BEHAVIOUR OF A TEST PARTICLE, DOPPLER EFFECT AND NEWTONIAN ANALOGUE OF FORCE IN THE COSMOLOGICAL MODEL

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ABSTRACT :

The present paper provides an investigation on behaviour of a test particle, Doppler effect and Newtonian analogue of force in the cosmological model. If a particle is initially at rest then from equation of geodesic it is found that for all such particles the components of spatial acceleration would vanish and the particle would remain permanently at rest.

Key Words : Geodesic, test particle, Doppler effect, model, spatial acceleration.

1. INTRODUCTION

Many researchers have focussed their mind to words the study of behaviour of test particle, Doppler effect and Newtonian analogue of force in the cosmological model [5, 8, 10,15]. The cosmological models in the presence of electromagnetic fields in general relativity have attracted a lot of interest in recent years. The behavior of magnetic fields in the models of universe has been studied by Roy and Prakash [8]. Magnetohydrodynamics (MHD) is the study of motion of an electrically conducting fluid in the presence of a magnetic field. Electric currents induced in the fluid as a result of its motion modify the field; at the same time their flow in the magnetic field produces mechanical forces which modify the motion. Magnetohydrodynamics own its peculiar interest and difficulty to this interaction between the field and the motion. It is well known that galaxies and interstellar spaces exhibit the presence of strong magnetic fields [15] which impart a sort of viscous effect to the fluid flow [1]. The magnetic field assumes an important role for the universe. The behavior of the magnetic field of a star was investigated by Cowling [2] and Wrubel [2(a)]. The important results obtained make possible the interpretation of magnetohydrodynamical processes in stars. When motions of stellar metter caused by the electromagnetic forces are taken into account, new properties may be revealed and the non-stability of the magnetohydrodynamical processes in stars may be studied. A cosmological model in the presence of magnetic field has been studied by Zeldovich [14] and later by Thorne [11]. Magnetic field in stellar bodies was discussed by Monoghan [4]. Roy and Prakash [8] have discussed cosmological model in presence of incident magnetic field and have also discussed there in behaviour of a test particle and Dopper effect in the model. Singh and Yadav [10] have constructed model taking cylindrically symmetric metric of Marder [3] where they have also discussed behaviour of a test particle, Dopper effect and Newtonian and logue of force in the model. The present all their in his earher paper [6] also found cosmological model for cytindrically symmetric space time with an electromagnetic field and which is given by

$$(1.1)\quad ds^2 = N^2\left[\left(\frac{a+T}{T}\right)dT^2 - dX^2\right]$$

$$-(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{2n} dY^2$$

$$-(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{-2n} dZ^2$$

where symbols have their usual meanings (as given in ref. [6]).

In this paper taking cylinarically symmetric cosmological model given by (1.1), we have discussed and studied behaviour of a test particle, Dopper effect and also have investigated Newtonian analogue of force in the model.

2. Behaviour of a Test Particle in the Model :

The motion of a test particle in the model (1.1) is given by the geodesic viz.

$$(2.1)\quad \frac{d^2x}{ds^2} = 0$$

$$(22)\quad \frac{d^2Y}{ds^2} + \frac{1}{2}\left[\frac{T^{1/2}+n(a+T)^{1/2}}{T^{1/2}(a+T)}\right]\frac{dY}{ds}.\frac{dT}{ds} = 0$$

$$(2.3)\quad \frac{d^2Z}{ds^2} + \frac{1}{2}\left[\frac{T^{1/2}-n(a+T)^{1/2}}{T^{1/2}(a+T)}\right]\frac{dZ}{ds}.\frac{dT}{ds} = 0,$$

$$(2.4)\quad \frac{d^2T}{ds^2} + \frac{1}{2N^2}\left[\frac{\left[T^{1/2}+(a+T)^{1/2}\right]^{2n}(T+nT^{1/2}(a+t)^{1/2}]}{(a+T)}\right]\left(\frac{dY}{ds}\right)^2$$

$$+\frac{1}{2N^2}\left[\frac{\left[T^{1/2}+(a+T)^{1/2}\right]^{-2n}\left[T-nT^{1/2}(a+T)^{1/2}\right]}{(a+T)}\right]\left(\frac{dZ}{ds}\right)^2$$

$$-\frac{a}{2T(a+T)}\left(\frac{dT}{ds}\right)^2 = 0.$$

If a particle is initially at rest, that is, if

(2.5) $$\frac{dX}{ds}=\frac{dY}{ds}=\frac{dZ}{ds}=0$$

then from equations of geodesic we find that for all such particles the component of spatial acceleration would vanish, namely

(2.6) $$\frac{d^2X}{ds^2}=\frac{d^2Y}{ds^2}=\frac{d^2Z}{ds^2}=0$$

and the particle would remain permanently at rest.

