Difference between Fuzzy and Crisp Transportation Problem Using Pentagonal Fuzzy Numbers with ranking by α-cut Method

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Abstract:

Uncertainty plays important role in science, technology, and the medical branch. In all types of fuzzy numbers, pentagonal fuzzy numbers are important due to their shape. In this article, crisp transportation problems converted into fuzzy transportation problems with pentagonal fuzzy numbers. Fuzzy North West Corner Method, fuzzy Least Cost Method, and Fuzzy Vogel's Method apply to obtain an initial basic feasible solution. The result and comparison of a crisp transportation problem and fuzzy transportation problem are discussed at the end. In real-life applications available forecast demand and supply are often uncertain because some information is incomplete or unavailable.

Keywords: Fuzzy Pentagonal, Error, Fuzzy Transportation, α- Cut, Ranking

Introduction

Arindam Chaudhuri [6] used trapezoidal fuzzy number for solving the transportation problem. Hiroaki Ishii [7] used random transportation costs to solve fuzzy transportation Problem. W.Ritha [10] developed Multi-objective two-stage fuzzy transportation problem. K. L. Bondar and Ashok Mhaske[1] Fuzzy Transportation Problem with Error by Using Lagrange's Polynomial. S. Nareshkumar [9] used modified Vogel's approximation method to study fuzzy transportation problems. Surjeet Singh Chauhan [8] applied improved VAM with robust ranking technique to find the solution of fuzzy transportation problem. Ashok S Mhaske, Kirankumar L Bondar used Fuzzy Transportation Problem by Using Triangular, Pentagonal and HeptagonalFuzzy Numbers With Lagrange's Polynomial to Approximate Fuzzy Cost for Nonagon and Hendecagon.

Basic Definitions

Pentagonal fuzzy number:

Pentagonal fuzzy number $A = (x_1, x_2, x_3, x_4, x_5)$ and its membership function is:

$$\mu_A(x) = \begin{cases} 0 & x < x_1 \\ \frac{x-x_1}{x_2-x_1} & x_1 \le x < x_2 \\ \frac{x-x_2}{x_3-x_2} & x_2 \le x < x_3 \\ 1 & x = x_3 \\ \frac{x_4-x}{x_4-x_3} & x_3 < x \le x_4 \\ \frac{x_5-x}{x_5-x_4} & x > x_5 \end{cases}$$

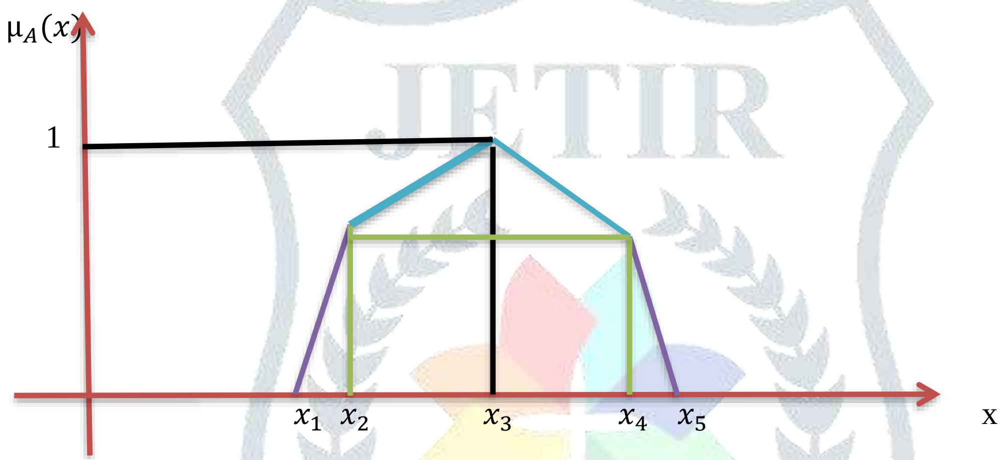

Figure 1: Pentagonal fuzzy number $[x_1, x_2, x_3, x_4, x_5]$

Operations on pentagonal fuzzy numbers:

Let $\bar{X} = (x_1, x_2, x_3, x_4, x_5)$, $\bar{Y} = (y_1, y_2, y_3, y_4, y_5)$ be any two pentagonal fuzzy pentagonal numbers then algebraic properties defined as below:

i) Addition: $\bar{X}\ (+)\ \bar{Y} = (x_1, x_2, x_3, x_4, x_5) + (y_1, y_2, y_3, y_4, y_5)$

