SOME NEW PROPERTIES OF 6X6 MAGIC SQUARE

D B OJHA

Associate Professor
Department Of Mathematics
University Of Rajasthan, Jaipur, India
Email:maildpost@gmail.com

Abstract : In this paper, we established some new results in the area of square magic and proved some new properties, in which one is M(A+α)+ M(A-α)= 2X M(A). Also, some new properties of Magic Constant of a Magic Square and it's derived matrix in the view of eigenvalues, Eigen vectors have been established. This paper is organized as introduction, definition & preliminaries, methodology and results in relation to 6X6 square magic.

Keywords: Magic Square, Magic Constant, Matrix, Eigen Value and Eigen Vector.

MSC Classification: 97A20, 97A40, 97A80.

1. INTRODUCTION :

The development of Magic Squares in India leads irresistibly to the conclusion that the Magic Square originated in India. The knowledge of these Squares might have gone outside India at any time between the first century and 10th century AD, but it appears to be the most probable that the west as well as China got the Magic Squares from India through the Arabs about tenth century. Many researchers in India and world have focused on square magic in last five years in different directions. In which few are [1, 2, 3, 4, 5, and 6].

In this paper, we established some properties of 6X6 square magic having virtue in association with its magic constant, Eigen values and Eigen vectors.

2. Definition & Preliminaries:

A Magic Square of order n is a square matrix having array of n^2 numbers such that the sum of each row and column as well as the main diagonal and main back diagonal [1], is the same number called Magic Constant (Magic Sum or Lowest Required Sum). If X is a Magic Square and each element of another same order square matrix by addition, subtraction, multiplication or division to the corresponding element of X by the same number (not 0 for multiplication or division), then new square matrix will be a Magic Square.

An upper bound for the number of normal Magic Squares of order n can be given by $n^2!/8(2n+1)$. There is only one distinct third order normal Magic Square with lowest sum.

3. Methodology:

Example 3.1: Let us consider a Sixth-order Magic Square.

A	30	29	28	9	8	7	M(A)=111
	12	11	10	27	26	25	
	18	17	16	24	14	22	
	19	20	21	13	23	15	
	31	32	33	4	5	6	
	1	2	3	34	35	36	

The Magic Constant of the Magic Square is M (A) =111.

Taking a combination of the two numbers 11 and 100 of the Magic Constant 111 such that 11+100. Now, adding 11 to each element of the Magic Square A, we get a Magic Square A_{11} (Say).

A_{11}	41	40	39	20	19	18	M(A)=177
	23	22	21	38	37	36	
	29	28	27	35	25	33	
	30	31	32	24	34	26	
	42	43	44	15	16	17	
	12	13	14	45	46	47	

A_{-11}	19	18	17	-2	-3	-4	M(A)=45
	1	0	-1	16	15	14	
	7	6	5	13	3	11	
	8	9	10	2	12	4	
	20	21	22	-7	-6	-5	
	-10	-9	-8	23	24	25	

A_{100}	130	129	128	109	108	107	M(A)=711
	112	111	110	127	126	125	
	118	117	116	124	114	122	
	119	120	121	113	123	115	
	131	132	133	104	105	106	
	101	102	103	134	135	136	

A_{-100}	-70	-71	-72	-91	-92	-93	M(A)= -489
	-88	-89	-90	-73	-74	-75	
	-82	-83	-84	-76	-86	-78	
	-81	-80	-79	-87	-77	-85	
	-69	-68	-67	-96	-95	-94	
	-99	-98	-97	-66	-65	-64	

Now take,
$M(A_{11}) + M(A_{-11})$ = 177+45= 222= 2*111= 2*M(A)
$M(A_{100}) + M(A_{-100})$ = 711-489= 222= 2*111= 2*M(A)
$M(A_{11}) + M(A_{100})$ = 177+711= 888= (6+2)*111= (n+m)*M(A), where n= order of square magic and m= number of partitions. Here n=6 and m=2.

4. Results and discussion:

(i) If M(Aα)= Magic Constant of Magic Square Aα
M(A-α)= Magic Constant of Magic Square A-α
M(A(M-α))= Magic Constant of Magic Square A(M-α)
M(A-(M-α))= Magic Constant of Magic Square A-(M-α)
M(A)= Magic Constant of Magic Square A
n = Order of Magic Square A
m = Number of partitions of M(A), Where M is any 5x5 square magic.

(ii)
It is possible to find a large number of essentially different matrices, which have different Eigen Values but nearly same Eigen vectors. But it is a difficult and time consuming task.
After Eigen decomposition process, the Eigen values and Eigen vectors of these obtained matrices are also linearly related together. Eigen vector of one matrix are the basis of an invariant subspace within the range of the corresponding linear map.

