Semi – symmetric non metric connection on a Riemannian manifold

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Abstract. In this paper we define a new semi-symmetric non-metric connection which generalizes the notation of the semi-symmetric non-metric connection introduced by Agashe and Chafle [1] and established its existence. Then we study some properties of the curvature tensor, Ricci tensor and Weyl projective curvature tensor with respect this semi-symmetric non-metric connection.

Keywords and Phrases. Riemannian manifold, semi-symmetric non-metric connection, curvature tensors, Ricci tensor and Weyl projective curvature tensor.

1. Introduction.

In, 1924 Friedmann and Schouten [10] introduced the idea of semi-symmetric connection on a differentiable manifold. A linear connection $\bar{D}$ on a differentiable manifold M is said to be semi-symmetric connection if the torsion tensor $\bar{T}$ of the connection $\bar{D}$ satisfies

$$\bar{T}(X, Y) = u(Y)X - u(X)Y, \qquad (1)$$

where u is a 1-form and U is a vector field given by

$$u(X) = g(X, U), \qquad (2)$$

for all vector fields X on M.

In 1932 Hayden [11] investigated the idea of a semi-symmetric metric connection on a Riemannian manifold. A semi-symmetric connection $\bar{D}$ is said to be a semi-symmetric metric connection if

$$(\bar{D}_X g)(Y, Z) = 0 \qquad (3)$$

A relation between the semi-symmetric metric connection $\bar{D}$ and Levi-Civita connection D on M discovered Yano [19] as follows

$$\bar{D}_X Y = D_X Y + u(Y)X - g(X, Y)U. \qquad (4)$$

The study of semi-symmetric metric connection was further developed by Amur and Puzara [2], Binh [4], De [7], Singh et al.[17], Ozgur et al.[13] and many others. After a long gap the study of a semi-symmetric connection $\bar{D}$ satisfying

$$(\bar{D}_X g)(Y, Z) \neq 0 \qquad (5)$$

was initiated by Prvanovic [14] with the name pseudo metric semi- symmetric connection and the followed by Andonic [3]. Later on, a semi-symmetric connection $\bar{D}$ is said to be a semi-symmetric non-metric connection if it satisfies the condition (4). Keeping the idea , in 1992

Agashe and Chafle [1] defined and studied a semi-symmetric non-metric connection $\bar{D}$ whose torsion tensor given by (1) and metric was $(\bar{D}_X g)(Y, Z) = -u(Y)g(X, Z) - u(Z)g(X, Y) \neq 0$. They proved the projective curvature tensor of the manifold is projectively flat if the curvature tensor of $\bar{D}$ vanishes. In 1994, Liang [12] studied another type of a semi-symmetric non-metric connection $\bar{D}$ for which $(\bar{D}_X g)(Y, Z) =$

2u (X) g (Y, Z), although he called as a semi-symmetric metric connection. The semi-symmetric non-metric connection

was further developed by several authors such as De and Kamilya [8], Chaubey and Ojha [5], De, Han, and Zhao [9], Chaubey and Yieldiz [6], Prasad and Singh [15] and other investigators. Recently Sengupta, De and Binh [17] and Prasad and Verma [16] generalized Agashe and Chafle [1] connection and obtained necessary and sufficient condition under which the Weyl projective curvature tensor of a semi-symmetric non-metric connection is equal to the projective curvature tensor of the Riemannian connection. In continuation of the above study, we define another type of semi-symmetric non-metric connection on a Riemannian manifold which generalizes Agashe and Chafle [1] connection by another way.

We organize our work as follows: after introduction in section 1, we define a new type of semi-symmetric non-metric connection on a Riemannian manifold and proves its existence in section 2. In section 3, we prove the relation between curvature tensor of D and semi-symmetric non-metric connection $\bar{D}$ and prove some basic properties of the curvature tensor $\bar{D}$. We also obtain necessary and sufficient condition for the Ricci tensor of the semi-symmetric non-metric connection $\bar{D}$ to be symmetric and skew-symmetric. Weyl projective curvature on Riemannian manifold admitting a type of semi-symmetric non-metric connection have been studied in section 4. Finally, in section five we define concircular curvature tensor with respect to semi-symmetric non-metric connection and obtain the condition connecting it concircular curvature tensor with respect to Levi-Civita connection.

