GARCH (1,1)- A Review

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***Abstract :** An investor's major risk comes from the fact that asset prices are volatile in essence. There's always an eternal effort by researchers to be able to measure accurately this volatility so as to make required returns. Researchers for ages are committed to solve this issue through introducing new models. There has been vast number of papers on measuring volatility in several markets through different models. One amongst the foremost popular family of models tested is the GARCH family which relies on the very fact that returns are auto-regressed, and therein family also, GARCH(1,1) has been quite an idol member. So, present study tries to review some papers from the extant literature available on GARCH(1,1) and to appreciate how it stands as compared to other models in the same family yet as other models across assets and countries.*

IndexTerms **- Volatility, GARCH, Autoregression, Asset, Stock Market.**

I. INTRODUCTION

Volatility of an asset has long been intriguing topic, both amongst researchers as well as practitioners. Volatility in simple terms is the fluctuations in the price of an asset over a given period of time. If one studies the returns of those asset prices over a fixed period of time, then one can commence with some peculiar features of the underlying process, which may be utilized for modelling and predicting the long run volatility of the asset prices. Some important features of financial data includes autocorrelation of squared returns, reversion of volatility to the mean, non-constant volatility, volatility clustering, etc. The models that capture these features are the family of ARCH and GARCH models. Out of the family of ARCH and GARCH models, GARCH(1,1) has been quite a preferred model by many researchers.

II. GARCH (1,1)

The Autoregressive Conditional Heteroscedasticity (ARCH) model, was presented by Engle in 1982 to explain the time-varying volatility. AR stands for autoregression and squared returns are autoregressive in nature. Next period's volatility is extracted from information during this period indicating conditionality. Hetero (different) scedasticity (scatter) means non constant volatility, i.e., the scatter of the error terms show an autoregressive pattern, suggestive of devouring a relationship with their own history. In other words, heteroscedasticity means 'changing variance', so conditional heteroscedasticity means changing conditional variance.

Since it's unearthing, ARCH modelling has become a growing industry, with many variations of the first model. One that has become prevalent is that the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, proposed by Bollerslev in 1986. The simplest GARCH model is the GARCH(1,1) model, which might be written as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_t^2 + \alpha_2 \sigma_{t-1}^2$$

which says that the conditional variance of u at time t be contingent not only on the squared error term in the prior time period [as in ARCH(1,1)] but also on its conditional variance in the prior time period. This model is often generalized to a GARCH(p,q) model in which there are p lagged terms of the squared error term and q terms of the lagged conditional variances. A GARCH(1,1) model is corresponding to an ARCH(2) model.

III. OBJECTIVES OF THE STUDY

The main objective of the current paper is to review the findings of varied researchers in relevance to GARCH(1.1). The varied objectives of the review are;

- To review performance of GARCH(1,1) as compared to other models across nations and products.
- To review the performance of GARCH(1,1) as compared to GARCH(p,q).
- To review the performance of GARCH(1,1) as compared to other models within the GARCH family.

IV. REVIEW OF LITERATURE

Akigray (1989) investigated the daily returns on the CRSP value-weighted and equal-weighted indices dated from January 1963 to December 1986. The author divided the full sample into four different periods of 6 years each consisting of 1500 observations and every period was studied separately. He compared the ARCH and GARCH models with the easy historical average model and also the EWMA model. Within the category of GARCH processes, the author found GARCH(1,1) to be the premier fit. After applying other models of GARCH (p=1,…5 and q=1,….5), no significant improvements in goodness-of-fit were found consistent with the likelihood ratio test. And then only GARCH (1,1) was used for comparison of forecasting ability. When the related models, ARCH and GARCH, were related amongst themselves, the GARCH model was found to be more grander. Using the quality statistical measures, ME, RMSE, MAE and MAPE, the GARCH(1,1) forecasts were far improved than the opposite three, and also the difference was more prominent in periods of sky-high fluctuations.

Awartani and Corradi (2005) tested the predictive abilities of GARCH(1,1) as compared to asymmetric GARCH models. The authors performed pairwise comparison of various models against GARCH(1,1) and also a joint comparison of all models against GARCH(1,1). The authors concluded that GARCH(1,1) is outperformed by asymmetric GARCH models, whether it's the case of 1 step ahead pairwise comparison or some longer forecast horizons.

