A TWO WAREHOUSE INVENTORY MODEL FOR DETERIORATING ITEMS WITH GENERALIZED EXPONENTIAL DECREASING STOCK DEPENDENT DEMAND, CONSTANT HOLDING COST AND TIME-VARYING DETERIORATION RATE

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ABSTRACT

In this paper, an inventory model is considered with generalized exponential decreasing stock-dependent demand rate and constant holding cost. Here we had considered two types of warehouses OW (Own Warehouse) and RW (Rented Warehouse) where holding cost for OW is zero and RW is dependent on time of storage and quantity of items stored. The deterioration rate is a time-varying, linear function of time. Shortages are not allowed. A numerical example is used to illustrate the application of the model and some sort of analysis of the optimal solution with respect to various parameters is carried out, to see the effect of parameter changes on the solution.

Keywords: Stock-dependent demand rate, Linear Deterioration, Generalize Exponential Decreasing Demand, Inventory, Constant Holding Cost, Rented Warehouse, Own Warehouse.

1.0 INTRODUCTION

In real life, the effect of deterioration is very important in many inventory systems. In general, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or loss of marginal value of a commodity that results in decreasing usefulness, Wee (1993). Most of physical goods undergo decay or deterioration over time. The proposed model in this paper is for the deteriorating item which has a time-dependent generalized exponential decreasing demand rate and time dependent, linear deterioration rate and a constant holding cost. The items that exhibit the above phenomenon are food items, photographic films, drugs, chemicals, pharmaceuticals, electronic components, blood kept in blood banks and so on. Therefore, the effect of deterioration on these items cannot be disregarded in their inventory systems.

Consequently, the production and inventory problem of deteriorating items has been extensively studied by researchers. Some of the researchers include Ghare and Schrader (1963) who are the first researchers to derive an economic order quantity model by assuming exponential decay for the item. Later, Covert and Philip (1973) extended Ghare and Schrader's (1963) model by considering the deterioration rate to be a two-parameter weibull distribution. Later, Shah and Jaiswal (1977) presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal (1978) corrected the analysis in Shah and Jaiswal's model (1977). Dave and Patel (1981) considered an inventory model for deteriorating items with time-proportional demand when shortages were not allowed. Authors such as Hollier and Mark (1983), Hariga and Benkherouf (1994), Wee (1995a, 1995b) all developed their models by considering demand to be an exponential demand.

Some recent work on deteriorating items include the work of Goyal and Giri (2001), in which they presented a very good survey on the recent trends in modelling of deteriorating inventory. Also Ouyang et al; (2005) developed an Economic Order Quantity (EOQ) inventory model for deteriorating items in which demand function is exponentially declining and with partial backlogging. Also Shah and Pandy (2008) in their study developed an optimal ordering policy for time dependent deterioration with associated salvage value where delay in payments is permissible. Another recent work also includes He and He (2010), who made an extension to consider the fact that some products may deteriorate during storage. They developed a production inventory model for deteriorating items with production disruptions. The inventory plans and optimal production were provided, in such a way that the manufacturer can minimize the loss caused by disruptions. Kumar et al (2012) in their research, developed a deterministic inventory model for deteriorating items, where they considered their demand as a quadratic function of time, no shortages are allowed and the effect of inflation rate in the model was assumed to be over a finite planning horizon taking a variable holding cost. Sing and Pattnayk (2013) also in their work presented an Economic Order Quantity (EOQ) model for deteriorating items with time-dependent quadratic demand and variable deterioration, under permissible delay in payment. Dash et al (2014) also developed an inventory model for deteriorating items having a time- dependent exponential declining demand rate and time-varying holding cost as a linear function of time. Shortages were not allowed. Aliyu and Sani (2016) developed an inventory model for deteriorating items with generalized exponential decreasing demand and linear time-varying holding cost. The rate of deterioration was considered to be a constant. Shortages were not allowed.

