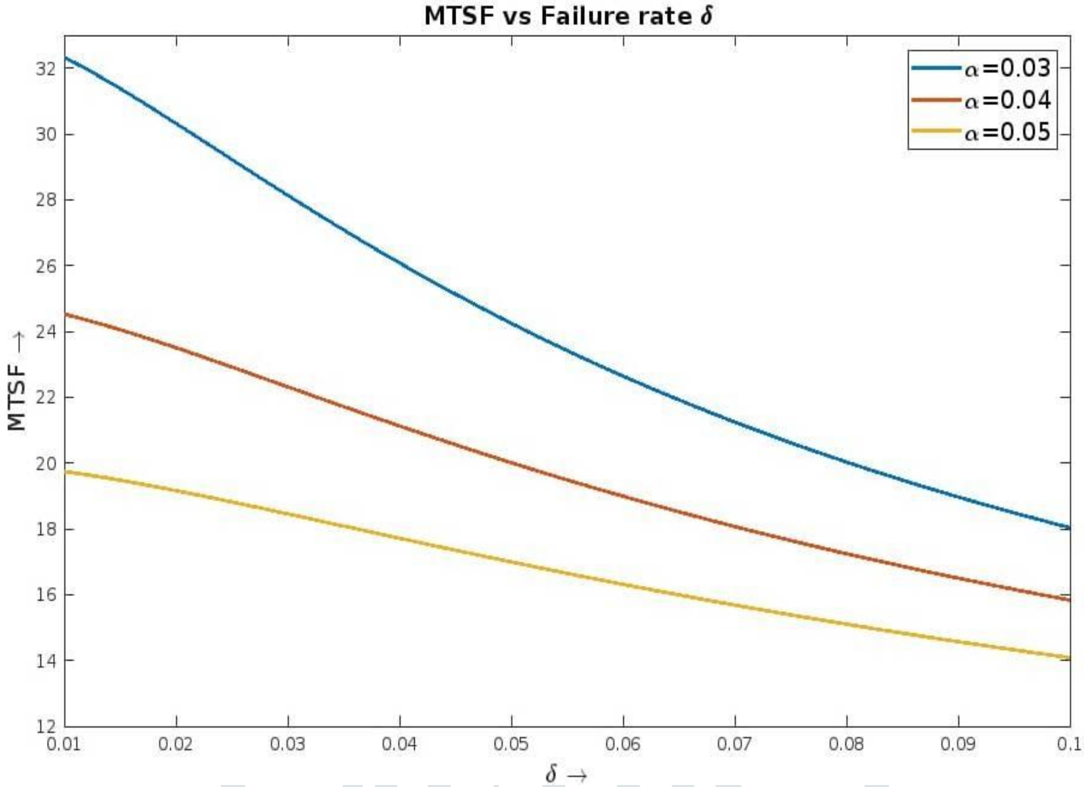

Figure 2

In fig. 3 curve represent the change in profit with respect to time for different values of failure rate of main unit when helping unit is operative, δ=0.001, 0.006, 0.100 and the other parameters kept fixed as α=0.04, β=0.03, ϕ=0.003, γ=0.001, C_0=4000 and C_1=600. We can see from the graph that the profit decreases as the time increases. It can also be interpreted by this graph that with increase of failure rate (δ), the profit decreases.

