Bending of composite and sandwich plate using new higher order shear deformation theory

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[1]Komal Nawale, [1]Dr C P Pise1 , [2]Amol More

[1]Designation 1Department of Civil Engineering, SKN Sinhgad College of engineering, Pandharpur
[2]Department of Civil Engineering, D Y Patil Institute of Engineering management and Research ,Pune

Abstract : This For the bending, buckling and free vibration response of the thick and thin laminated composite and sandwich plates, the novel higher-order shear deformation theory is created. The recommended theories presume nonlinear variation of the transverse shear strain across the thickness of the plate and provide stress-free top and bottom surface of the plate for transverse shear. The governing differential equation derived by Hamilton's theory of the laminated composite and sandwich plates is solved using the technique of analytical solution, i.e. For the cross-ply plates with simply assisted boundary conditions along all the edges, Navier solution methodology. A wide range of numerical examples of the bending, buckling and free vibration response are considered in order to illustrate the accuracy and applicability of the proposed higher-order shear deformation theories. Using analogous single layer theories and three-dimensional precise solutions, the present findings are considered to be in strong agreement with those published earlier in the literature.

***IndexTerms* - Bending, Naviers Equation, Hamilton Principal , Shear deformation Theories**

I. INTRODUCTION

Composite materials has high strength to weight ratio, low specific modulus, high corrosion resistance, high stiffness to weight ratio and fatique resistance which enable its use in several engineering application[1-2]. The overgrowing demand of laminated composite plate in different structural application makes vital for the researcher to predict the accurate behaviour of plates in different working conditions.

Fiber-reinforced composites (FRC) have an extensive array of applications. They range from structural to recreational use. The aerospace and automotive industries look to composites to improve fuel economy due to its high strength to weight ratio.

The sports industry looks to composites to improve sports equipment technologies. Automobile, Defence, Naval, Civil and most recently in Micro-electromechanical applications due to high strength to weight ratio, flexibility, impact resistance and, manufacturability. The composite structures are generally classified based on the mixing of two or more material as Fibrous composite, particulate composite and, laminated composite. The Fibrous composite are prepared with matrix of one material with fiber of other material e.g. Fiber reinforced composite. Particulate composite are made up of powder sized particles of one material with matrix of other material e.g. metal matrix composites. Laminated composites are made up of different combination of layers so as to provide strength in required directions which can be a combination of layer of fibrous composite and particulate composite stacked over each other. The layers can be made up of various combination of material like isotropic, orthotropic or metallic and nonmetallic in nature or some laminates have piezoelectric properties and porous in nature depending on the application [1-2].

The behaviour of laminated composite plates are fundamentally predicted using three different approaches i.e. equivalent single layer (ESL) theories and layer-wise (LW) or Zig-Zag (ZZ) theories[3-5]. The Equivalent single layer theory was developed for the based on the single laminated structure which uses multi laminated structure as three dimensional problem as two dimensional problem. There are certain drawbacks of ESL theories such as Classical laminated plate theory (CLPT) which is based on Kirchhoff’s hypothesis can only be useful to predict the behaviour for thin plates accurately. The effect of transverse shear stress are not taken into consideration which affecting the behaviour of high thickness composite plates.

The Reissner [6]had taken into account of gross shear deformation into kinematics of Classical Laminated theory. The shear correction factor was introduced by Mindlin[7] which is determined comparing with Elasticity solution. The addition of shear deformation into classical laminated plate theory is also called as Mindilin reissner plate theory or The First order shear deformation theory. In FSDT, assumed that the shear deformation which is normal to midplane, after deformation became straight not perpendicular to midplane. The shear correction factor does not give accurate results as well as does not satisfied the surface conditions, also the shear correction factor depends on the ply angle sequence, boundary condition as well as Loading[8]. Many researcher studied the influence of transverse shear deformation using FSDT [9-12].

Many Higher order shear deformation theories are discovered by researcher to vanish the shear correction factor. The Reddy[13] developed higher order shear deformation based on polynomial transverse shear function which predicts deformation, natural frequency and buckling load of composite laminate with small amount of error. There are many researcher who developed polynomial shear deformation theories in terms of Taylor series expansion. The transverse shear strain shape function was introduced by Ambartsumyan[[14]] which predicts deformation of laminated plate using third order polynomial function. A Transverse shear strain function of higher order polynomial function was consider by many researcher [10], [15]considering boundary condition top and bottom surface of plate with zero transverse shear stress.