In this paper, we had considered two warehouses OW and RW with zero holding cost in OW and time varying holding cost in RW. The present model is applicable to seasonal items like mangoes, Apples etc. whose demand decreases after its season and hence one should store the access quantity. It is also applicable to the items for which farmers doesn't get the good price in its season and so one should store them to use it later.

2.0 ASSUMPTIONS AND NOTATION

In formulating the mathematical model, the following notation and assumptions are employed.

2.1 ASSUMPTIONS

1. The inventory system considers a single item only.
2. The demand rate R(q) is deterministic and is a generalized exponential decreasing function of inventory level q. We can express it as follows:
   $R(q(t)) = Re^{h-\beta q(t)}$; $Where\ Constants\ R > 0, 0 < \beta < 1$
3. The deterioration rate is considered to be a linear function of time.
4. Lead time is zero.
5. There are no shortages.
6. The inventory system is considered over an infinite time horizon.
7. The holding cost is constant.

2.2 NOTATION

A = The fixed ordering cost per order
R = Constant Demand rate
S = Fixed order level (It should be maintained for good functioning)
I(t) = The inventory at any time t, $0 \leq t \leq T$
I_0 = Initial order quantity
$R(q(t))$ = The exponential stock dependent demand rate
$q(t)$ = On hand inventory at time t.
D(t) = The deterioration rate is a linear function of time given as D(t) = a + bt .
c = The cost of unit item.
T = The cycle time.
Q_0 = Initial stock.
TC = The total cost per unit time.
T* = The optimal length of the cycle.
Q^* = The economic order quantity
TC* = The minimum total cost per unit time.
N = Number of distinct time periods with different holding cost rates

Ti = End time of period i, where i =1,2,3…..n. $T_0 = 0$.

hi = Holding cost of the item in period i

h(t) = Holding cost of an item at time t, h(t) = h_i if $t_{i-1} \leq t \leq t_i$

β = Demand parameter indicating elasticity in relation to the inventory level

W = Capacity of OW.

R = Amount of goods stored in RW.

ho = Holding cost per unit per unit time in OW.

hr = Holding cost per unit per unit time in RW, hr > ho.

Iow(t) = Inventory level at any time t in OW.

Irw(t) = Inventory level at any time t in RW.

2.3 MATHEMATICAL MODEL

In the development of the model, a company place an order of I_0 units and the order level reaches to S ($S > W$) units out of which W units are kept in OW and ($S - W$) = R units are kept in RW, is being assumed. Initially, the demands are not using the stocks of RW until the stock level drops to ($W - K$) units at the end of $T1$. At this stage, K ($K < W$) units are transported from RW to OW. As a result, the stock level of OW again becomes W and the stocks of OW are used to meet further demands.

This process continues until the stock level in RW is fully exhausted. After the last shipment, only W units are used to satisfy the demand during the interval [Tn-1, Tn].