3. The Doppler Effect in the Model

The path of light in the model (1.1) is given by

(3.1) $$N^2\left(\frac{dX}{dT}\right)^2+(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{2n}\left(\frac{dY}{dT}\right)^2$$

$$+(a+T)\left[T^{1/2}+(a+T)^{1/2}\right]^{-2n}\left(\frac{dZ}{dT}\right)^2=\frac{N^2(a+T)}{T}$$

and for the case when the velocity is along z-axis, equation (3.1) gives

(3.2) $$\frac{dZ}{dT}=\pm N\frac{\left[T^{1/2}+(a+T)^{1/2}\right]^n}{T^{1/2}}=\pm\psi(T)$$

Hence the light pulse leaving a particle at (0,0, z) at time T_1 would arrive at a later time T_2 given by

(3.3) $$\int_{T_1}^{T_2}\psi(T)dT=\int_0^Z dZ$$

Hence

(3.4) $$\psi_2(T)\delta T_2=\psi_1(T)\delta T_1+\frac{dZ}{dT}\delta T_1$$

$$=\psi_1(T)\delta T_1+u_Z\delta T_1$$

where $\frac{dZ}{dT} = u_Z$ is the Z-component of the velocity of the particle at the time of emission and $\psi_1(T)$ and $\psi_2(T)$ are the values of $\psi(T)$ for T = T_1 and T = T_2 respectively. From the above equation we get

$$(3.5)\quad \delta T_2 = \left[\frac{\psi_1(T) + u_Z}{\psi_2(T)}\right]\psi T_1$$

The proper time interval δT_1^0 between successive wave crests as messured by the local observer moving with the source is given by

$$(3.6)\quad \delta T_1^0 = \left\{\frac{N^2(a+T)}{T} - N^2\left(\frac{dX}{dT}\right)^2 - (a+T)\right.$$

$$\left[T^{1/2} + (a+T)^{1/2}\right]^{2n}\left(\frac{dY}{dT}\right)^2$$

$$\left.-(a+T)\left[T^{1/2} + (a+T)^{1/2}\right]^{-2n}\left(\frac{dZ}{dT}\right)^2\right]^{1/2} \delta T_1$$

This can be written as

$$(3.7)\quad \delta T_1^0 = \left[\frac{N^2(a+T)}{T} - u^2\right]^{1/2} \delta T_1$$

where u is the velocity of source at the time of emission.

Similarly we may write

$$(3.8)\quad \delta T_2^0 = N\left(\frac{a+T}{T}\right)^{1/2} \delta T_2$$

As the proper time interval between the reception of the two successive wave crests by an observer at rest at origin. Hence following Tolman [12], the red shift in this case is given by

(3.9) $$\frac{\lambda+\delta\lambda}{\lambda}=\frac{\delta T_2^0}{\delta T_1^0}=\frac{\left(\frac{a+T}{T_1}\right)^{1/2}\left\{\frac{N\left[T_1^{1/2}+(a+T_1)^{1/2}\right]^n+u_z}{T_1^{1/2}}\right\}}{\left[N^2\left(\frac{a+T}{T}-u^2\right)^{1/2}\frac{\left[T_1^{1/2}+(a+T_2)^{1/2}\right]^n}{T_2^{1/2}}\right]}$$

4. Newtonian Analogue of Force in the Model

Here we study the effect of electromagnetic field in the force terms R_μ and S_μ (Narlikar and Singh [5]). It is well known that Christoffel three – index symbols for the Riemannian metric of general relativity

$$ds^2=g_{\mu\upsilon}dx^\mu dx^\upsilon$$

defined by

(4.1) $$[\mu\upsilon,\sigma]=\frac{1}{2}\left[\frac{\partial g_\mu\sigma}{\partial x^\upsilon}+\frac{\partial g_\upsilon\sigma}{\partial x^\mu}-\frac{\partial g_\mu\upsilon}{\partial x^\sigma}\right]$$

and

(4.2) $$\Gamma\sigma_{\mu\upsilon}=g^{\sigma1}[\mu\upsilon,1]$$

are not tensor, but the difference of two such symbols of the same kind is a tensor. Thus if $\left\{\upsilon^\mu_\sigma\right\}$ be Christoffel three-index symbols for another metric.

(4.3) $$ds^2=y_{\mu\upsilon}dx^\mu dx^\upsilon,$$

the difference

(4.4) $$\Gamma^\mu_{\upsilon\sigma}-\left\{\upsilon^\mu_\sigma\right\}=\Delta^\mu_{\upsilon\sigma}$$

is a tensor.

