Some more expressions from Rational Number Series

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Abstract

The author had submitted a paper on ‘Rational Number Series’[1]. After this a paper on ‘A few expressions from Rational Number Series’[2] was submitted. Some more expressions got evolved in the past few days. These are being outlined in this paper.

Keywords

Expressions, rational number series, Arithmetic Geometric Progression, Gamma function;

Introduction

The expression $\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{mn}$ can be used to generate many expressions which are interesting. ‘A few expressions from Rational Number Series’[2] has been developed from this basic expression. The very same expression is also used for making expressions in this paper too.

Expression 1

$$\sum_{k=0}^{n} \binom{n}{k} \left(\frac{n}{(n+1)} - \frac{(n-1)}{n} \right)^{-1} = \sum_{k=1}^{n} \binom{k}{1} \left(\frac{(1+n)}{n} + \frac{(n-1)}{n} \right)^{(n+1)}$$

Expression 2

$$\sum_{k=0}^{n} \binom{n}{k} \left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{(mn)} \right) = \left(2^{(n+1)}\right)(m) \sum_{k=1}^{n} \binom{k}{1}$$

Expression 3

$$\int_{1}^{\infty} \left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1)}{(mn)} \right) dn = \log(2) \sum_{1}^{\infty} \left(\frac{(mn+m-1)}{(mn+m)} - \frac{(mn-1}{(mn)} \right)$$

Expression 4

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)} i^{n} = (1 - \frac{\pi}{4}) - i(\frac{\log(2)}{2})$$

Expression 5

$$\sum_{i=0}^{\infty} \frac{n}{(n+1)^2} i^n = \frac{(\pi - 4C - \log(4))}{8} + i\left(\frac{\pi^2}{48} - \frac{\log(2)}{2}\right)$$

C – Catalan's constant

Expression 6

$$\left(\frac{AGP(1st\ term)!}{AGP(2nd\ term)!} - \frac{AGP\big((n-1)th\ term\big)!}{AGP(nth\ term)!}\right)$$
$$= \Gamma(common\ ratio\ of\ the\ GP)\left(\frac{AGP((n-1)th\ term)}{AGP(nth\ term)} - \frac{AGP(1st\ term)}{AGP(2nd\ term)}\right)$$

where AGP stands for Arithmetic Geometric Progression and Γ(x) is the gamma function

Expression 7

$$\sum_{n=1}^{\infty}\left(\frac{n}{(n+1)} - \frac{(n-1)}{n}\right) = \sum_{n=1}^{\infty}\frac{2}{(2n-1)(2n+1)} = 1$$

$$\int_{1}^{\infty}\left(\frac{2n}{(2n+1)} - \frac{(2n-2)}{(2n-1)}\right)dn = \int_{1}^{\infty}\frac{2}{(2n-1)(2n+1)}dn = 0.54931 \ldots.$$

Conclusion

In total seven expressions have been submitted in this paper. The concept of Rational Number Series can be more widely used.

References

1. Kirtivasan Ganesan, *Rational Number Series,* June 2019 http://www.jetir.org/papers/JETIR1907J15.pdf (www.jetir.org (ISSN -2349-5162))
2. Kirtivasan Ganesan, *A few expressions from Rational Number Series,* December 2020 http://www.jetir.org/papers/JETIR2022024.pdf (www.jetir.org (ISSN -2349-5162))