A SHORT REVIEW OF LITRATURE IN PARTITION THEORY

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ABSTRACT

The history of partition theory is very popular. Although it is believed that the first discovery of any depth was done in the 18th century, while some problems in partition came back in the middle ages when El Euilar proved many never-forgotten very essential and beautiful division theorem. In 20th century is a time in which Sh.Shrinivas Ramanujan considered as a great mathematician who had played an an important role in the area of maths born 22th December, 1887 in a small village in Tamil Nadu (India),Sh.Srinivasa Ramanujan could not get higher education in mathematics .He died when he was 32 years old, in a short span of time, he is still remembered due to his excellent discoveries in the area of maths on, "partition theory, continuing fractions, q series, elliptic functions, definite integrals and mock- theta function".If we consider ,number theory research field , " the principle of division of numbers" is a necessary and wide part in maths, that was launched within the 18th century approximately .Leonard Euler was followed by Gauss, Jacobi, Schur, McMahon, and Andrews etc.. Discussion and study of the theory of division of numbers, together with Prof. G.H. Hardik, Ramanujan's work is known with a significant change in this area, Ramanujan and Hardy invented the circle theory during his tenure the first first estimate of the division of numbers beyond 200.

Keywords- Generating Function, analysis, Congruences , Division.

Introduction:

Mr. Leonhard Euler:

His tenure 1707 to 1783. he made the first presentation on the division of integers in St. Petersburg Academy, Euler had an important contribution in the field of integer division. Euler's construction work for an integer division, which is important to it

$$\sum_{i=0}^{i=\infty} \mathrm{p(i)}\, y^i = \frac{1}{\prod_{n=1}^{\infty} 1-y^n} \text{ , where } |y| < 1(\text{By Euler's})\ldots\ldots\ldots..(\mathrm{I})$$

In form of example we consider the number five.To find partition number, we draw the coefficient of y^5 from the right of the equation one.

"[1^{st}factor{$1+y^1+y^2+y^3+\cdots$}][2^{nd}factor{$1+y^1+y^2+y^3+\cdots$}]………………….(II)"

Which is known as Euler's Generating Function

By solving eq^n II after multiply ,we have $1+y+2y^2+3y^3+5y^4+7y^5+11y^6+15y^7+\cdots$

Hence the number of partitions of 5 is 7.

Mr. Percy Alexander McMahon:

His tenure from 1854 to 1929 is believed that Mr. McMahom is an English mathematician who is famous due to skills of his work in th field of maths , “division of numbers and knowledgeable combinatorics” . In a article written by McMahon , it has been stated that the values of p (i) of n = 1,2,3, ..., 200, came out by hand through his work.. McMahon proved value of partition of number two hunderes which was verified – “p (200) = 3, 972,999,029,388”.

Mr. Srinivasan Ramanujan :

whose tenure is considered from 1887 to 1920, it is said that Srinivasan Ramanujan is an Indian mathematician and he has no counterpart in the field of pure mathematics, contributing to his mathematical field, especially-“ mathematical analysis, number theory, infinite series” . Mr. Srinivasan Ramanujan is a non-converging asymptomatic series with Godfrey Harold Hardy. Were very skilled in the calculations, which are important in calculating the number of integers of integer .

Sir Godfrey Harold Hardy:

His tenure from 1877 to 1947, is considered by Mr. Godfrey Hardy as an English mathematician, he is known for number theory .He contributes, solve the problem, Hardy proved Ramanujan's congratulatory theory, while working together Or Hardy and Ramanujan expanded subject to both work together

$$p(i) \sim \frac{1}{4i\sqrt{3}} e^{\pi} \sqrt{\frac{2i}{3}} \quad \text{if } i \to \infty \text{…………………(III)}$$

which is knon as Hardy Ramanujan's Asymptotic Expression

For every m we have

(5m+4)≡0 (mod 5),

(7m+5)≡0 (mod 7),

and p(11m+6)≡0 (mod 11)………………………………..(IV)

which is knon as Ramanujan’s Congruences

further

$$p(i) = \frac{1}{2\sqrt{2}} \sum_{u=1}^{v} A_u(i)\sqrt{u}\,\frac{d}{di}\, e^{\pi} \sqrt{\frac{2(i-\frac{1}{24})}{3u}} \quad ,$$

$$\text{when } A_u(i) = \sum_{\substack{0\le m\le u \\ (m,u)=1}} e^{\pi j\{s(m,u)-\frac{1}{u}2im]} \quad \text{………………(V)}$$