The equation of the geodesic may now be cast in the form

(4.5) $$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\upsilon\sigma}\frac{dx^\upsilon}{ds}\frac{dx^\sigma}{ds}=-\Delta^\mu_{\upsilon\sigma}\frac{dx^\mu}{ds}\frac{dx^\sigma}{ds}$$

In a region free from gravitation the geodesics are given by

(4.6) $\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\upsilon\sigma}\frac{dx^\upsilon}{ds}\frac{dx^\sigma}{ds}=0$

Hence the gravitational effects depend only on the residual term-$\Delta^\mu_{\upsilon\sigma}\frac{dx^\upsilon}{ds}\frac{dx^\sigma}{ds}$. Since $\left(\frac{dx^\mu}{ds}\right)$ is arbitrary, the gravitational effect have to be associated with the tensor $\Delta^\mu_{\upsilon\sigma}$ only. It can be seen that (Rosen [7])

(4.7) $\Delta^\mu_{\upsilon\sigma}=\frac{1}{2}g^{\mu 1}(g_{\upsilon 1,\sigma}+g_{\sigma 1,\upsilon}-g_{\upsilon\sigma,1})$

where comme (,) denotes co-variant differentiation with respace to y – metric. A co-variant differentiation with respect to g – metric will be denoted by a semicolon (:).

From (4.7) we have

(4.8) $g_{\mu m}\Delta^\mu_{\upsilon\sigma}=\frac{1}{2}\left[g_{\upsilon m,\sigma}+g_{\sigma m,\upsilon}-g_{\upsilon\sigma,m}\right]$

If we write

(4.9) $g_{\mu m}\Delta^\mu_{\upsilon\sigma}=\Delta_{\upsilon\sigma m}$

then $\Delta^\mu_{\upsilon\sigma}$ and $\Delta_{\mu\upsilon\sigma}$ as given by (4.7) and (4.8) are associated tensor.

The Δ-tensors thus defined contain only the first order partial derivatives of the metric potential $g_{\mu\upsilon}$. Since Δ-tensors are defined against the background of an arbitrary flat substratum, they would very with the choice of the later for the same gravitational field. The importance of the Δ's consists in the fact that the gravitational force of the Newtonian theory appears through them.

The vectors R_μ and S_μ are defined as follows (Nerlikar and Singh [5]):

(4.10) $R_{\mu} = \Delta^{\upsilon}_{\upsilon\mu} = \frac{H_{,\mu}}{H}$,

(4.11) $S_{\mu} = \Delta^{\mu}_{\upsilon\sigma}\ g^{\upsilon\sigma}\ g_{\mu\mu}$

$$= g^{\upsilon\sigma} g_{\upsilon\mu,\sigma} - \frac{H_{,\mu}}{H}$$

where $H = \sqrt{\frac{g}{y}}$

For the line element (1.1) we have

(4.12) $g_{11} = -N^2$,

$$g_{22} = -(a+T)\left[T^{1/2} + (a+T)^{1/2}\right]^{2n},$$

$$g_{33} = -(a+T)\left[T^{1/2} + (a+T)^{1/2}\right]^{-2n},$$

$$g_{44} = \frac{N^2(a+T)}{T}$$

(4.13) $g^{11} = -\frac{1}{g^2}$

$$g^{22} = -(a+T)^{-1}\left[T^{1/2} + (a+t)^{1/2}\right]^{-2n},$$

$$g^{33} = -(a+T)^{-1}\left[T^{1/2} + (a+T)^{1/2}\right]^{2n},$$

$$g^{44} = \frac{1}{N^2}\frac{T}{(a+T)},$$

(4.14) $g = -N^4 \frac{(a+T)^3}{T}$

The corresponding flat metric $y_{\mu\upsilon}$ is taken to be that of special relativity

(4.15) $ds^2 = -dX^2 - dY^2 - dZ^2 + dT^2$

Thus

(4.16) $y_{\mu\upsilon} = [-1, -1, -1, 1]$

and

(4.17) $y = -1$

From equation (4.14) and (4.17)

(4.18) $H = \sqrt{\frac{g}{y}} = \frac{N^4 (a+t)^3}{T}$

From (2.3.40) and (2.3.41) we get

(4.19) $R_\mu = \left[0, 0, 0, \frac{2T - a}{T(a+t)}\right]$

and

(4.20) $S_\mu = \left[0, 0, 0, \frac{a - 2T}{T(a+T)}\right]$

Thus we find that R_μ and S_μ both are null force vectors. R_4 and S_4 have no Newtonian analogues.

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