2. Semi-symmetric non-metric connection $\bar{D}$.

Let (M, g) be a Riemannian manifold with Levi-Civita connection D. We define a linear connection $\bar{D}$ on M by

$$\bar{D}_X Y = D_X Y + u(Y)X - a(Y)X \tag{5}$$

where u and a are 1-forms associated vector fields U and A by

$$g(X, U) = u(X) \tag{6}$$

and

$$g(X,A) = a(X). \tag{7}$$

Using (5), the torsion tensor $\bar{T}$ of M with respect to connection $\bar{D}$ is given by

$$\bar{T}(X,Y) = [u(Y)X - u(X)Y] - [a(Y)X - a(X)Y] \tag{8}$$

A linear connection satisfying (8) is called semi-symmetric connection. From (5), we get

$$(\bar{D}_X g)(Y, Z) = -u(Y)g(X, Z) - u(Z)g(X, Y) + a(Y)g(X, Z) + a(Z)g(X, Y). \tag{9}$$

A linear connection $\bar{D}$ defined by (5) satisfies (8) and (9) and therefore we call $\bar{D}$ as a semi-symmetric non-metric connection. Conversely, we show that a linear connection $\bar{D}$ defined on M satisfying (8) and (9) is given by (5).

Now we define a tensor field H of type (1, 2) by the expression

$$\bar{D}_X Y = D_X Y + H(X, Y) \tag{10}$$

where D is the Levi-Civita connection on (M, g). Then we have

$$\bar{T}(X,Y) = H(X, Y) - H(Y, X) \tag{11}$$

Also, we have

$$\bar{D}_X g(Y, Z) = (\bar{D}_X g)(Y, Z) + g(\bar{D}_X Y, Z) + g(Y, \bar{D}_X Z) \tag{12}$$

Equation (12) gives

g(H (X, Y), Z) + g(H (X, Z), Y) = u(Y) g(X, Z) + u(Z) g (X, Y) - a(Y) g(X, Z) - a(Z) g (X, Y) (13)

In view of (13), we get

g(H (Y, X), Z) + g(H (Y, Z), X) = u(X) g(Y, Z) + u(Z) g (X, Y) - a(X) g(Y, Z) - a(Z) g (X, Y) (14)

g(H (Z, X), Y) + g(H (Z, Y), X) = u(X) g(Y, Z) + u(Y) g (Z, X) - a(X) g(Z, Y) - a(Y) g (Z, X) (15)

From (11), (13), (14), and (15), we obtain

g($\overline{T}$(X, Y), Z) + g($\overline{T}$ ‘ (X, Y), Z) + g($\overline{T}$ ′(Y, X), Z) = 2g(H (X, Y), Z) – 2u(Z) g(X, Y) + 2a(Z) g(X, Y) (16)

Hence from (16), we get

H(X, Y) = $\frac{1}{2}$ [$\overline{T}$(X, Y) + $\overline{T}$ ‘ (X, Y) + $\overline{T}$ ′(Y, X)] + g(X,Y)U – g(X, Y)A (17)

where g($\overline{T}$(Z, X), Y) = g($\overline{T}$ ‘ (X, Y), Z) (18)

Then

T'(X, Y) = u(X)Y – g(X, Y)U – a(X)Y + g(X,Y)A (19)

In view of (8), (17) and (19), we get

H(X, Y) = u(Y)X – a(Y)X (20)

Hence from (10) and (20), we get

$\overline{D}_X$Y = D_XY + u(Y)X – a(Y)X.

This proves the existence of the defined connection.

Further for a 1-form π on M , we get

$\overline{D}_X$(π (Y)) = ($\overline{D}_X$ π) (Y) + π ($\overline{D}_X$Y). (21)

From (5) and (21), we get

($\overline{D}_X$ π) (Y) = (D_X π) (Y) – u(Y) π(X) + a(Y) π(X) (22)

In view of (22), we get

($\overline{D}_X$ π) (Y) - ($\overline{D}_Y$ π) (X) = (D_X π) (Y) - (D_Y π) (X) + π(X) [a(Y) – u(Y)] + π(Y) [u(X)- a(X)] (23)

If π(X) [a(Y) – u(Y)] + π(Y) [u(X) - a(X)] = 0 then from equation (23), we get

($\overline{D}_X$ π) (Y) - ($\overline{D}_Y$ π) (X) = (D_X π) (Y) - (D_Y π) (X).

Hence we have the following theorem:

Theorem (2.1). The 1-form π is closed with respect to semi-symmetric non-metric connection if and only if 1-form π is closed with respect to Levi-Civita connection, provided π(X) [a(Y) – u(Y)] + π(Y) [u(X) - a(X)] = 0.