{ which is known as By Hardy Ramanujan’s Asymptotic Exp. **}**

Sir Hans Rademacher ji;

His tenure is considered from 1892 to **1969.**Hans Redemacher is in a famous German mathematician contribute their particularly in this topic. Sir Radmacher together with Hardy-Ramanujan's finalized below result.

$$p(i) = \frac{1}{\pi\sqrt{2}} \sum_{u=1}^{\infty} A_u(i)\sqrt{u}\,\frac{d}{dy}\left[\frac{\sinh\left(\frac{\pi}{u}\sqrt{\frac{2}{3}}\left(y-\frac{1}{24}\right)\right)}{\left(y-\frac{1}{24}\right)}\right]_{y=i}$$

$$\text{when } A_u(i) = \sum_{\substack{g \bmod k \\ (g,u)=1}} \omega_{g,u} e^{-2\pi j i g/u} \quad , \omega_{g,u} = \sqrt[24]{1} \quad \textit{such as}$$

$$\omega_{g,u} = \begin{cases} (\frac{-u}{g})e^{-\pi j\{\frac{1}{4}(2-ug-g)+\frac{1}{12}(u-u^{-1})(2g-g^{*}+g^{2}g^{*})} when\ g\ is\ oddd \\ (\frac{-u}{g})e^{-\pi j\{\frac{1}{4}(u-1)+\frac{1}{12}(u-u^{-1})(2g-g^{*}+g^{2}g^{*})} when\ g\ is\ even \end{cases}$$

$$gg^{*} \equiv -1(\,mod\ u)$$

$\left(\frac{c}{d}\right)$ is the Jacobi−Legendre sym.= $\begin{cases} 0 \text{ if } c = 0 \text{ (mod d)} \\ 1 \text{ if } 0 \neq c = y^2 \text{ (mod d) } \forall\, y \\ -1 \text{ otherwise} \end{cases}$ ……….(V1)

Which is known as **Hardy-Ramanujan Rademacher's Result**

By using aboue eq^{n} (VI) , we have by satrting eight terms partition of no 200 such as 3,972,998,993,185.896 + 36,282.978 − 87.555 + 5.147 + 1.424 + 0.071 + 0.000 + 0.043 =3,972,999,029,338.004 which is equal to MacMahon p(200) within 0.004.

Mr. Arthur Oliver Lonsdale Atkins:

His tenure from 1925 to 2008, is considered as Atkin is a British mathematician who taking help of computers 1st time while doing research . "Andrew Wiles, the final theorem of Fermat", which shows pending for more than 300 years is a result of combine efforts of Joseph Lehner & Atkin.work with topic integer division Mr. Arthur Oliver Lonsdale Atkins try to sole all questions.

Mr. Freeman Dyson:

His tenure since 1923. Mr. Freeman is a " British physicist and mathematician" who is popular due to involvement in area related to " physics such as stronomy and atomic engineering" .and the main reason related to existence of two congratulations and guessed & Division related crank function which is helpful to Ramanujan's congratulatory Modulo 11 (Equation IV) .these results are verified by George Andrews and Frank Gervan successfully.

Mr. George Andrews:

His tenure since 1938 until now, working as an American mathematician .A whole discussion about problem of partition is available in his book, "The Doctrine of Partition" Up to till date, the previous mathematician has expanded a complete contributions and their involvements on latest theorems and outcomes in this field i.e partition theory. Andrews is fully responcible for collecting Ramanujan's lost notes …..

Conclusions -

Partition theory is one of the branches of mathematics subject which gives a new direction to the subject of mathematics. Not every person can live without its appreciation, which has some information and interest in it. "The discrete disciplines or classified, atomic studies of molecular and substances, proving the theory of numbers, or problems associated with all sources etc" are the main applications of the principle of division.

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