Analysis of Propped Cantilever Beam Carries "U.V.L" Zero at Fixed End and Maximum at Propped End and Analysis of Fixed Beam Carries "U.V.L" Maximum at Centre by Mohr's Theorem

[1]R.M.KARTHIKEYAN, [2]C.MUNUSAMY, [3]AK.MD MUBARAK

[1]Associate Professor,[2]Senior Lecturer,[3] Post graduate Student
[1]Department of Civil Engineering, S.N.S.College of Technology, Coimbatore
[2]Department of Civil Engineering, Nanjiah Lingammal Polytechnic College, Mettupalayam-Coimbatore
[3] Department of Civil Engineering, S.N.S.College of Technology, Coimbatore

Abstract: : Analysis of propped cantilever beam carries uniformly varying load zero at fixed end and maximum at propped end and also analysis of fixed beam carries uniformly varying load maximum at centre by using Mohr's theorem are not given in any other references. Article which is shows procedure for analyzing the indeterminate beam of uniformly varying load acting on the location given in above title.

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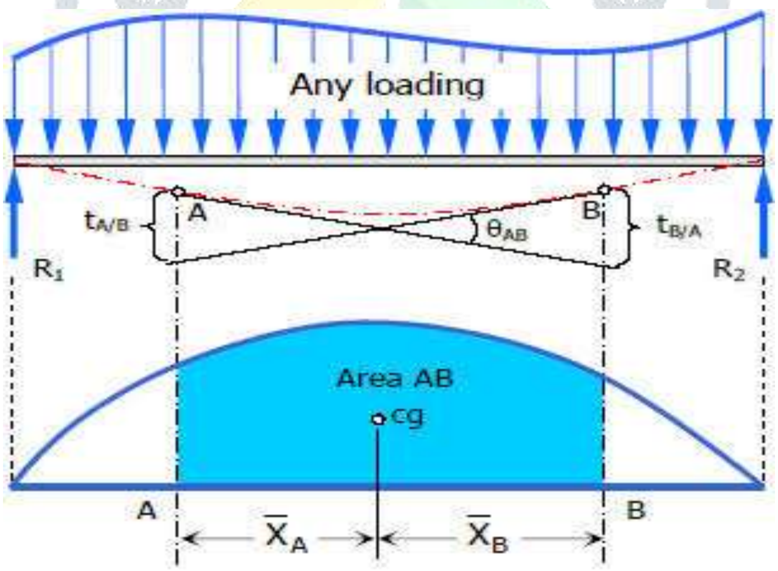
IndexTerms **- Analysis of propped cantilever beam carries uniformly varying load zero at fixed end and maximum at propped end, analysis of fixed beam carries uniformly varying load maximum at centre**

I. INTRODUCTION

Analysis of propped cantilever beam carries uniformly varying load zero at propped end and maximum at fixed end by moment area method (Mohr's Theorem) which is analysed in books, other references and if the uniformly varying load zero at fixed end and maximum at propped end and also analysis of fixed beam carries uniformly varying load maximum at centre by using Mohr's theorem are not given in any other references and books. In this analysis direct area method is used to find out the area constant and centre of gravity of bending moment diagram is using for analysis.

II. METHODOLOGY

Mohr's area moment theorems



Moment Diagram

Mohr's -1 theorem:

$$\mathbf{i_{AB} = \frac{(AREA\ OF\ B.M.D)_{AB}}{EI}}$$

The change in angle of slope between the tangents at any two points (A and B) on the elastic curve is equal to the area of bending moment diagram in between those two points divided by flexural rigidity (EI).

Mohr's -2 theorems:

$$Y_{AB} = \frac{(\text{AREA OF B.M.D})_{AB}\, X_B}{EI}$$

The tangential deviation of any point B on the elastic curve from a tangent at any other point A on the elastic curve, perpendicular to the original axis of the beam is equal to the moment of inertia of BMD in between those two points about B divided by flexural rigidity.

Direct area method is the combination of double integration method and Mohr's theorem. In this method used to find out the area constant, centre of gravity by equation of slope and deflection formula generated from double integration method.

In Mohr's theorem I substituting the generated slope formula from double integration method to find out the area constant of bending moment diagram and also in Mohr's theorem II substituting the generated deflection formula from double integration method to find out the centre of gravity of bending moment diagram.

From double integration method:

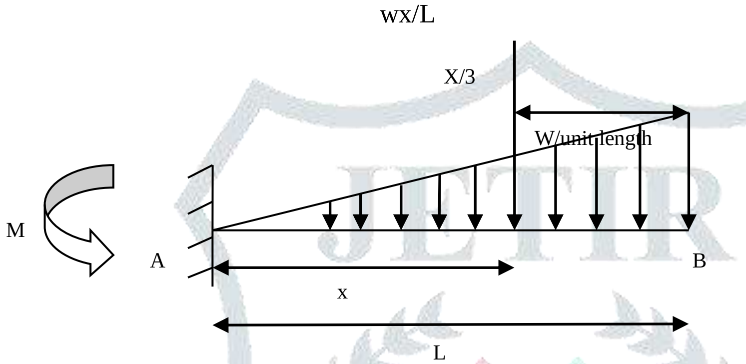


FIG.3

$$\text{(V) Reaction at A} = \frac{wL}{2}$$

$$\text{(M) Bending moment at A} = \frac{wL^2}{3}$$

Slope and deflection formulae generated from double integration method shown below:
Slope at distance 'X' from A:

$$i_x = -\frac{wL^2x}{3EI} + \frac{wLx^2}{4EI} - \frac{wx^4}{24EIL}$$

Deflection at distance 'X' from A:

$$Y_x = -\frac{wL^2x^2}{6EI} + \frac{wLx^3}{12EI} - \frac{wx^5}{120EIL}$$

We know that maximum slope and deflection occurs at free end at cantilever beam. So substitute x = L in the slope deflection formula generated from double integration method.