3. Curvature tensor, Ricci tensor and scalar curvature tensor of M with respect to semi-symmetric non-metric connection $\bar{D}$.

Analogous to the definition of a Riemannian manifold M with respect to the Riemannian connection D, we define curvature tensor M with respect to the semi-symmetric non-metric connection $\bar{D}$ by

$$\bar{R}(X,Y)Z = \bar{D}_X \bar{D}_Y Z + \bar{D}_Y \bar{D}_X Z - \bar{D}_{[X,Y]} Z \quad (24)$$

From (5) and (24) we get

$$\bar{R}(X,Y)Z = R(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X - \beta(X,Z)Y + \beta(Y,Z)X \quad (25)$$

where

$$R(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z$$

is the curvature tensor of M with respect to the semi-symmetric non-metric connection, α and β defined on M by

$$\alpha(X,Z) = (D_X u)(Z) - u(X)u(Z) \quad (26)$$

and

$$\beta(X,Z) = (D_X a)(Z) - u(X)a(Z) - u(Z)a(X) + a(X)a(Z) \quad (27)$$

From (26) and (27), we get

$$\alpha(X,Y) - \alpha(Y,X) = du(X,Y) \quad (28)$$

and

$$\beta(X,Y) - \beta(Y,X) = da(X,Y) \quad (29)$$

Thus we see that both the tensor α (X, Y) and β(X, Y) are symmetric if and only if both 1-form are closed. From (25), we get

$$\bar{R}(X,Y)Z + \bar{R}(Y,X)Z = 0 \quad (30)$$

And

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = [du(Z,Y)X + du(X,Z)Y + du(Y,X)Z] - [da(Z,Y)X + da(X,Z)X + da(Y,X)Z] \quad (31)$$

Expression (31) call as the Bianchi's first identity with respect to semi-symmetric non-metric connection $\bar{D}$. In particular if the 1-form u and a are closed then (31) reduces to

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0. \quad (32)$$

Let ′R and ′$\bar{R}$ are two tensor field of type (0, 4) defined on the Riemannian manifold M by the relations

$$'R(X,Y,Z,W) = g(R(X,Y)Z, W) \quad (33)$$

And

$$'\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y)Z, W) \quad (34)$$

From (25), (33) and (34), we get

$$'\bar{R}(X,Y,Z,W) + {}'\bar{R}(X,Y,W,Z) = [\alpha(X,Z) - \beta(X,Z)]g(Y,W) + [\alpha(X,W) - \beta(X,W)]g(Y,Z) - [\alpha(Y,Z) - \beta(Y,Z)]g(X,W) - [\alpha(Y,W) - \beta(Y,W)]g(X,Z)$$

Analogous to the definition of Ricci tensor of a Riemannian manifold with respect to Levi-Civita connection D. We define Ricci tensor of M with respect to semi-symmetric non-metric connection $\bar{D}$ by

$\overline{Ric}$ (Y,Z) = $\sum_i^n \bar{R}$ (E_i, Y, Z, E_i) (35)

where { E_i }, 1 ≤ i ≤ n is the set of orthonormal vector fields on M. Then from (35), (25), (33) and (34), we get

$\overline{Ric}$ (Y,Z) = Ric (Y, Z) – (n – 1) [α(Y, Z) – β(Y, Z)] (36)

where Ric (Y, Z) denotes the Ricci tensor of M with respect to Levi-Civita connection D. From (36) , we see that Ricci tensor Ric is symmetric if and only if

da(Y, Z) = du(Y, Z) (37)

Moreover from (36), we get

$\overline{Ric}$ (Y,Z) + $\overline{Ric}$ (Z,Y) = 2 Ric (Y, Z) – (n – 1) [{ α(Y, Z) + α(Z, Y)} + { β(Y, Z) + β(Z, Y)}] (38)

Analogous to the definition of scalar curvature tensor r of a Riemannian manifold with respect to Levi-Civita connection D. We define scalar curvature tensor $\bar{r}$ of M with respect to semi-symmetric non-metric connection $\bar{D}$ by

$\bar{r}$ = $\sum_i^n \overline{Ric}$ (E_i, E_i) (39)

From (36) and (39), we get

$\bar{r}$ = r – (n-1) [p – q] (40)

where p = trace α and q = trace β and r denotes the scalar curvature tensor of M with respect to the Levi-Civita connection D.

