ON RADIO GEOMETRIC MEAN Dd-DISTANCE NUMBER OF SOME BASIC GRAPHS

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Abstract

A Radio geometric mean Dd-distance labeling of a connect graph G is an injective function f from the vertex set $V(G)$ to the $\mathbb{N}$ such that for two distinct vertices u and v of $G, D^{Dd}(u,v) + \lceil\sqrt{f(u)f(v)}\rceil \geq 1 + diam^{Dd}(G), where\ D^{Dd}(u,v) = D(u,v) + \deg(u) + \deg(V)$, $D^{Dd}(u,v)$ denotes the Dd-distance between u and v $diamD^{Dd}(G)$ denotes the Dd-diameter of G. The radio geometric mean Dd-distance number of f, $rgmn^{Dd}(f)$ is the maximum label assigned to any vertex of G . The radio geometric mean Dd-distance number of, $rgmn^{Dd}(G)$is the minimum value of G, $rgmn^{Dd}(G)$ is the minimum value of $rgmn^{Dd}\ (f)$ taken over all radio geometric mean Dd-distance labeling f of G. In this paper we find the radio geometric mean Dd-distance number of some basic graphs.

Keywords: Dd-distance, Radio geometric mean Dd-distance, Radio geometric mean Dd-distance number.

Introduction

By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Radio labeling (multi-level distance labeling) can be regarded as an extension of distance two labeling which is motivated by the channel assignment problem introduced by Hale [4]. Chartrand et al [3] introduced the concept of radio labeling of graph. Chartrand [3] gave the upper bound for the radio number of path.

The concept of Dd-distance was introduced by A. Anto Kinsley and P. Siva Ananthi [1]. For a connected graph G, the Dd-length of a connected $u-v$ path is defined as $D^{Dd}(u,v) = D(u,v) + \deg(u) + \deg(V)$, The Dd-radius, denoted by $r^{Dd}\ (G)$ is the minimum Dd- eccentricity among all vertices of G. That is r^{Dd} (G) $= min\{e^{Dd}(G) : v \in V(G)\}$. Similarly the Dd-diameter, $D^{Dd}(G)$ is the maximum Dd-eccentricity among all vertices of G. We observe that for any two vertices u , v of G, We have $d(u,v) \leq D^{Dd}(u,v)$. The equality holds if and only if u, v are identical. If G is any connected graph then the Dd-distance is a metric on the set of vertices of G. We can check easily $r^{Dd}(G) \leq D^{Dd}(G) \leq 2r^{Dd}(G)$. The lower bound is clear from the definition and the upper bound follows from the triangular inequality.

In [5], Radio geometric mean labeling was introduced by V. Hemalatha et al [6]. A radio geometric mean labeling is a one to one mapping f from $V(G)$ to $\mathbb{N}$ satisfying the condition
$d(u,v) + \lceil\sqrt{f(u)f(v)}\rceil \geq 1 + diam(G)$, for every $u, v \in G$. The span of labeling f is the maximum integer that f maps to a vertex of G. The radio geometric mean number of G, $rgmn(G)$ is the lowest span taken over all radio geometric mean labeling of the graph G. The above condition is called radio geometric mean condition.

Further we are introduced the concept of radio geometric mean Dd-distance. The radio geometric labeling is a function $f: V(G) \to \mathbb{N}$ such that $D^{Dd}(u,v) + \lceil\sqrt{f(u)f(v)}\rceil \geq 1 + diam^{Dd}(G)$. It is denoted by $rgmn^{Dd}(G)$. where $rgmn^{Dd}(G)$ is called the radio geometric mean Dd- distance number. The radio geometric mean Dd-distance number of f, $rgmn^{Dd}\ (f)$ is the maximum label assigned to any vertex of G . The radio geometric mean Dd-distance number of, $rgmn^{Dd}(G)$is the minimum value of G, $rgmn^{Dd}(G)$ is the minimum value of $rgmn^{Dd}\ (f)$ taken over all radio geometric mean Dd-distance labeling f of G. In this paper we find the radio geometric mean Dd-distance number of some basic graphs.

Main results

Theorem 2.1: The Radio geometric mean Dd-distance number of a complete graph K_n, $rgmn^{Dd}\ (K_n)\ = n.$

Proof: Let $V(K_n) = \{v_1, v_2, v_3, v_4, \dots, v_n\}$ be the vertex set and $E(K_n) = \{v_i\, v_j, 1 \le i, j \le n, i \ne j\}$ be the edge set. Its $diam^{Dd}(K_n) = 3(n-1)$. we define the vertex label $f\ (v_i) =\ i, 1\ \le i \le n.$

$$Compute \text{ the } pair\ (v_i, v_j) = 3(n-1), 1 \le i, j \le n, i \ne j\ are\ adjacent$$

$$D^{Dd}(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \ge 1 + diam^{Dd}(K_n)$$

$Hence, rgmn^{Dd}(K_n)\ = n.$

Theorem 2.2: The Radio geometric mean Dd-distance number of a star graph $K_{1,n}$,

