A few expressions from Rational Number Series

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Abstract

The author had submitted a paper on 'Rational Number Series'[1]. Rational numbers of the form $\frac{n}{(n+1)}$ was used in Rational Number series to get expressions. In the near past, this idea was developed a little more. Working further and using a general expression of the type $\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}$ a lot of interesting results have been obtained. In this paper a few expressions using the general expression $\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}$ are being submitted.

Keywords

Expressions, rational number series, consecutive numbers;

Introduction

The expression $\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}$ can be used to generate many expressions which are interesting. Putting m equal to 1 gives $\frac{n}{(n+1)} - \frac{(n-1)}{n}$. This expression $\frac{n}{(n+1)} - \frac{(n-1)}{n}$ involves three consecutive numbers (n-1),n and (n+1). Putting m equal to 2 gives $\frac{(2n+1)}{(2n+2)} - \frac{(2n-1)}{(2n)}$. And this expression $\frac{(2n+1)}{(2n+2)} - \frac{(2n-1)}{(2n)}$ involves four consecutive numbers (2n-1),2n,(2n+1) and (2n+2). Putting m equal to any other numbers greater than 1 and 2 do not result in consecutive numbers.

The above are some of the points relevant to the expressions which are submitted below.

Expression 1

$$\left(\frac{(mn-1)!}{mn!} - \frac{(mn+m-1)!}{(mn+m)!}\right) = \left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}\right)$$

Expression 2

$$\left(\frac{1}{mn} - \frac{1}{(mn+1)}\right) = \left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}\right)$$

Expression 3

$$\left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{(mn)}\right)^a \sum_{(k=1)}^{a} \binom{a}{k} (-1)^{(k+1)} (mn^2+m)^a a^{(a-k)} = m^a \left(\frac{(mn+m-1)^a}{(mn+m)^a} - \frac{(mn-1)^a}{(mn)^a}\right)$$

Expression 4

$$\left(\frac{n}{(n+1)}-\frac{(n-1)}{n}\right)=\sum_{k=1}^{m}\left(\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{mn}\right)$$

Expression 5

$$\left(\frac{(mn+m-1)^k}{(mn+m)^k}-\frac{(mn+m-3)^k}{(mn+m-2)^k}\right) \\ \triangleq \frac{a}{(mn+m-1)^2}-\frac{b}{(mn+m-1)^3}+\frac{c}{(mn+m-1)^4}-\frac{d}{(mn+m-1)^5}$$

(Where a, b, c and d are in Arithmetic Progression with the first term being 2k and the common ratio being 2k(k-2). Here k is a positive number. It may be an integer or a fraction.)

Expression 6

$$\left(\frac{\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{(mn)}}{\frac{n}{(n+1)}-\frac{(n-1)}{n}}\right)=\left(\frac{\frac{n}{(n+1)}-\frac{(n-1)}{n}}{\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{(mn)}}\right)\left(\frac{1}{(m^2+1)}+\frac{1}{(m^2+1)^2}+\frac{1}{(m^2+1)^3}+\ \dots\ \infty\right)$$

Expression 7

$$\left(\frac{\frac{n}{(n+1)}-\frac{(n-1)}{n}}{\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{(mn)}}\right) \\ =\left(\frac{\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{(mn)}}{\frac{n}{(n+1)}-\frac{(n-1)}{n}}\right)^0 \\ +\ ((m-1)^2)\left(\left(\frac{\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{(mn)}}{\frac{n}{(n+1)}-\frac{(n-1)}{n}}\right)^1+\left(\frac{\frac{(mn+m-1)}{(mn+m2)}-\frac{(mn-1)}{(mn)}}{\frac{n}{(n+1)}-\frac{(n-1)}{n}}\right)^2+\ \dots\ \infty\right)$$

Conclusion

In total seven expressions have been submitted in this paper. More expressions can be made by using the basic expression of $\frac{(mn+m-1)}{(mn+m)}-\frac{(mn-1)}{mn}$.

References

1. Kirtivasan Ganesan, *Rational Number Series,* June 2019 http://www.jetir.org/papers/JETIR1907J15.pdf (www.jetir.org (ISSN -2349-5162))