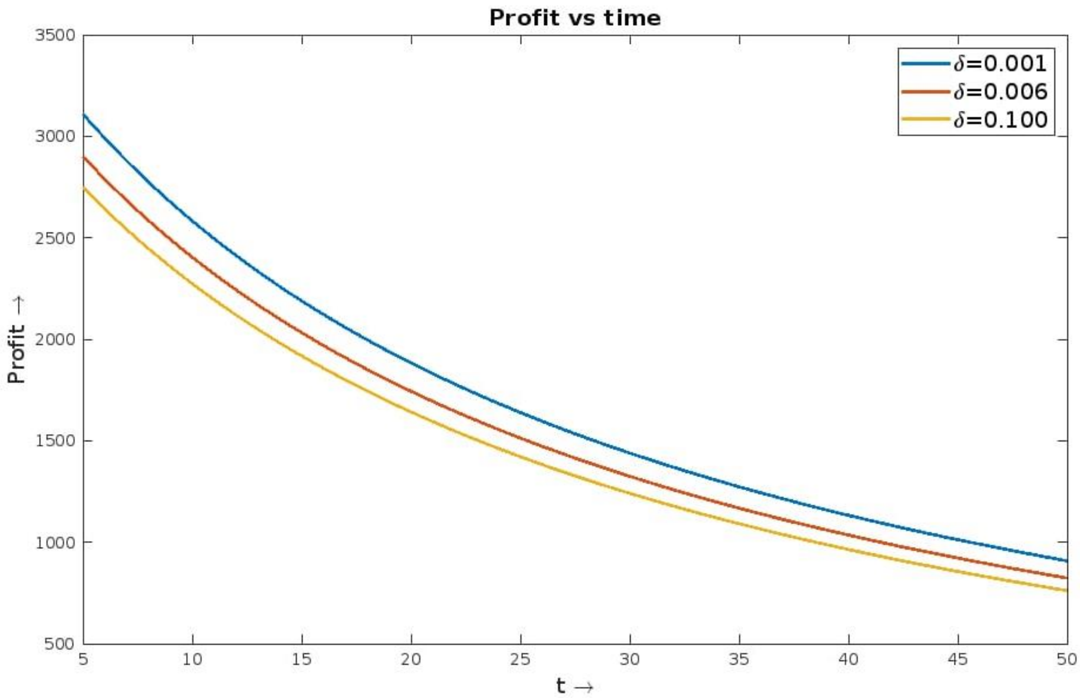

Figure 3

REFERENCES

1. Agnihotri, R.K., Kumar, V. and Ansari, M.A. (1993). Stochastic analysis of a redundant system with two types of failure. *Microelectron. Reliab.*, 33(10), 1467-1471.
2. Agarwa1, S.C., Sahani, M and Bansal, S. (2010). Reliability characteristic of cold-standby redundant system. *IJRRAS*, 3(2), 193-199.

3. Agnihotri, R.K., Satsangi, S.K. and Agrawal, S.C. (1995). Two non-identical operative unit system with repair and inspection. *Microelectron. Reliab.*, 35(5), 869-874.
4. Dhillon B.S. and Misra, R.B. (1984). Reliability evaluation of systems with critical human error. *Microelectron. Reliab.*, 24(4), 743-759.
5. Dhillon, B.S. and Anude, O.C. (1993). Common-cause failure analysis of a non-identical unit parallel system with arbitrarily distributed repair times. *Microelectron. Reliab.*, 33(1), 87-103.
6. Gupta R., Goel, C.K. and Tomer, A. (2010). A two dissimilar unit parallel system with administrative delay in repair and correlated life times. *ITMSC*, 3(1), 103-112.
7. Kumar, P. Bharti, A. and Gupta, A. (2012). Reliability analysis of a two non-identical unit system with repair and replacements having correlated failures and repairs. *Journal of Informatics and Mathematical Sciences*, 4(3), 339–350.
8. Rizwan, S.M., Khurana, V. and Taneja, G. (2010). Reliability analysis of a hot standby industrial system. *International Journal of Modeling and Simulation*, 30(3), 315-322.
9. Sridharan, V. and Mohanavadivu, P. (1997). Reliability and availability analysis for two non-identical unit parallel systems with common cause failures and human errors. *Microelectron. Reliab.*, 37(5), 747-752.
10. Singh, S.K. and Nair, S.G. (1996). On the characteristics of a system having master and helping unit. *Microelectron. Reliab.*, 36(2), 255-259.