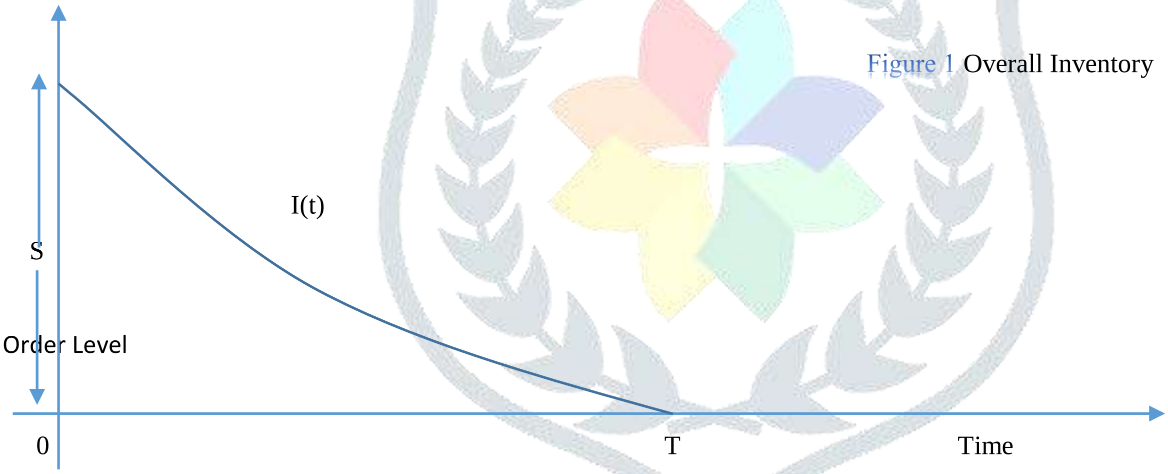

Figure 1 Overall Inventory

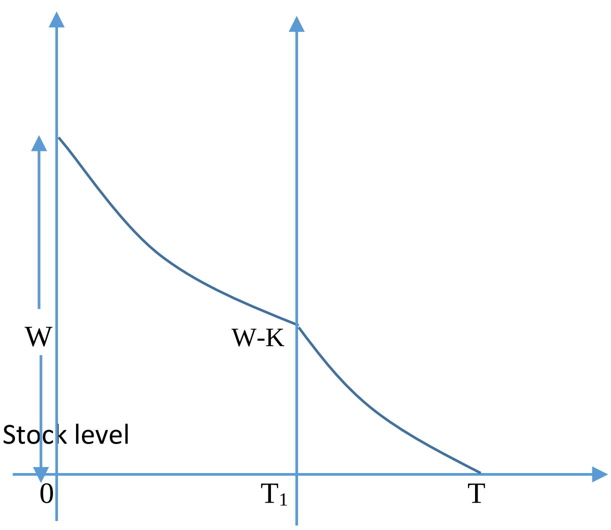

Figure 2 : Own Warehouse

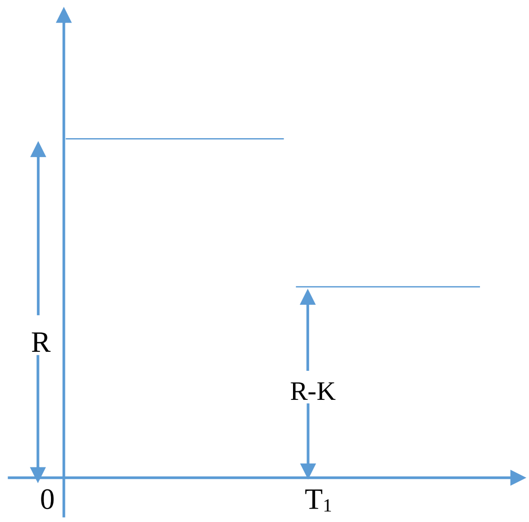

Figure 3 : Rented Warehouse

The above process is shown in the Figures 1, 2 and 3. As we can observe from Figure 1, the inventory level gradually decreases from initial stage due to the effect of both demand and deterioration. The differential equation which describes the state of inventory level I (t) in the interval [0, *T*] , is given by the equation as follows;

$$\frac{dI}{dt} + D(t)I(t) = -R(t); \qquad 0 \le t \le T$$

Where $R(t) = M \cdot e^{-\beta t}; M > 0, 0 < \beta < 1, t\ denotes\ time$

$$\frac{dI}{dt} + (a + bt)I(t) = -M \cdot e^{-\beta t} \tag{1}$$

On solving the above equation, we get

$$I(t) = \frac{-M}{-\beta+a+bt} e^{-\beta t} + Ce^{-at-\frac{bt^2}{2}} \tag{2}$$

Applying the boundary condition *I* (*t*) = 0 when *t* = *T* in Equation (2), where T is the length of ordering cycle and t is the current time we are concerned with. Therefore,

$$I(T) = 0 = \frac{-M}{-\beta + a + bT} e^{-\beta T} + Ce^{-aT-\frac{bT^2}{2}} \quad \Longrightarrow \quad \frac{M}{-\beta + a + bT} e^{-\beta T} = Ce^{-aT-\frac{bT^2}{2}}$$

$$\Longrightarrow C = \frac{M}{-\beta+a+bT} e^{-\beta T} e^{aT+\frac{bT^2}{2}} \quad \Longrightarrow C = \frac{M}{-\beta+a+bT} e^{-\beta T+aT+\frac{bT^2}{2}}$$