The a parabolic higher order shear deformation function was developed by Levinson and Murthy in which The displacement function are selected such that its behaviour varies with the plate thickness and the slope of the function would be continuous which also called as third order shear deformation theory (TSDT). The Phung -Van et al.[16] utilized TSDT for static and free vibration with cell based smoothed discrete shear gap method. Further Akbarzadeh et al [17]used TSDT on plates with structural gap and overlap to predict the characteristics using hybrid Fourier-Galerkin Method. Nasihatgozara and Khalili[18] used combination of FSDT and HSDT to predict the characteristics of sandwich plate which utilized FSDT for face sheet and HSDT for the core material. They applied Hamilton Principal and Energy method to determine motion equation which are solved using differential quadrature method.

Furthermore Kant and Pandya [19]uses same finite element method with seven variable polynomial to predict the behaviour of unsymmetrical ply angle sequence. Lo et al developed higher order theory using principal of potential energy which consist of eleven variable in differential to be determined for displacement equation. The extensive research on higher order shear deformation theories are well documented in a review papers of Noor, Reissner, Reddy, Carrierra and Kant [20-23].

The non-polynomial function are similarly utilized as a shear strain function for HSDT extensively in recent decade to predict the characteristics of static deformation, buckling and free vibration of laminated as well as the sandwich composite plates. Levy, Stein and Tourtier[24] uses sine function as shear deformation function which also referred to as Trigonometric sine shear deformation function. D B Singh et al[25] TSDT with third order power expansion which gives accurate results than TSDT. Moreover, the shear deformation function in terms of trigonometric secant function was employed by Grover et al[26] which also accurately predict the behaviour of laminated and sandwich plates under the different loading conditions.

Mantari et al[27] introduced Tangent function as shear strain function for isotropic, composite and sandwich plates which shows good agreement with other TSDT. The Trigonometric tangent shear deformation theory are further developed and modified by D B Singh et al[28] to study of composite, laminated , and 3D braided composite plates. Furthermore inverses tangent shear strain function is also used by Suganyadevi et al. for predicting the static, bucking and free vibration response. Grover et al[29] utilized Cotangent (cot) function as shear strain function.

Soldetos et al. [[30]]proposed unified hyperbolic sine shear strain function which is able to change transverse strain distribution, This Hyperbolic sine shear deformation Theory(HSSDT) employed on composite and sandwich laminated plate static , buckling and Free vibration. The HSSDT is modified by El meiche et al for functionally graded composite plates. Inverse hyperbolic sine shear deformation theory was developed by Grover et al.[31] to study static and buckling response of the laminated plates.

Karama et al [32]used exponential function as shear strain function which gives far better results than polynomial as well as trigonometric shear deformation theory with compared elasticity solution. The comparison of polynomial , trigonometric, hyperbolic trigonometric and Exponential shear deformation was carried out by Aydogdu[33].Mantari et al [34] used exponential shear stress parameter to develop higher shear deformation theory.

Unified structure for modelling and analysis of composite plate are developed by Nguyen et al [35] which is polynomial form of higher shear deformation theory in unified formulation with consideration of thickness stretching consideration. The various non-polynomial shear deformation theories are constructed in diagram which also called as Best Theory Diagrams for composite laminated plates. Best theory diagrams used to calculate number of variables required to reduce the fixed error based on axiomatic/asymptotic method which gives accurate results for composite plates.

The literature review suggests the need of predicting accurate nature of bending, buckling and vibration characteristics of laminated and sandwich composite plate. There are several methods which are already developed in the last century for predicting the behaviour of composite plate with accurate prediction but there is still scope for improvement to minimize the error. The new higher order shear deformation theories are developed so that the error in results can be minimized.

In This Paper, A simple isotropic and Composite plates simply supported plates with sinusoidal and uniform distributed loading with standard boundary conditions are considered for the analysis using New Higher order shear deformation theory using Naviers solution. The computed results for bending are compared with previously published results. The proposed theory can be then use for sandwich as well as piezoelectric plates..

II. TYPE MATHEMATICAL MODEL

The Displacement field in Higher Order Shear Deformation Theory is given as per Eq.1.

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\phi_x(x,y,t)$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\phi_y(x,y,t)$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{1}$$

Where f(z) is the function of transverse shear strain, which must satisfy the conditions of transverse sharing of the plate's top and bottom surfaces without stress. The derivative of f(z) must disappear on the top and bottom surfaces of the plate to fulfill the condition of transverse shear stress-free top and bottom surfaces of the plate. In nature, f(z) can be polynomial, trigonometric, hyperbolic and logarithmic.