$= (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4, x_5 + y_5)$

ii) Subtraction: $\bar{X}\ (-)\ \bar{Y} = (x_1, x_2, x_3, x_4, x_5) - (y_1, y_2, y_3, y_4, y_5)$ $= (x_1 - y_5, x_2 - y_4, x_3 - y_3, x_4 - y_2, x_5 - y_1)$

iii) Multiplication: $\bar{X}\ (\times)\ \bar{Y} = (x_1, x_2, x_3, x_4, x_5) \times (y_1, y_2, y_3, y_4, y_5)$

$= (x_1 \times y_1, x_2 \times y_2, x_3 \times y_3, x_4 \times y_4, x_5 \times y_5)$

α- Cut for pentagonal fuzzy number:

For any $\alpha \in [0, 1]$

$$\frac{x_1^\alpha - x_1}{x_2 - x_1} = \alpha, \quad \frac{x_2^\alpha - x_2}{x_3 - x_2} = \alpha \quad \frac{x_4 - x_4^\alpha}{x_4 - x_3} = \alpha, \quad \frac{x_5 - x_4^\alpha}{x_5 - x} = \alpha$$

$$x_1^\alpha = (x_2 - x_1)\,\alpha + x_1, \quad x_2^\alpha = (x_3 - x_2)\,\alpha + x_2,$$

$$x_4^{\alpha} = -(x_4 - x_3)\,\alpha + x_4,\quad x_5^{\alpha} = -(x_5 - x_4)\,\alpha + x_5$$

Thus $\bar{X}_{\alpha} = [x_1^{\alpha} \quad x_3^{\alpha} \quad x_4^{\alpha} \quad x_5^{\alpha}]$

Ordering two pentagonal fuzzy numbers:

Pentagonal fuzzy numbers $\bar{A}$ and $\bar{B}$ can be order by finding α cut say $\bar{A}_{\alpha}$ and $\bar{B}_{\alpha}$. If α cut $\bar{A}_{\alpha}$ is less than or equal to each α-cut of $\bar{B}_{\alpha}$ for any $\alpha = 0.5$ then fuzzy number $\bar{A} \leq \bar{B}$.

$\bar{A}_{\alpha} = [p_1 = (x_2 - x_1) * \alpha + x_1,\ p_2 = (x_3 - x_2) * \alpha + x_2,\ p_3 = -(x_4 - x_3) * \alpha + x_3,\ p_4 = -(x_5 - x_4) * \alpha + x_5]$

$\bar{B}_{\alpha} = [q_1 = (y_2 - y_1) * \alpha + y_1,\ q_2 = (y_3 - y_2) * \alpha + y_2,\ q_3 = -(y_4 - y_3) * \alpha + y_3,\ q_4 = -(y_5 - y_4) * \alpha + y_5]$

Then $\bar{A} \leq \bar{B}$ if $\frac{(p_1+p_2+p_3+p_4)}{4} \leq \frac{(q_1+q_2+q_3+q_4)}{4}$ otherwise $\bar{B} \leq \bar{A}$

Method for solving fuzzy transportation problem:

Crisp transportation problem is converted into fuzzy transportation problem using Pentagonal, Fuzzy numbers. Fuzzy North West Corner Method (FNWCM), Fuzzy Matrix Minima Method (FMMM) and Fuzzy Vogel's Approximation Method (FVAM) methods are used to obtain the initial basic feasible solution to fuzzy transportation.

Mat-lab Code:

```
clc
'Enter Your first Pentagonal number\n'
a1=input('');
b1=input('');
c1=input('');
d1=input('');
e1=input('');
a=[a1  b1  c1   d1  e1]
 'Enter Your second Pentagonal number\n'
a2=input('');
b2=input('');
c2=input('');
d2=input('');
e2=input('');
b=[a2  b2  c2   d2  e2]
'Enter Your Choice 1= Addition 2= Subtraction  3 = Multiplication ,4 = order'
ch=input('');
switch(ch)
   case 1
'Addition of two fuzzy Pentagonal number is'
c = a + b
   case 2
'subtraction of two fuzzy Pentagonal is(a-b)'
c=[a1-e2  b1-d2  c1-c2   d1-b2   e1-a2]
   case 3
 'Multiplication of two fuzzy Pentagonalis(a-b)'
c=[a1*a2  b1*b2  c1*c2  d1*d2   e1*e2]
```

```
    otherwise
disp('unknown method')
end
```

Numerical example:

Consider the following crisp transportation problem

Table 1: Crisp transportation problem

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	6	3	5	4	23
$\bar{O}_2$	5	9	2	7	17
$\bar{O}_3$	5	7	8	6	9
Demand	8	13	18	10	

Table 2: Comparison of minimum transportation cost

	NWCR	LCM	VAM
Minimum Transportation cost	190	161	160

Fuzzy Transportation Problem by using Pentagonal Fuzzy Number

Table 3: FNWCM for pentagonal fuzzy number

Destination	Supply
[2 3 4 5 6]	(21 22 23 24 22)
[5 6 7 8 9] **(-11 -5 2 7 13)**	(15 16 17 18 19)
[4 5 6 7 8] **(2 8 14 21 22)**	(7 8 9 10 11)
(9 10 11 12 13)	
Minimum Fuzzy Transportation Cost = [-50,25,135,284,487] = **199.6**	

Origin	[3 4 5 6 7] **(5 6 7 8 9)**	[1 2 3 4 5] **(11 12 13 14 15)**	[3 4 5 6 7] **(-4 0 4 7 8)**
	[2 3 4 5 6]	[7 8 9 10 11]	[0 1 2 3 4] **(7 11 15 19 23)**
	[2 3 4 5 6]	[5 6 7 8 9]	[6 7 8 9 10]
Demand	(5 6 7 8 9)	(11 12 13 14 15)	(16 17 18 19 20)

Table 4: FLCM for pentagonal fuzzy number

Destination	Supply
[2 3 4 5 6] **(8 9 10 11 12)**	(21 22 23 24 25)
[5 6 7 8 9]	(14 15 16 18 19)
[4 5 6 7 8]	(6 8 10 11 12)
(8 9 10 11 12)	
Minimum Fuzzy Transportation Cost = [11,24,39,56, 58] + [-6,4,20,42,48] + [16,27,40,55,65] + [0,15,32,54,55] + [18,28,40,54,59] + [12,7,32,54,79] = [-	

Origin	[4 5 6 7 8]	[1 2 3 4 5] **(10 11 12 13 14)**	[23 4 5 6 7] **(-5 -2 1 4 7)**
	[3 4 5 6 7]	[7 8 9 10 11]	[0 1 2 3 4] **(14 15 16 18 19)**
	[3 4 5 6 7] **(5 6 7 8 9)**	[5 6 7 8 9]	[6 7 8 9 10] **(-4 -2 1 4 6)**
Demand	(6 7 8 9 10)	(11 12 13 14 15)	(16 17 18 19 20 21)

Table 5: FVAM for pentagonal fuzzy number

Destination	
[2 3 4 5 6] **(4 8 12 17 18)**	(21 22 23 24 25)
[5 6 7 8 9]	(15 16 17 18 19)
[4 5 6 7 8] **(-2 1 4 6 7)**	(7 8 9 10 11)
(8 9 10 11 12)	
Minimum Fuzzy Transportation Cost = [24,39,56, 58] + [0,8,20,42 44] + [8,24,48,85 , 87] + [0,15,32,54,58] + [118,28,40,54] + [-12,-8,5,24,42] = [29,104,203,333,356] = **167.8**	

	[2 3 4 5 6 7] **(0 2 4 7 8)**	[0 1 2 3 4] **(14 15 16 18 19)**	[6 7 8 9 10]	(16 17 18 19 20)	
	[1 2 3 4] 5 **(11 12 13 14 15)**	[7 8 9 10 11]	[5 6 7 8 9]	(11 12 13 14 15)	
	[4 5 6 7 8]	[3 4 5 6 7]	[3 4 5 6 7] **(6 7 8 9 10)**	(6 7 8 9 10)	
	Origin			Demand	

Table 6: Result comparison

North West Corner Method		
Fuzzy Number	**IBFS Solution**	Error
Pentagonal Fuzzy Number	199.6	9.6
Crisp	190	
Least Cost Method		
Pentagonal Fuzzy Number	175.65	14.65
Crisp	161	
VAM		
Pentagonal Fuzzy Number	160	7.8
Crisp	167.8	

Conclusion and Future Work

Fuzzy pentagonal numbers are used to convert the crisp transportation problem into a fuzzy transportation problem. The fuzzy transportation problem is solved by using fuzzy NWCR, LCM and VAM methods. Error calculated between fuzzy and crisp transportation problem. Through the numerical example, we can conclude that minimum fuzzy transportation cost obtained from pentagonal fuzzy numbers close to cost obtained by crisp transportation problem. In future, we want to extend our research work for trapezoidal, hexagonal etc. fuzzy numbers.

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