Put this value of C in equation (2), We get

$$I(t) = \frac{-M}{-\beta+a+bt} e^{-\beta t} + \left(\frac{M}{-\beta+a+bT} e^{-\beta T+aT+\frac{bT^2}{2}}\right) e^{-at-\frac{bt^2}{2}}$$

$$\therefore\ I(t) = M\left[\frac{-e^{-\beta t}}{-\beta+a+bt} + \frac{e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{-\beta+a+bT}\right]$$

$$\therefore\ I(t) = M\left[\frac{-e^{-\beta t}(-\beta+a+bT)+(-\beta+a+bt)e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{(-\beta+a+bt)(-\beta+a+bT)}\right] \tag{3}$$

The initial order quantity can be obtained by putting the boundary condition *I* (*0*) = I_0 into Equation (3) as follows:

$$I(0) = I_0 = M\left[\frac{-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}}{(-\beta+a)(-\beta+a+bT)}\right] \tag{4}$$

Total demand during the cycle time [0, T] is given as follows:

$$Total\ Demand\ = \int_0^T R(t)dt = \int_0^T Re^{h-\beta q(t)}dt = -\frac{R}{\beta}\left[\frac{e^{h-\beta q(t)}}{q'(t)}\right]_0^T$$

$$= -\frac{R}{\beta}\left[\frac{e^{h-\beta q(T)}}{q'(T)} - \frac{e^{h-\beta q(0)}}{q'(0)}\right] = -\frac{Re^h}{\beta}\left[\frac{1}{q'(T)} - \frac{e^{\beta I_0}}{q'(0)}\right] \tag{5}$$

The number of deteriorated units is given as initial order quantity minus the total demand in the cycle period [0, T].

Thus, the number of deteriorated units $= I_0 - Total\ Demand = I_0 - \int_0^T R(t)dt$

$$= M\left[\frac{-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}}{(-\beta+a)(-\beta+a+bT)}\right]+\frac{Re^h}{\beta}\left[\frac{1}{q'(T)}-\frac{e^{\beta I_0}}{q'(0)}\right] \quad (6)$$

Thus, the total deterioration cost for the cycle period is given as follows:

$$DC = Total\ number\ of\ deteriorated\ units * c$$

$$= c\left(M\left[\frac{-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}}{(-\beta+a)(-\beta+a+bT)}\right]+\frac{Re^h}{\beta}\left[\frac{1}{q'(T)}-\frac{e^{\beta I_0}}{q'(0)}\right]\right) \quad (7)$$

The total inventory holding cost (IHC) for the cycle period is given as follows:

$$\text{IHC} = \int_0^T (h_0+h_r)\, I(t)dt$$

$$= \int_0^T (h_0+h_r)\, M\left[\frac{-e^{-\beta t}(-\beta+a+bT)+(-\beta+a+bt)e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{(-\beta+a+bt)(-\beta+a+bT)}\right]dt$$

$$= \int_0^T (h_0+h_r)\, M\left[\frac{-e^{-\beta t}}{(-\beta+a+bt)}+\frac{e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{(-\beta+a+bT)}\right]dt$$

$$= -(h_0+h_r)M\left\{\int_0^T \frac{e^{-\beta t}}{(-\beta+a+bt)}dt+\int_0^T \frac{e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{(-\beta+a+bT)}dt\right\} = -(h_0+h_r)M\{A+B\} \quad (8)$$

