# Cosmological model with zero mass scalar field in modified gravity

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**Abstract:** In this context, we study the exact matter dominated power law solutions of Bianchi type- $V$ cosmological models contains two fluid source with zero mass scalar field in the metric version of $f(R)$ gravity. The field equations are solved by taking expansion scalar proportional to shear scalar, which provides the relation between the metric coefficients. The physical behavior of the solutions has been discussed using some physical quantities.

**Keywords:** Two fluid source; $f(R)$ gravity; Cosmology.

## 1. Introduction

Recent cosmological observations indicating that our universe experiences an accelerated expansion due to mysterious energy with negative pressure [1, 2] and other observations such as cosmic microwave background (CMB) anisotropies measured with WMAP satellite [3] and large-scale structure [4] suggest that nearly two-third of our universe consists of negative pressure dubbed as dark energy, and the remaining consists of relativistic dark matter and baryons [5]. A very important parameter for the dark energy (DE) investigation is that of the equation of state parameter (EoS) which is usually parameterized of the form $\omega = p/\rho$, where $p$ and $\rho$ be the pressure and density. One can see that a value of EoS parameter $\omega < -1/3$ is required for accelerated cosmic expansion. The primary candidates in this category are scalar field models such as Quintessence [6, 7] and K-essence [8]. In quintessence models the range of EoS parameter is $-1 < \omega < -1/3$, and the DE density decreases by a scale factor $a(t)$ as $\rho \propto a^{-3(1+\omega)}$ [9]. A specific exotic form of DE denoted phantom energy, with $\omega < -1$ [10].

The alternative way to explain cosmic acceleration is a classical generalization of general relativity (GR). Among the various modifications of Einstein's GR, one is $f(R,T)$ gravity [11]. In this theory the Gravitational Lagrangian is given by an arbitrary function of the Ricci scalar $(R)$ and trace of the stress energy tensor $(T)$. Several authors have investigated the aspect of cosmological models in this gravity [12, 17]. Second, Einstein [18] has presented another form of gravity called teleparallel gravity, namely $f(T)$ gravity to explain the current accelerating expansion without introducing dark energy, which allows one to say gravity is not due to curvature, but due to torsion. In this theory some authors [19- 23] have discussed several features of cosmological models. In a classical generalization of general relativity (GR) one replaces the Ricci scalar $R$ in the Einstein-Hilbert action by an arbitrary function of $R$ belongs to the well-known $f(R)$ modified gravity. Considering a viable $f(R)$

gravity models Nojiri and Odintsov [24] shows that the unification of early-time inflation and late-time acceleration.

Deriving the exact solution from a power law $f(R)$ cosmological model Capozziello et al. [25] achieve dust matter and DE phase. Using the same theory Azadi et al. [26] studied vacuum solution in cylindrically symmetric space time. Miranda et al. [27] discussed a viable singularity-free $f(R)$ gravity without a cosmological constant. Sharif and Yousaf [28] studied the impact of DE and dark matter models on the dynamical evolution of collapsing self-gravitating systems in this gravity.

Zero-mass scalar field which is fundamental challenge to look into the yet unsolved problem of the unification of the gravitational and quantum theories. In the last few decades there has been renewed interest focused on the theory of gravitation representing zero-mass scalar fields coupled with gravitational field. The zero-mass scalar field has acquired particular importance. For isotropic background in the presence of a gravitational field 'Big-Bang' of universe at the initial stage can be avoided by introducing a zero-mass scalar field [29].

## 2 Some Basics of $f(R)$ Gravity

The $f(R)$ theory of gravity is the generalization of General Relativity. The three main approaches in $f(R)$ theory of gravity are "Metric Approach", "Palatine formalism" and "affine $f(R)$ gravity". In metric approach, the connection is the Levi-Civita connection and variation of the action is done with respect to the metric tensor. While, in Palatine formalism, the metric and the connection are independent of each other and variation is done for the two mentioned parameters independently. In metric-affine $f(R)$ gravity, both the metric tensor and connection are treating independently and assuming the matter action depends on the connection as well.