The above discussion helps us to state the following propositions:

Theorem (3.1). For a Riemannian manifold M with respect to semi-symmetric non-metric connection $\bar{D}$, we have

(i) The curvature tensor $\bar{R}$ is given by

$\bar{R}$ (X,Y) Z = R(X, Y)Z + α(X, Z)Y - α(Y, Z)X - β(X, Z)Y + β(Y, Z)X

(ii) $\bar{R}$ (X,Y) Z + $\bar{R}$ (Y,X) Z = 0

(iii) $\bar{R}$ (X,Y) Z + $\bar{R}$ (Y,Z) X + $\bar{R}$ (Z,X) Y = 0, if and only if α(X, Z) – β(X, Z) = 0

(iv) $'\bar{R}$ (X,Y,Z,W) + $'\bar{R}$ (X,Y,W,Z) = 0, if and only if α(X, Z) – β(X, Z) = 0

(v) Ricci tensor Ric of the manifold M is equal to the Ricci tensor $\overline{Ric}$ of the manifold with respect to semi-symmetric non-metric connection $\bar{D}$ if and only if α(Y, Z) – β(Y, Z) = 0.

(vi) Ricci tensor $\overline{Ric}$ of the manifold with respect semi-symmetric non-metric connection $\bar{D}$ is symmetric if and only if da(Y, Z) = du(Y, Z)

(vii) If a Riemannian manifold of dimension n admits a semi-symmetric non-metric connection $\bar{D}$, then a necessary and sufficient for the Ricci tensor $\bar{D}$ to be skew-symmetric is that Ricci tensor of D is given by

2 Ric (Y, Z) – (n – 1) [{ α(Y, Z) + α(Z, Y)} + { β(Y, Z) + β(Z, Y)}] = 0.

(viii) A necessary and sufficient condition for the scalar curvature $\bar{r}$ of $\bar{D}$ and r of D to coincide is that trace α = trace β.

4. Projective curvature tensor of a Riemannian manifold with respect semi-symmetric non-metric connection $\bar{D}$.

Projective curvature tensor of a Riemannian manifold M with respect Riemannian connection D is given by [20]

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1} [\, Ric\,(Y, Z)X - Ric\,(X, Z)Y \,] \quad (41)$$

Analogous to the definition of Projective curvature tensor of a Riemannian manifold M with respect to Levi-Civita connection D, we define Projective curvature tensor $\bar{P}$ of M with respect to semi-symmetric non-metric connection $\bar{D}$ by

$$\bar{P}(X, Y)Z = \bar{R}(X, Y)Z - \frac{1}{n-1} [\, \overline{Ric}\,(Y, Z)X - \overline{Ric}\,(X, Z)Y \,] \quad (42)$$

From (25), (36), (41) and (42), we get

$\bar{P}(X, Y)Z = P(X, Y)Z.$

This leads us to state the following theorem:

Theorem(4.1). If a Riemannian manifold admits a semi-symmetric non-metric connection, then the projective curvature tensor with respect to the semi-symmetric non-metric connection $\bar{D}$ is equal to the projective curvature tensor with respect to the Levi-Civita connection D.

Let M be Riemannian manifold satisfying

$$\bar{R}(X, Y)Z = 0 \quad (44)$$

Equation (44) gives

$$\overline{Ric}\,(Y,Z) = 0 \text{ and } \bar{r} = 0 \quad (45)$$

In view of (42), (43), (44) and (45), we get

$$P(X, Y, Z) = 0 \quad (46)$$

Hence we have the following theorem:

Theorem(4.2): If in a Riemannian manifold the curvature tensor of a semi-symmetric non-metric connection vanishes. Then manifold is projectively flat.

Let $\overline{Ric}\,(Y,Z) = 0$ (47)

Hence in view of (36) and (47), we get

$$\alpha(Z, Y)\} = \frac{1}{n-1} Ric\,(Y, Z) + \beta(Y, Z) \quad (48)$$

Hence in view of (25) and (48), we get

$$\bar{R}(X, Y)Z = P(X, Y, Z) \quad (49)$$

Thus we have the following theorem:

Theorem(4.3). If in a Riemannian manifold the Ricci tensor of a semi-symmetric non-metric connection vanishes, then the curvature tensor of a semi-symmetric non-metric connection $\bar{D}$ is equal to the projective curvature tensor of the manifold.