Banumathy K. and Azhahaiah R. (2015) studies the volatility pattern of Nifty index dated from January 2003 to December 2012. They tested GARCH(1,1), GARCH-M(1,1), EGARCH and TGARCH models. Identifying the most effectively fitted model in keeping with AIC and SIC criterion result in the conclusion that GARCH(1,1) was the finest in capturing the uniform volatility.

Boudoukh, Richardson and Whitelaw (1997) investigated the historical variance, the exponentially weighted moving average model, GARCH(1,1) and MDE model. The asset on which these tests were performed was the rate on three-month treasury- bills for the period 1983-1992. The GARCH(1,1) estimates were calculated through maximum likelihood method. They compared realised and forecasted volatility in two ways. First, they compare the out-of-sample performance over the whole period using the MSE of the forecasts. Second, they regress realised volatility on the forecasts and show the regression coefficients and coefficient of determination. The authors found that GARCH(1,1) was the worst performer for a brief estimation window.

Day and Lewis (1992) compared the implied volatilities from call options on the S&P 100 index to GARCH and EGARCH models. Although a more general GARCH(p, 4) model could have been used, but the results came out to be persistent with the results of Akgiray (1989) who finds that, within the class of GARCH family, the GARCH(1, 1) specification supply the best fit. They cast-off the closing prices and contract volumes for call options on S&P 100 index and the daily closing prices of the underlying index dated 1983 to1989. The out-of-sample comparisons designated that weekly volatility is tough to predict. The results furnished limited evidence that, in certain instances, GARCH models supply better forecasts than EGARCH models.

Doidge and Wei in 1998, put in seven volatility models to test the efficiency of the Toronto 35 index options market. The, GARCH (1,1), EGARCH (1,1), implied volatility, a simple average model and a weighted average model (both based on average of implied and GARCH(1,1) volatility models), a GARCH/IV model and also a historical volatility model . To test the effectiveness of the volatility models, the authors used a 1-month forecast horizon and calculated 3 statistical error measures, MAE, MAPE and RMSE. The combination forecasts came out to be the most precise, followed by the GARCH forecasts. Between the two GARCH forecasts, GARCH (1,1) outperformed EGARCH (1,1) by a slight margin.

Ederington and Guan (2000) compared the historical variance model, the EWMA, the GARCH (1,1) and EGARCH models. Just like in previous studies, the authors found that financial markets have longer memories than reflected in GARCH (1, 1) model estimates. But this had little influence on their out-of-sample forecasting ability. The authors found that GARCH (1,1) provided better forecasts than the historical standard deviation and EWMA. Between GARCH and EGARCH there was no clear winner.

Hansen and Lunde (2001) evaluated 330 different volatility models from the family of ARCH/GARCH type of models using daily exchange rate data (DM/$) and IBM stock prices. The authors used two different benchmark models to compare the different volatility models. These were the basic ARCH (1) and GARCH (1,1) models. Compared with a simple ARCH (1) model, it was found that ARCH (1) was significantly outpaced by other models. Moreover, not much evidence was found that the GARCH (1,1) model was outperformed by other models.

Jorion in 1995, compared the implied volatilities, MA and a GARCH (1,1) model for the three currencies: the German deutsche mark, the Japanese yen and the Swiss franc. The out-of-sample results indicated that the implied volatilities are better than MA and GARCH in predicting future volatility for the three assets. The result was compared sharply with those of Canina and Figlewski (1993), who reported implied volatilities to be poor performers than the time series models in the US stocks.

Lin Zhe (2018) conducted a test by applying ARCH, GARCH(1,1), TGARCH(1,1) and EGARCH(1,1) models on these composite index in China stock market. The author concluded that the EGARCH(1,1) outperformed all other models tested on the China stock market. The SSE was shown to possess significant properties of clustering and time-variability.