The action for this theory is given by

$$S = \int \sqrt{-g}\left(f(R) + L_m\right) d^4 x. \tag{1}$$

Here $f(R)$ is a general function of the Ricci Scalar, $g$ is the determinant of the metric $g_{\mu\nu}$ and $L_m$ is the matter Lagrangian.

It is noted that this action is obtained just by replacing $R$ by $f(R)$ in the standard Einstein–Hilbert action.

The corresponding field equation from this action are found

$$F(R)\, R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \Box F(R) = T^M_{\mu\nu}, \tag{2}$$

where $\Box \equiv \nabla^\mu \nabla_\mu$, $F(R) \equiv \frac{df(R)}{dR}$, $\nabla_\mu$ is the covariant derivative and $T^M_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian $L_m$.

## 3. Metric and Energy Momentum Tensor

We consider a spatially homogeneous Bianchi type $V$ cosmological model of the form

$$ds^2 = -dt^2 + A^2(t) dx^2 + B^2(t) e^{-2x} dy^2 + C^2(t) e^{-2hx} dz^2, \tag{3}$$

where the metric potentials $A, B, C$ are the functions of time $t$ and $m$ be the constant.

Some geometrical parameters related with the metric potential for the metrics (3) are defined as

The average scale factor $a$ and the volume scale factor $V$ are defined as

$$a = \sqrt[3]{ABC}, \qquad V = a^3 = ABC, \tag{4}$$

The average Hubble parameter $H$ is given by

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

The expansion scalar $\theta$, Anisotropy parameter $\Delta$ and shear scalar $\sigma$ are defined as

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{6}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i^2}{H}\right)^2, \tag{7}$$

$$\sigma^2 = \frac{3}{2}\Delta H^2. \tag{8}$$

The energy momentum tensor for the fluid coupled with zero-mass scalar field is given as

$T_{\mu\nu}^{M} = \left[T_{\mu\nu}^{F} + T_{\mu\nu}^{\phi}\right]$The energy momentum tensor $T_{\mu\nu}^{F}$ of a fluid with two fluids (barotropic and dark energy) is given by the equation

$$T_{\mu\nu}^{F} = (\rho + p)\, u_\mu u_\nu - p\, g_{\mu\nu}, \tag{9}$$

where $\rho$ and $p$ are density and pressure of the fluid. In a co-moving system, the unit time vector $u^\mu$ is given by $u^\mu u_\mu = 1$.

For two fluid sources pressure, energy density and equation of state parameter can be parameterized as

$$p = (p_m + p_d), \qquad \rho = (\rho_m + \rho_d), \tag{10}$$

$$\omega_m = p_m / \rho_m, \tag{11}$$

and

$$\omega_d = p_d / \rho_d. \tag{12}$$

where $p_m$ and $\rho_m$ are pressure and energy density of barotropic fluid, $p_d$ and $\rho_d$ are pressure and energy density of DE respectively also

And the contribution to $T_{\mu\nu}^{M}$ from the minimally coupled zero-mass scalar field $\phi$ is

$$T_{\mu\nu}^{\phi} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}. \tag{13}$$

where $\phi$ being a zero-mass scalar field which satisfy the equation

$$\phi_{;\mu}^{\mu} = 0. \tag{14}$$

### 4. Field Equations and their Solutions

In the presence of barotropic and dark energy source given in equation (9), the field equations (2) corresponding to the metric (3) lead to the following set of linearly independent differential equations

$$\left[H^2(4-q)+\frac{2}{A^2}\right]F+\frac{1}{2}f(R)+2H\dot{F}+\ddot{F}=\left(-p+\frac{\dot{\phi}^2}{2}\right), \tag{15}$$

$$\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)F+\frac{1}{2}f(R)-3H\dot{F}=\left(\rho-\frac{\dot{\phi}^2}{2}\right), \tag{16}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{17}$$

$$\ddot{\phi}+3H\dot{\phi}=0. \tag{18}$$

Here the overhead dot denotes differentiation with respect to *t*.