$rgmn^{Dd}\ (K_{1,n}) =\ 2n-2, n \ge 3.$

Proof: Let $V(K_{1,n}) =\ \{v_0, v_1, v_2, v_3, \dots, v_n\}$ be the vertex set, where v_0 is the central vertex and $E(K_{1,n}) = \{v_0 v_i, 1 \le i \le n\}$ be the edge set. Its $diam^{Dd}(K_{1,n}) = n+2$. we define the vertex label $f(v_i) = n+i-2, 0 \le i \le n.$

$\boldsymbol{Case}(\boldsymbol{i})$: $Compute$ the $pair\ (v_0, v_i) = n+2, 1 \le i \le n\ are\ adjacent$

$$D^{Dd}(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \ge 1 + diam^{Dd}(K_{1,n})$$

$\boldsymbol{Case}(\boldsymbol{ii})$: $Compute$ the $pair\ (v_i, v_j) = 4, 1 \le i, j \le n\ , i \ne j\ are\ non\ adjacent$

$$D^{Dd}(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \ge 1 + diam^{Dd}(K_{1,n})$$

Hence, $rgmn^{Dd}\ (K_{1,n}) =\ 2n-2, n \ge 3.$
Note: $rgmn^{Dd}(K_{1,n}) = n+1, if\ n = 1{,}2.$

Theorem 2.3: The Radio geometric mean Dd-distance number of a Subdivision of a star graph $S(K_{1,n})$, $rgmn^{Dd}\ (S(K_{1,n})) =\ 3n-2, n \ge 3.$

Proof: Let $V\left(S(K_{1,n})\right) = \{v_0, v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$ be the vertex set, where v_0 is the central vertex and $E\left(S(K_{1,n})\right) = \{v_0 v_i, v_i u_i 1 \le i \le n\}$ be the edge set. Its $diam^{Dd}(S(K_{1,n})) = n+3$. we define the vertex label $f(v_0) = 1, f(v_i) = n+2i-3, 1 \le i \le n, f(u_i) = n+2i-2,, 1 \le i \le n$.

$\boldsymbol{Case}(\boldsymbol{i})$: $Compute$ the $pair\ (v_0, v_i) = n+3, 1 \le i \le n\ are\ adjacent$

$$D^{Dd}(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \ge 1 + diam^{Dd}(S(K_{1,n}))$$

$\boldsymbol{Case}(\boldsymbol{ii})$: $Compute$ the $pair\ (v_0, u_i) = n+3, 1 \le i \le n\ are\ non\ adjacent$

$$D^{Dd}(v_0, u_i) + \left\lceil \sqrt{f(v_0)f(u_i)} \right\rceil \ge 1 + diam^{Dd}(S(K_{1,n}))$$

$\boldsymbol{Case}(\boldsymbol{iii})$: $Compute$ the $pair\ (v_i, v_j) = 6, 1 \le i, j \le n\ , i \ne j\ are\ non\ adjacent$

$$D^{Dd}(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \ge 1 + diam^{Dd}\left(S(K_{1,n})\right)$$

$\boldsymbol{Case}(\boldsymbol{iv})$: $Compute$ the $pair\ (v_i, u_j) = 4{,}1 \le i, j \le n\ , i \ne j\ are\ non\ adjacent$

$$D^{Dd}(v_i,u_j)+\left\lceil\sqrt{f(v_i)f(u_j)}\right\rceil \geq 1+diam^{Dd}\left(S(K_{1,n})\right)$$

$\boldsymbol{Case(v)}$: $Compute$ the $pair$ $(u_i,u_j)=6, 1\leq i,j\leq n, i\neq j$ $are\ non\ adjacent$

$$D^{Dd}(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil \geq 1+diam^{Dd}(S(K_{1,n}))$$

$\boldsymbol{Case(vi)}$: $Compute$ the $pair$ $(v_i,u_i)=4, 1\leq i\leq n$, $are\ adjacent$

$$D^{Dd}(v_i,u_i)+\left\lceil\sqrt{f(v_i)f(u_i)}\right\rceil \geq 1+diam^{Dd}(S(K_{1,n}))$$

Hence, $rgmn^{Dd}\left(S(K_{1,n})\right)= 3n-2, n\geq 3.$

Note: $rgmn^{Dd}\left(S(K_{1,n})\right)=2n+1, if\ n=1,2.$

Theorem 2.4: The Radio geometric mean Dd-distance number of a Fan graph F_n,

$rgmn^{Dd}(F_n)=2n-4, n\geq 6.$

Proof: Let $V(F_n)=\{v_0,v_1,v_2,v_3,\dots,v_n\}$ be the vertex set and $E(F_n)=\{v_0v_j, v_iv_{i+1}, 1\leq i\leq n-1, 1\leq j\leq n\}$ be the edge set. Its $diam^{Dd}(F_n)=2n+2$. we define the vertex label $f(v_0)=1, f(v_i)=n+i-4, 1\leq i\leq n$.