Madhusudan Karmakar (2004) The author investigated the daily closing prices of Nifty from the period January 1991 to December 2003 by applying five models, namely, random walk model, historical mean model, moving average model, simple regression model and the typical GARCH (1,1) model. For comparison, the author used two methods: firstly four traditional error statistics that's, ME, MAE, RMSE and the MAPE and secondly, a regression based efficiency test was used. No select model was clearly superior. ME, MAE and MAPE ranked random walk model as the best one. RMSE statistics ranked GARCH (1,1) to be the best model. According to the regression based efficiency test, GARCH (1,1) came out to be the best model followed by random walk model.

Naimy and Hayek (2018) applied GARCH(1,1), EWMA and EGARCH(1,1) for the first time on Bitcoin/USD exchange rate for the period from April 2013 to March 2016. The data was divided into in-sample and out-of-sample categories. The in-sample period consisted of 788 observations starting from 1 April 2013 to 31 March 2015 to estimate the model parameters, while the out-of-sample period consisted of 305 observations starting from 1 June 2015 to 31 March 2016 to forecast the future volatility. The study concludes that the EGARCH(1,1) has better predictive power than GARCH(1,1) and EWMA according to the early-stage behaviour of Bitcoin.

Pandey (2005) tested various models from the traditional unconditional volatility models, as well as the GARCH and EGARCH models from the conditional volatility estimators. They used data from the period 1st January 1996 to 31st December 1998 to estimate GARCH (1,1) and EGARCH model for forecasting. He tested the ability of these estimators and models to forecast one-day, five-days and monthly volatility. Using the Schwarz Information Criterion, the author found that the best model in the GARCH (p, q) class for $p \in [1, 5]$ and $q \in [1, 5]$ was GARCH (1, 1). Volatility estimates given by GARCH/EGARCH models indicated lower bias than other models in terms of both MSE and MAE criteria. As far as forecasting of five-day and one-month volatility ahead was concerned, the extreme value estimators were much better than conditional volatility models.

Vasudevan R. D. and Vetrivel S.C. (2016) modelled and forecasted the SENSEX returns using GARCH(1,1), EGARCH(1,1) and TGARCH(1,1) models. The data period covered was from July 1997 to December 2015. First data was divided into in sample and out of sample period. The in sample period was used to know the model estimates and the out of sample period was used for predicting the future volatility. The evaluation measures indicated that the asymmetric models of EGARCH and TGARCH perform superiorly than the symmetric GARCH(1,1) model.

Walsh and Tsou (1998) compared historical volatility model, EWMA, an improved extreme-value method (IEV), ARCH/GARCH class of models. The data used included three price indices collected every five minutes from 1 January 1993 to 31 December 1995. From this, hourly, daily and weekly values were found as the value at the close of each hour, day or week. The forecast errors were then compared using four loss functions: MSE, RMSE, MAE and MAPE. The authors concluded that the larger number of coefficients in the GARCH (1, 2) worsen the prediction as compared to the GARCH (1, 1) model. The hourly data examination proved the EWMA and GARCH (1,1) models as best with slight differences as per loss function used. The results for the daily data were also same, though the weekly data results pointed out towards EWMA as the best predictor for weekly volatility.

IV. CONCLUSIONS

Since it's unearthing, ARCH modelling has become a growing industry, with hundreds of variations of the original model. One that has become prevalent is the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, proposed by Bollerslev in 1986. The main objective of the present paper was to review the findings of various researchers in respect to GARCH(1,1). Out of all the papers reviewed in the present study, GARCH(1,1) won by majority as the preferred model. GARCH(1,1) outperformed GARCH(p,q) and there was not much need felt by researchers to go for higher values of p and q. the performance of GARCH(1,1) was found to be better amongst the traditional volatility models. Moreover, the GARCH(1,1) was found to be superior by many studies as in the category of symmetric volatility models. Though amongst the asymmetric models, EGARCH(1,1) especially provided a good competition for the GARCH(1,1) model. The performance criterion used to measure the performance of a model also does play a significant role in the conclusion derived from he tests conducted. Also, the assets tested covered assets ranging from stock indices, stocks, exchange rates, interest rates to even Bitcoin. And the research studies were conducted more on developed market as compared to developing markets which provide a direction to future research of conducting more test on the developing markets in order to know the performance of GARCH(1,1).

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