**Exact Matter Dominated Power Law Solution of the Field Equation**

The universe is characterized by power law scale factors with constant exponent. Due to the complexity of the field equation, it is very difficult to evaluate explicit analytical forms of the scale factor, so we have chosen the scale factor of the form Setare et al. [20], Chirde & Shekh [21]

$$a(t)=a_0 t^{n_1}, \tag{19}$$

$n_1$ is any positive real number.

In this type of solution, the expansion of the universe depends on $n_1$.

(i) $n_1>0$ leads to an expanding universe;

(ii) $n_1<0$ describes a contracting universe.

The equation (18) yield

$$\phi=\left(\frac{c_1}{1-3n_1}\right)t^{1-3n_1}+c_2. \tag{20}$$

Since the twice contracted Bianchi identity assures the conservation of the energy momentum tensor $T^{(F)\mu\nu}_{;\nu}$, we could be sure that the fluid would satisfy the energy-momentum conservation law. The conservation equation yield

$$\dot{\rho}+3(p+\rho)H=0. \tag{21}$$

We consider the interaction between dark viscous and barotropic fluids. Therefore, the general form of conservation Eqn. (21) leads to bulk viscous and dark fluid separately as

$$\dot{\rho}_m+3H(p_m+\rho_m)=Q, \tag{22}$$

$$\dot{\rho}_d+3H(p_d+\rho_d)=-Q. \tag{23}$$

where, the quantity $Q > 0$ expresses the interaction between the fluids. Here we emphasize that the continuity Eqns. (22) and (23) imply that the interaction term $Q$ should be proportional to a quantity with units of inverse of time $Q \propto {}^{1}\!/_{t}$. In our study, we consider $Q$ of the form

$$Q = 3H\sigma\rho_m , \tag{24}$$

where, $\sigma$ is the coupling constant.

Solving Eqn. (24) with the help of Eqn. (19), we get

Energy density and pressure of barotropic fluid is

$$\rho_m = \frac{c_4}{t^{3n_1(1+\omega_m-\sigma)}} , \tag{25}$$

$$p_m = \frac{c_4\omega_m}{t^{3n_1(1+\omega_m-\sigma)}} . \tag{26}$$

**4.2 Field equations and their solutions:**

The field of $f(R)$ gravity provided in equation equations (15)-(17) will reduce to

$$\left[H^2(4-q)+\frac{2}{A^2}\right]F+\frac{1}{2}f(R)+2H\dot{F}+\ddot{F}=\left(-p+\frac{\dot{\phi}^2}{2}\right), \tag{27}$$

$$\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)F+\frac{1}{2}f(R)-3H\dot{F}=\left(\rho-\frac{\dot{\phi}^2}{2}\right), \tag{28}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 , \tag{29}$$

Above equation (29) gives

$A^2 = BC$, (without loss of generality we consider integration constant is one). (30)

We need one additional constraint to solve these field equations. We use a physical condition that expansion scalar $\theta$ proportional to the shear scalar $\sigma$ which gives

$$B = C^n . \tag{31}$$

Using equations (19), (30) and (31) metric potential comes out to be

$$A = a_0 t^{n_1} , \tag{32}$$

$$B = a_0^{\frac{2n}{(n+1)}} t^{\frac{2nn_1}{(n+1)}} , \tag{33}$$

$$C = a_0^{\frac{2}{n+1}} t^{\frac{2n_1}{(n+1)}} . \tag{34}$$

Using the above equations (32) to (34), the cosmological model takes the form

$$ds^2 = -dt^2 + a_0^{\frac{6n}{2n+1}} t^{\frac{6nn_1}{(2n+1)}} dx^2 + a_0^{\frac{6n}{2n+1}} t^{\frac{6nn_1}{(2n+1)}} e^{-2x} dy^2 + a_0^{\frac{6}{2n+1}} t^{\frac{6n_1}{(2n+1)}} e^{-2hx} dz^2 . \tag{35}$$