$$A = \int_0^T \frac{e^{-\beta t}}{(-\beta+a+bt)}dt = be^{-\beta t}[\ln(-\beta+a+bt)]_0^T + \beta b\int_0^T e^{-\beta t}\ln(-\beta+a+bt)\,dt$$

$$= be^{-\beta t}[\ln(-\beta+a+bt)]_0^T + \beta b\left(\left[\frac{e^{-\beta t}}{-\beta}\right]_0^T \ln(-\beta+a+bt) - \int_0^T \frac{b}{(-\beta+a+bt)}\frac{e^{-\beta t}}{-\beta}dt\right)$$

$$A = be^{-\beta t}(\ln(-\beta+a+bT)-\ln(-\beta+a)) + \beta b\left(\left(\frac{e^{-\beta T}-1}{-\beta}\right)\ln(-\beta+a+bt)+\frac{b}{\beta}\int_0^T \frac{e^{-\beta t}}{(-\beta+a+bt)}dt\right)$$

$$A = be^{-\beta t}(\ln(-\beta+a+bT)-\ln(-\beta+a)) + \beta b\left(\left(\frac{e^{-\beta T}-1}{-\beta}\right)\ln(-\beta+a+bt)+\frac{b}{\beta}A\right)$$

$$(1-b^2)A = be^{-\beta t}(\ln(-\beta+a+bT)-\ln(-\beta+a)) + \beta b\left(\left(\frac{e^{-\beta T}-1}{-\beta}\right)\ln(-\beta+a+bt)\right)$$

$$A = \frac{be^{-\beta t}}{(1-b^2)}(\ln(-\beta+a+bT)-\ln(-\beta+a)) - b\left(\left(\frac{e^{-\beta T}-1}{(1-b^2)}\right)\ln(-\beta+a+bt)\right) \quad (9)$$

$$B = \int_0^T \frac{e^{-\beta T+a(T-t)+\frac{b(T^2-t^2)}{2}}}{(-\beta+a+bT)}dt$$

$$= \frac{e^{-\beta T+aT+\frac{bT^2}{2}}}{(-\beta+a+bT)}\int_0^T e^{-at-\frac{bt^2}{2}}dt = \frac{e^{-\beta T+aT+\frac{bT^2}{2}}}{(-\beta+a+bT)}\left[\frac{e^{-at-\frac{bt^2}{2}}}{-a-bt}\right]_0^T$$

$$B = \frac{e^{-\beta T+aT+\frac{bT^2}{2}}}{(-\beta+a+bT)}\left(\frac{e^{-aT-\frac{bT^2}{2}}}{-a-bT}+\frac{1}{a}\right) \quad (10)$$

Total Cost = TC = Ordering Cost + Deterioration Cost + Inventory Holding Cost

$$TC = \frac{A}{T}+c\left(M\left[\frac{-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}}{(-\beta+a)(-\beta+a+bT)}\right]+\frac{Re^h}{\beta}\left[\frac{1}{q'(T)}-\frac{e^{\beta I_0}}{q'(0)}\right]\right)-(h_0+h_r)M\{A+B\} \quad (11)$$

Our main objective of this paper is to find minimum variable cost per unit time. The necessary and sufficient conditions to minimize TC are respectively,

$$\frac{dTC}{dT} = -\frac{A}{T^2} + c\left(M\left[\frac{\left((-\beta+a+bT)\left(-(b)+(-\beta+a)(-\beta+a+bT)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)-\left(-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)b\right)}{(-\beta+a)(-\beta+a+bT)^2}\right] - \frac{Re^h}{\beta}\left[\frac{1}{q'(T)^2}\right]\right) -$$

$$(h_0+h_r)M\left\{\frac{be^{-\beta t}}{(1-b^2)}\left(\frac{b}{(-\beta+a+bT)}\right) - b\left(\left(\frac{-\beta e^{-\beta T}}{(1-b^2)}\right)\ln(-\beta+a+bt)\right) + \frac{e^{-\beta T+aT+\frac{bT^2}{2}}}{(-\beta+a+bT)}\left(\frac{e^{-aT-\frac{bT^2}{2}}}{-a-bT}\right)\right\}$$