The shear strain Function proposed in this paper is given by Eq. 2

$$f(z)=g(z)+\Omega z=\sin\left(\frac{z}{h}\right)-\frac{z^3}{h^2}+\frac{1}{h}\cos\left(\frac{1}{2}\right)-\frac{3}{4h^2} \tag{2}$$

- **Strain-Displacement Relationships**

Considering small deformation theory, linear strain-displacement relationship, according to the considered displacement field, for the plate are expressed as:

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w}{\partial x^2}+f(z)\frac{\partial \phi_x}{\partial x} \tag{3a}$$

$$\varepsilon_{yy}=\frac{\partial v}{\partial y}=\frac{\partial v_0}{\partial y}-z\frac{\partial^2 w}{\partial y^2}+f(z)\frac{\partial \phi_y}{\partial y} \tag{3b}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}=\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}-2z\frac{\partial^2 w}{\partial x\partial y}+f(z)\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right) \tag{3c}$$

$$\gamma_{xz}=\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=\frac{\partial f(z)}{\partial z}\phi_x \tag{3d}$$

$$\gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}=\frac{\partial f(z)}{\partial z}\phi_y \tag{3e}$$

Assuming homogeneous and linear behaving material and plane stress condition in the lamina, the constitutive stress-strain relations for any *k*th lamina in the composite or sandwich plate is expressed as:

$$\sigma=Q.\varepsilon \tag{4}$$

where Q is a constitutive matrix depending up the fibre orientations and elastic properties within the *k*th lamina.

The governing differential equations of the plate are obtained using Hamilton's participle, in the mathematical form it is expressed as:

$$\int_{\tau_1}^{\tau_2}\left(\delta K-\delta U+\delta V\right)d\tau=0 \tag{6}$$

where N_{xx}^b, N_{yy}^b and N_{xy}^b are the applied in-plane loads. Substituting the variations of kinetic energy, potential energy and work-done due to external loads with Eq. (6) followed by integration by parts and collecting coefficients of δu_0, δv_0, δw_0, $\delta\phi_x$ and $\delta\phi_y$, the five Euler-Lagrange equations are expressed as

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \tag{7a}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}=0 \qquad \text{(7b}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+q=0 \qquad) \tag{7c}$$

$$\frac{\partial M_{xx}^0}{\partial x}+\frac{\partial M_{xy}^0}{\partial y}-Q_{xz}^0=0 \tag{7d}$$

$$\frac{\partial M_{xy}^0}{\partial x}+\frac{\partial M_{yy}^0}{\partial y}-Q_{yz}^0=0 \tag{7e}$$

Subsequently stiffness coefficients Eqs. (71-7d) can be revised as:

$$A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}=\sum_k\bar{Q}_{ij}\int_{z_k}^{z_{k+1}}1,z,z^2,g(z),zg(z),g^2(z)\,dz \qquad i,j=1,2,6 \tag{8}$$

$$L_{ij},K_{ij}=\sum_k\bar{Q}_{ij}\int_{z_k}^{z_{k+1}}\frac{\partial g(z)}{\partial z},\left(\frac{\partial g(z)}{\partial z}\right)^2dz \qquad i,j=4,5 \tag{9}$$

The in-plane force and moment resultants due to stresses in the plate and laminated stiffness coefficients are obtained by integrations over the thickness of the plate and expressed as:

$$\begin{Bmatrix}N_{ij}\\M_{ij}\\M_{ij}^0\end{Bmatrix}=\sum_k\int_{z_k}^{z_{k+1}}\sigma_{ij}\begin{Bmatrix}1\\z\\f(z)\end{Bmatrix}dz \qquad i,j=x,y \tag{10a}$$

$$\begin{Bmatrix}Q_{xz}^0\\Q_{yz}^0\end{Bmatrix}=\sum_k\int_{z_k}^{z_{k+1}}\frac{\partial f(z)}{\partial z}\begin{Bmatrix}\sigma_{xz}\\\sigma_{yz}\end{Bmatrix}dz \tag{10b}$$

III. SOLUTION

The following assumed trigonometric series satisfy the simply supported boundary conditions:

$$u_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos\alpha x.\sin\beta y.\sin\omega t \tag{11a}$$

$$v_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin\alpha x.\cos\beta y.\sin\omega t \tag{11b}$$

$$w_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\alpha x.\sin\beta y.\sin\omega t \tag{11c}$$

$$\phi_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Phi_{X\,mn} \cos\alpha x.\sin\beta y.\sin\omega t \tag{11d}$$

$$\phi_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Phi_{Y\,mn} \sin\alpha x.\cos\beta y.\sin\omega t \tag{11e}$$

where $\alpha = m\pi/a$ and $\beta = n\pi/b$, m and n are the half-wave numbers along x and y-directions, respectively.