Discussion on Example 3.1

M(A)=111, multiplicity =1
β_1 = 111, β_2 = 11.06, β_3 = -5.53+12.44i , β_4 = -5.53_12.44i, β_5 = -8.72e-15,
β_6 = 8.90e-16.

w_1 =(-0.41, -0.52, 0.14, 0.27, 0.64, 0.55),
w_2 =(-0.41, 0.11, -0.079, -0.34, -0.46, -0.77)
w_3 = (-0.41, 0.21, -0.31, 0.05, -0.2, 0.23)
w_4 = (-0.41, -0.21, 0.31, -0.05, -0.41, -0.16)
w_5 = (-0.41, -0.32, 0.15, 0.54, 7.22e-16, -4.9e-17)
w_6 = (-0.41, 0.73, -0.22, -0.47, 0.41, 0.16)

M(A_{11}) =177, multiplicity =1
β_1 = 177, β_2 = 11.06, β_3 = -5.53+12.44i , β_4 = -5.53_12.44i, β_5 = -8.72e-15,
β_6 = 8.90e-16.

w_1 =(-0.41, -0.52, 0.14, 0.27, 0.64, 0.55),
w_2 =(-0.41, 0.11, -0.079, -0.34, -0.46, -0.77)
w_3 = (-0.41, 0.21, -0.31, 0.05, -0.2, 0.23)
w_4 = (-0.41, -0.21, 0.31, -0.05, -0.41, -0.16)
w_5 = (-0.41, -0.32, 0.15, 0.54, 7.22e-16, -4.9e-16)
w_6 = (-0.41, 0.73, -0.22, -0.47, 0.41, 0.16)

M(A_{-11})=45
β_1 =45, β_2 = 11.06, β_3 = -5.53+12.44i , β_4 = -5.53_12.44i, β_5 = -8.72e-15,
β_6 = 8.90e-16.

w_1 =(-0.41, -0.52, 0.14, 0.27, 0.64, 0.55),
w_2 =(-0.41, 0.11, -0.079, -0.34, -0.46, -0.77)
w_3 = (-0.41, 0.21, -0.31, 0.05, -0.2, 0.23)
w_4 = (-0.41, -0.21, 0.31, -0.05, -0.41, -0.16)
w_5 = (-0.41, -0.32, 0.15, 0.54, 7.22e-16, -4.9e-16)
w_6 = (-0.41, 0.73, -0.22, -0.47, 0.41, 0.16)

M(A_{100}) =711
β_1 =711, β_2 = 11.06, β_3 = -5.53+12.44i , β_4 = -5.53_12.44i, β_5 = -8.72e-15,
β_6 = 8.90e-16.

w_1 =(-0.41, -0.52, 0.14, 0.27, 0.64, 0.55),

w_2 =(-0.41, 0.11, -0.079, -0.34, -0.46, -0.77)

w_3 = (-0.41, 0.21, -0.31, 0.05, -0.2, 0.23)

w_4 = (-0.41, -0.21, 0.31, -0.05, -0.41, -0.16)

w_5 = (-0.41, -0.32, 0.15, 0.54, 7.22e-16, -4.9e-16)

w_6 = (-0.41, 0.73, -0.22, -0.47, 0.41, 0.16)

$M(A_{-100})$= -489

β_1 =-489, β_2 = 11.06, β_3 = -5.53+12.44i , β_4 = -5.53_12.44i, β_5 = -8.72e-15,

β_6 = 8.90e-16.

w_1 =(-0.41, -0.52, 0.14, 0.27, 0.64, 0.55),

w_2 =(-0.41, 0.11, -0.079, -0.34, -0.46, -0.77)

w_3 = (-0.41, 0.21, -0.31, 0.05, -0.2, 0.23)

w_4 = (-0.41, -0.21, 0.31, -0.05, -0.41, -0.16)

w_5 = (-0.41, -0.32, 0.15, 0.54, 7.22e-16, -4.9e-16)

w_6 = (-0.41, 0.73, -0.22, -0.47, 0.41, 0.16)

5. Conclusion:

By our methodology, one can find the another matrix of order 6x6, which have different Eigen Values but nearly same Eigen vectors if matrix is square and have property of square magic.

REFERENCES

[1] D. B. Ojha and B.L. Kaul, "Generalization of 4×4 magic square", International Journal Of Applied Engineering Research, Dindigul, vol.1,No.4,2011, 706-714.

[2] D. B. Ojha and B.L. Kaul, " Generalization of 6×6 magic square", Journal of Education and Vocational Research,Vol. 2,No. 1, pp. 18-23, July 2011.

[3] D. B. Ojha, "Generalization of Magic Square" Applied Mathematics, Elixir Appl. Math. 60 (2013) 15977-15979.

[4] D. B. Ojha, B. L. Kaul, " Generalization of 5×5 Magic Square", Journal of Education and Vocational Research, Vol. 2, No. 1, pp. 18-23, July 2011.

[5] Bibhutibhusan Datta and Awadhesh Narayan Singh (Revised by Kripa Shankar Shukla), Magic Squares in India, Indian Journal of History of Science, 27(1), 1992.

[6] W.W. Horner, Addition-multiplication magic squares, Scripta Math. 18 (1952) 300-303.