With the values of metric potentials given in equations (32) and (34),

Ricci scalar,

$$R=\frac{\varepsilon_5}{(n+1)^2t^2}+\frac{6}{a_0^2t^{2n_1}}. \tag{36}$$

Density of dark fluid

$$\rho_d=\frac{\varepsilon_6}{2(2n+1)^4t^4}+\frac{\varepsilon_7}{a_0^2(n+1)^2t^{2n_1+2}}-\frac{12n_1\alpha\varepsilon_5}{(2n+1)^2t^4}-\frac{72\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{18\alpha}{a_0^4t^{4n_1}}-\frac{c_1^2}{2a_0^6t^{6n_1}}-\frac{c_3}{t^{3n_1(1+\omega_m-\sigma)}}. \tag{37}$$

Density of combined fluid,

$$\rho=\frac{\varepsilon_6}{2(2n+1)^4t^4}+\frac{\varepsilon_7}{a_0^2(n+1)^2t^{2n_1+2}}-\frac{12n_1\alpha\varepsilon_5}{(2n+1)^2t^4}-\frac{72\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{18\alpha}{a_0^4t^{4n_1}}-\frac{c_1^2}{2a_0^6t^{6n_1}}. \tag{38}$$

Pressure of dark fluid

$$p_d=\frac{\varepsilon_8}{(n+1)^2t^4}-\frac{\alpha\varepsilon_5^2}{2(n+1)^4t^4}-\frac{\varepsilon_9}{a_0^2(n+1)^2t^{2n_1+2}}+\frac{48\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{24\alpha}{a_0^4t^{4n_1}}+\frac{c_1}{2a_0^6t^{6n_1}}-\frac{c_4\omega_m}{t^{3n_1(1+\omega_m-\sigma)}}, \tag{39}$$

Pressure of combined fluid,

$$p=\frac{\varepsilon_8}{(n+1)^2t^4}-\frac{\alpha\varepsilon_5^2}{2(n+1)^4t^4}-\frac{\varepsilon_9}{a_0^2(n+1)^2t^{2n_1+2}}+\frac{48\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{24\alpha}{a_0^4t^{4n_1}}+\frac{c_1}{2a_0^6t^{6n_1}}. \tag{40}$$

EoS parameter of Dark fluid

$$\omega_d=\frac{\left(\frac{\varepsilon_8}{(n+1)^2t^4}-\frac{\alpha\varepsilon_5^2}{2(n+1)^4t^4}-\frac{\varepsilon_9}{a_0^2(n+1)^2t^{2n_1+2}}+\frac{48\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{24\alpha}{a_0^4t^{4n_1}}+\frac{c_1}{2a_0^6t^{6n_1}}-\frac{c_4\omega_m}{t^{3n_1(1+\omega_m-\sigma)}}\right)}{\left(\frac{\varepsilon_6}{2(2n+1)^4t^4}+\frac{\varepsilon_7}{a_0^2(n+1)^2t^{2n_1+2}}-\frac{12n_1\alpha\varepsilon_5}{(2n+1)^2t^4}-\frac{72\alpha n_1^2}{a_0^2t^{2n_1}}-\frac{18\alpha}{a_0^4t^{4n_1}}-\frac{c_1^2}{2a_0^6t^{6n_1}}-\frac{c_3}{t^{3n_1(1+\omega_m-\sigma)}}\right)}. \tag{41}$$

**Conclusion:**

In the present paper the analysis of exact matter dominated power law solutions of Bianchi type- $V$ cosmological models contains two fluid source with zero mass scalar field in the metric version of $f(R)$ gravity has been proposed. The field equations are solved by taking expansion scalar proportional to shear scalar, which provides the relation between the metric coefficients. In the derived model, it is observed that the energy density for both fluids are inversely related with time which shows with the expansion of the model energy density decreases. Initially when t = 0, the spatial volume of the model is zero and expands exponentially as model expands.

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