For the bending response of the laminated composite and sandwich plates, the inertia terms and the terms associated with the in-plane applied loads are omitted. The transverse load q can be expanded using the Fourier series:

$$q = \sum_{m}^{\infty}\sum_{n}^{\infty} q_{mn} \sin\alpha x.\sin\beta y \tag{12a}$$

For the sinusoidal load $q = q_0 \sin\alpha x \sin\beta y$:

$$q_{mn} = q_0 \qquad m = n = 1 \tag{12b}$$

For the uniformly distributed load $q = q_0$:

$$q_{mn} = 16q_0/(\pi^2 mn) \qquad m = 1, 2, 3, 4\ldots\ldots \text{ and } n = 1, 2, 3, 4\ldots\ldots \tag{12c}$$

IV. RESULTS AND DISCUSSION

The bending of symmetric and anti-symmetric simply supported cross-ply laminated and sandwich plates under different types of transverse loads, such as sinusoidal load (SSL), uniformly distributed load (UDL) and concentrated point load (CPL), is investigated using proposed higher-order shear deformation theories. For the cross-ply laminated plates with [0/90/90/0], [0/90/0] and $[0/90/0....]_n$ lamination schemes are considered with the following material properties:

$E_1 = 25E_2$; $G_{12} = 0.5E_2$; $G_{13} = 0.5E_2$; $G_{23} = 0.2E_2$ and $\nu_{12} = 0.25$

The central deflection and stresses are presented in the non-dimensional using the following identities:

$$\bar{w}_0 = w_0\left(\frac{100h^3E_2}{b^4q_0}\right),\quad \bar{\sigma}_{xx} = \sigma_{xx}\left(\frac{h^2}{b^2q_0}\right),\quad \bar{\sigma}_{yy} = \sigma_{yy}\left(\frac{h^2}{b^2q_0}\right),\quad \bar{\sigma}_{xy} = \sigma_{xy}\left(\frac{h^2}{b^2q_0}\right)$$

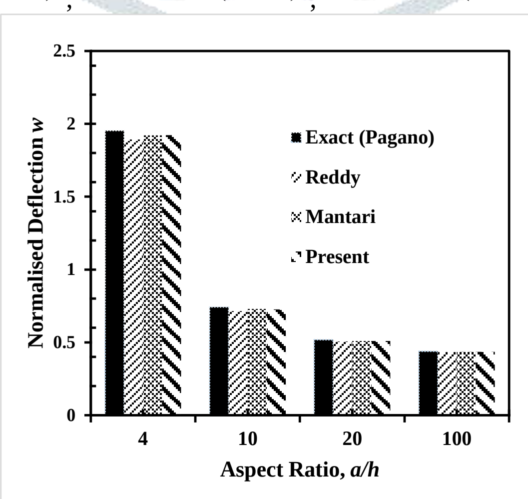

Fig 1. Variation of Central deflection with aspect ratio a/h

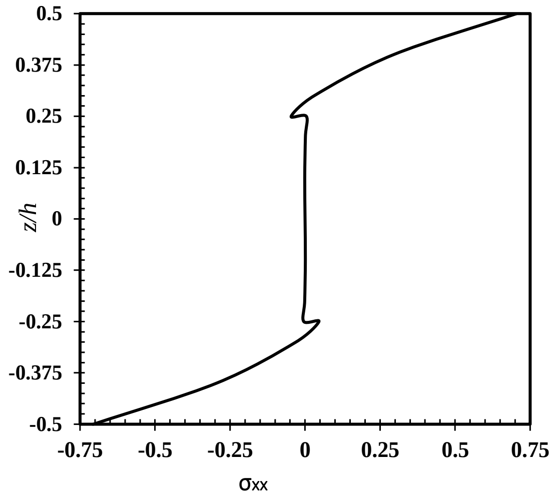

Fig 2 Variation of normal stress σ_{xx}, across thickness for [0/90/90/0] laminated plate

For the various values of a/h predicted using a transverse shear strain function, the non-dimensional central deflection as shown in Fig 1 is similar to that obtained via Reddy's third-order shear deformation[13]. The average error obtained for all the parameters for the respective period thickness ratio shows that the current function gives an average error of 2.75 percent, less than 4.26 percent for Mantari et al.[26] and 7.25 percent for Reddy[13], compared to the Ref.[35] elasticity solution. The variance of bending and transverse shear stresses with the thickness coordinate is shown in Fig 2 by thickness. The action of stresses for the theories proposed is very similar to each other, even comparable to those previously published according to theories of higher-order shear deformation

IV. CONCLUSION

The newly developed higher-order shear deformation theory is accessed for the bending, buckling and free vibration response of the laminated composite and sandwich plates. The proposed plate theory assume nonlinear variation of transverse shear strain through the plate thickness and thus delivers transverse shear stress-free top and bottom surface of the plate.

The governing equation of plate are solved using Navier solution technique with simply supported boundary conditions to investigated the accuracy of the proposed theories. From the proposed higher-order shear deformation theory predicted better deflection of laminated composite plates

The proposed higher-order shear deformation theory with combination of trigonometric and polynomial function as a transverse shear strain function predicted results close of those of predicted with TSDT of Reddy [12].

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