$\boldsymbol{Case(i)}$: $Compute$ the $pair$ $(v_0,v_i)=2n+2, 1\leq i\leq n$ $are\ adjacent$

$$D^{Dd}(v_0,v_i)+\left\lceil\sqrt{f(v_0)f(v_i)}\right\rceil \geq 1+diam^{Dd}(F_n)$$

$\boldsymbol{Case(ii)}$: $Compute$ the $pair$ $(v_1,v_j)=n+5, 2\leq j\leq n-1$

$$D^{Dd}(v_1,v_j)+\left\lceil\sqrt{f(v_1)f(v_j)}\right\rceil \geq 1+diam^{Dd}(F_n)$$

$\boldsymbol{Case(iii)}$: $Compute$ the $pair$ $(v_1,v_n)=n+4,$ $are\ non\ adjacent$

$$D^{Dd}(v_1,v_n)+\left\lceil\sqrt{f(v_1)f(v_n)}\right\rceil \geq 1+diam^{Dd}(F_n)$$

$\boldsymbol{Case(vi)}$: $Compute$ the $pair$ $(v_i,v_n)=n+5, 2\leq i\leq n-1$ $are\ non\ adjacent$

$$D^{Dd}(v_i,v_n)+\left\lceil\sqrt{f(v_i)f(v_n)}\right\rceil \geq 1+diam^{Dd}(F_n)$$

$\boldsymbol{Case(v)}$: $Compute$ the $pair$ $(v_i,v_j)=n+6, 3\leq j\leq n-1, 2\leq i\leq n-1$

$$D^{Dd}(v_i,v_j)+\left\lceil\sqrt{f(v_i)f(v_j)}\right\rceil \geq 1+diam^{Dd}(F_n)$$

Hence, $rgmn^{Dd}(F_n)= 2n-4, n\geq 6.$

Note: $rgmn^{Dd}(F_n)=n+1\ if\ n=1,2,3,4,5.$

Theorem 2.5: The Radio geometric mean Dd-distance number of a bistar graph $B_{n,n}$,

$rgmn^{Dd}(B(n,n))= 4n-2, n\geq 3.$

Proof: Let $V(B_{n,n})=\{v_0, v_1, v_2, v_3,\dots,v_n, u_0, u_1, u_2, u_3,\dots,u_n\}$ be the vertex set, where v_0, u_0 are the central vertex and $E(B_{n,n})=\{v_0v_i, u_0u_i, v_ou_o, 1\leq i\leq n\}$ be the edge set. Its $diam^{Dd}(B_{n,n}))=2n+3$. we define the vertex label $f(v_0)=3, f(v_i)=2n+i-2, f(u_i)=3n+i-2 . f(u_o)=2$.

$\boldsymbol{Case(i)}$: $Compute$ the $pair$ $(v_0, v_i) = n + 3, 1 \le i \le n$ are $adjacent$

$$D^{Dd}(v_0, v_i) + \left\lceil \sqrt{f(v_0)f(v_i)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n}),$$

$\boldsymbol{Case(ii)}$: $Compute$ the $pair$ $(u_0, u_i) = n + 3, 1 \le i \le n$ are $adjacent$

$$D^{Dd}(u_0, u_i) + \left\lceil \sqrt{f(u_0)f(u_i)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

$\boldsymbol{Case(iii)}$: $Compute$ the $pair$ $(v_0, u_i) = n + 4, 1 \le i \le n$ are non $adjacent$

$$D^{Dd}(v_0, u_i) + \left\lceil \sqrt{f(v_0)f(u_i)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

$\boldsymbol{Case(iv)}$: $Compute$ the $pair$ $(u_0, v_i) = n + 4, 1 \le i \le n$ are non $adjacent$

$$D^{Dd}(u_0, v_i) + \left\lceil \sqrt{f(u_0)f(v_i)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

$\boldsymbol{Case(v)}$: $Compute$ the $pair$ $(v_i, v_j) = 4, 1 \le i, j \le n$, $i \ne j$ are non $adjacent$

$$D^{Dd}(v_i, v_j) + \left\lceil \sqrt{f(v_i)f(v_j)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

$\boldsymbol{Case(vi)}$: $Compute$ the $pair$ $(u_i, u_j) = 4, 1 \le i, j \le n$, $i \ne j$ are non $adjacent$

$$D^{Dd}(u_i, u_j) + \left\lceil \sqrt{f(u_i)f(u_j)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

$\boldsymbol{Case(vii)}$: $Compute$ the $pair$ $(u_0, v_0) = 2n + 3$ are $adjacent$

$$D^{Dd}(u_0, v_0) + \left\lceil \sqrt{f(u_0)f(v_0)} \right\rceil \ge 1 + diam^{Dd}(B_{n,n})$$

Hence, $rgmn^{Dd}\left((B_{n,n})\right) = 4n - 2, n \ge 3.$

3. References

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