Theorem(4.4). The Projective curvature tensor of a semi-symmetric non-metric connection $\bar{D}$ satisfies the following algebraic properties:

$\bar{P}(X, Y)Z + \bar{P}(Y, X)Z = 0.$

And

$\bar{P}$(X, Y)Z + $\bar{P}$(Y, Z)X + $\bar{P}$(Z, X)Y = 0. (50)

5. Concircular curvature tensor of a Riemannian manifold with respect semi-symmetric non-metric connection $\bar{D}$.

Concircular curvature tensor of a Riemannian manifold M with respect Riemannian connection D is given by [20]

$$C(X, Y)Z = R(X, Y)Z - \frac{r}{n\,(n-1)}\,[g(Y, Z)X - g(X, Z)Y] \qquad (51)$$

Analogous to the definition of Concircular curvature tensor of a Riemannian manifold M with respect to Levi-Civita connection D, we define Concircular curvature tensor $\bar{C}$ of M with respect to semi-symmetric non-metric connection $\bar{D}$ by

$$\bar{C}(X, Y)Z = \bar{R}(X, Y)Z - \frac{\bar{r}}{n-1}\,[g(Y, Z)X - g(X, Z)Y] \qquad (52)$$

From (25), (40), (51) and (52), we get

$$\bar{C}(X, Y)Z = C(X, Y)Z + \alpha(X, Z)Y - \alpha(Y, Z)X - \beta(X, Z)Y + \beta(Y, Z)X + \frac{p-q}{n}\,[g(Y, Z)X - g(X, Z)Y] \qquad (53)$$

If $\bar{C}$(X, Y)Z = C(X, Y)Z, then from (5.3), we get

$$\alpha(X, Z)Y - \alpha(Y, Z)X - \beta(X, Z)Y + \beta(Y, Z)X + \frac{p-q}{n}\,[g(Y, Z)X - g(X, Z)Y] = 0 \qquad (54)$$

Contracting (54) with respect to X, we get $\alpha(Y, Z) - \beta(Y, Z) = \frac{p-q}{n}\, g(Y, Z)$

This leads us to state the following theorem:

Theorem(5.1). A necessary condition for concircular curvature tensor C of the manifold with respect to the Levi-Civita connection D is equal to the Concircular curvature tensor $\bar{C}$ with respect to the semi-symmetric non-metric connection $\bar{D}$ if $\alpha(Y, Z) - \beta(Y, Z) = \frac{p-q}{n}\, g(Y, Z)$.

In consequences of (44), (45), (52) and (53), we get

$$C(X, Y)Z = -\alpha(X, Z)Y + \alpha(Y, Z)X + \beta(X, Z)Y - \beta(Y, Z)X - \frac{p-q}{n}\,[g(Y, Z)X - g(X, Z)Y] \qquad (55)$$

If C(X, Y)Z = 0, then from (55) we get

$$-\alpha(X, Z)Y + \alpha(Y, Z)X + \beta(X, Z)Y - \beta(Y, Z)X - \frac{p-q}{n}\,[g(Y, Z)X - g(X, Z)Y] = 0.$$

Contracting above equation with respect X, we obtain

$$\alpha(Y, Z) - \beta(Y, Z) = \frac{p-q}{n}\, g(Y, Z).$$

Thus we have the following theorem:

Theorem(5.2): If in a Riemannian manifold the curvature tensor of a semi-symmetric non-metric connection vanishes. Then manifold is concirculaly flat if

$$\alpha(Y, Z) - \beta(Y, Z) = \frac{p-q}{n}\, g(Y, Z).$$

From (53), we have the following theorem:

Theorem(5.3). The Concircular curvature tensor $\bar{C}$ of a semi-symmetric non-metric connection $\bar{D}$ satisfies the following algebraic properties:

$$\bar{C}(X, Y)Z + \bar{C}(Y, X)Z = 0 \qquad (56)$$

$\bar{C}$(X, Y)Z + $\bar{C}$(Y, Z)X + $\bar{C}$(Z, X)Y = [du (Z, Y)X + du(X, Z)Y + du(Y, X)Z] - [da (Z, Y)X

$+ da(X, Z)Y + da(Y, X)Z]$ (57)

From (31) and (57), we get

$$\bar{C}(X, Y)Z + \bar{C}(Y, Z)X + \bar{C}(Z, X)Y = \bar{R}(X, Y)Z + \bar{R}(Y, Z)X + \bar{R}(Z, X)Y \quad (58)$$

In view of (58) we can state the following theorem:

Theorem(5.4). Cyclic sum of Concircular curvature tensor $\overline{C}$ of a semi-symmetric non-metric connection $\overline{D}$ vanishes if and only if cyclic sum of Riemannian curvature tensor of a semi-symmetric non-metric connection $\overline{D}$ vanishes.

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