(12)

Let us consider $\frac{dTC}{dT} = 0$

The value of T which we obtain, gives the minimum cost once it satisfies the following condition

$$\frac{d^2TC}{dT^2} > 0. \quad (13)$$

$$\frac{2A}{T^3} + c\left(\frac{M}{(-\beta+a)}\left[\frac{\left(\left\{(b)\left(-(b)+(-\beta+a)(-\beta+a+bT)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)+(-\beta+a+bT)\left((-\beta+a+bT)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)-\left(-b+(-\beta+a+bT)(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)b\right\}(-\beta+a+bT)^2 - 2b(-\beta+a+bT)\left((-\beta+a+bT)\left(-(b)+(-\beta+a)(-\beta+a+bT)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)-\left(-(-\beta+a+bT)+(-\beta+a)e^{-\beta T+a(T)+\frac{bT^2}{2}}\right)b\right)\right)}{(-\beta+a+bT)^4}\right] - \frac{Re^h}{\beta}\left[\frac{1}{q'(T)^2}\right]\right)$$

$$-(h_0+h_r)M\left\{\frac{be^{-\beta t}}{(1-b^2)}\left(\frac{b}{(-\beta+a+bT)^2}\right) - b\left(\left(\frac{-\beta^2 e^{-\beta T}}{(1-b^2)}\right)\ln(-\beta+a+bt)\right) + \frac{e^{-\beta T+aT+\frac{bT^2}{2}}}{(-\beta+a+bT)^2}\left(\frac{e^{-aT-\frac{bT^2}{2}}}{-a-bT^2}\right)\right\} = 0 \quad (14)$$

Equation (14) is highly nonlinear and therefore difficult to solve by any analytic method. Likewise, the same problem will exist in trying to check the inequality in (13) above. However, in all our examples below, we use direct search method to obtain the root of the equation an also confirm that the sufficient condition (13) is satisfied.

2.4 Numerical Example

Example-1

To illustrate the model developed an example is considered based on the following values of parameters:

A = \$ 4000 per order, K =300, C = \$100 per unit, β =0.01 , a = 0.2 , b = 0.01 , i = 0.1 per rupees per unit time, and h = 2.

Substituting and simplify the above parameters into Equation (14), gives T* = 0.192595421 (87 days).

On substitution of this optimal value T* in equations (11) and (4), We obtain the minimum total cost per unit time TC* = \$ 2734948.2356 and

economic order quantity $I_0^* = 567.46127931$ units.

Note that the T* value satisfies $\frac{d^2TC}{dT^2} > 0$.

Example-2

Applying the same values as in example 1, with h changed to 3, the solutions we obtain are as follows: T* = 0.113582498 (67 days), TC* = \$ 163745.2356 and I_0^* = 929.1394946 units

Example 3

Also using the same values as in example 1, with h changed to 4, the solutions, are T* = 0.91235582347 (52 days), TC* = $ 243685.4527 and I_0^* = 1425.245786 units

2.5 Conclusion

In this paper, an inventory model with two warehouse is developed which determines the optimal order quantity of an on-hand inventory due to a generalized exponential decreasing demand rate. The deterioration rate is time varying linear function of time and the stockholding cost is a constant for both the warehouses. The model has been solved analytically by minimizing the total inventory cost. A numerical example has been given to show the application of the model. The analysis shows that T*, TC* and I_0^* are sensitive to changes in the parameters, A, R, b, C, i and h. However, they are not very sensitive to changes in the parameters a and β. Moreover, it has been shown that the values T*, TC* and I_0^* all increase with increase in the parameter A, but all of them decrease with increase in the parameter a. For future research one can go for variable holding cost.

2.6 References

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