Maximum Slope at free end:

$$i_{max} = \frac{wL^3}{8EI}$$

Maximum deflection at free end:

$$Y_{max} = \frac{11wL^4}{120EI}$$

From this,
By using Mohr's theorem-I find out area constant 'X' refer FIG.2.

$$i = \frac{Area\ of\ BMD}{Flexural\ rigidity}$$

$$\frac{wL^3}{8EI} = \boldsymbol{X} \; x \; L \; x \; H \; x \; \frac{1}{EI}$$

$$\frac{wL^3}{8EI} = \boldsymbol{X} \; x \; L \; x \; \frac{wL^2}{3} \; x \; \frac{1}{EI}$$

$$\text{Area constant for the curve (X)} = \frac{3}{8}$$

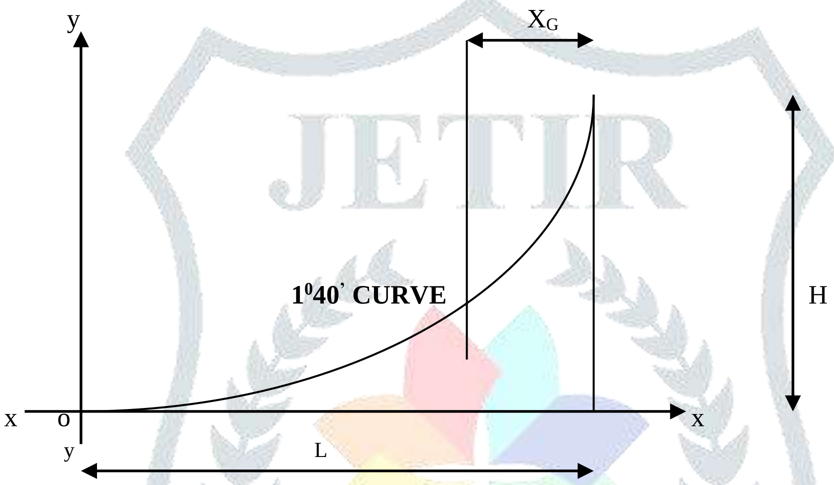
By using Mohr's theorem-II find out centre of gravity constant 'X_B' refer FIG.2.

$$Y_{max} = \frac{\text{Area of BMD } xX_B}{\text{Flexural rigidity}}$$

$$\frac{11wL^4}{120EI} = \frac{wL^3}{8EI} \; x \; X_B$$

$$\text{Centre of gravity for the curve } (X_B) = \frac{11L}{15}$$

Degree of curve::



$$\text{Area of the curve (A)} \quad = \frac{LH}{n+1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$\text{We know that area constant} = \frac{3}{8}$$

Area of curve = area constant x B x H

$$\text{Area of curve} = \frac{3}{8} \; x \; L \; x \; H$$

Substitute in the equation (1) to find degree of curve:

$$\frac{3LH}{8} = \frac{LH}{n+1}$$

From this,
n = 1.67 =**$1^0 40'$**
So the degree of curve is **$1^0 40'$**

From this,

Area and Centre of gravity For 1^0 40' curve to length 'L' :

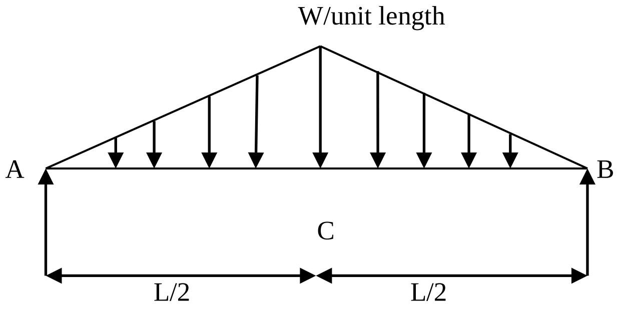
As well as, we know that maximum slope and deflection formula for simply supported beam carries an uniformly varying load is maximum at centre from double integration method :

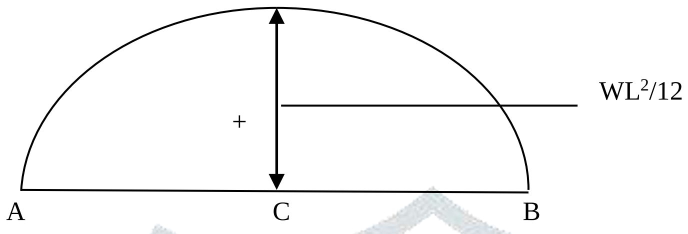
Maximum Slope at free end:

$$i_{max} = \frac{5wL^3}{192EI}$$

Maximum deflection at free end:

$$Y_{max} = \frac{wL^4}{120EI}$$





BMD

By using Mohr's 1 theorem find out area constant (X) :

$$\mathbf{i_B = \frac{(AREA\ OF\ B.M.D)_{CB}}{EI}}$$

$$\frac{5wL^3}{192EI} = \frac{X\ x\ B\ x\ H}{EI}$$

$$\frac{5wL^3}{192EI} = \frac{X\ x\frac{L}{2}\ x\ \frac{wL^2}{12}}{EI}$$

$$\frac{5wL^3}{192EI} = \frac{\frac{XwL^3}{24}}{EI}$$

$$X = \frac{5\ x\ 24}{192}$$

$$\boldsymbol{X = \frac{5}{8}}$$

By using Mohr's -2 theorem find out centroid (X_B) :

$$\mathbf{Y_C = \frac{(AREA\ OF\ B.M.D)_{CB}\ X_B}{EI}}$$

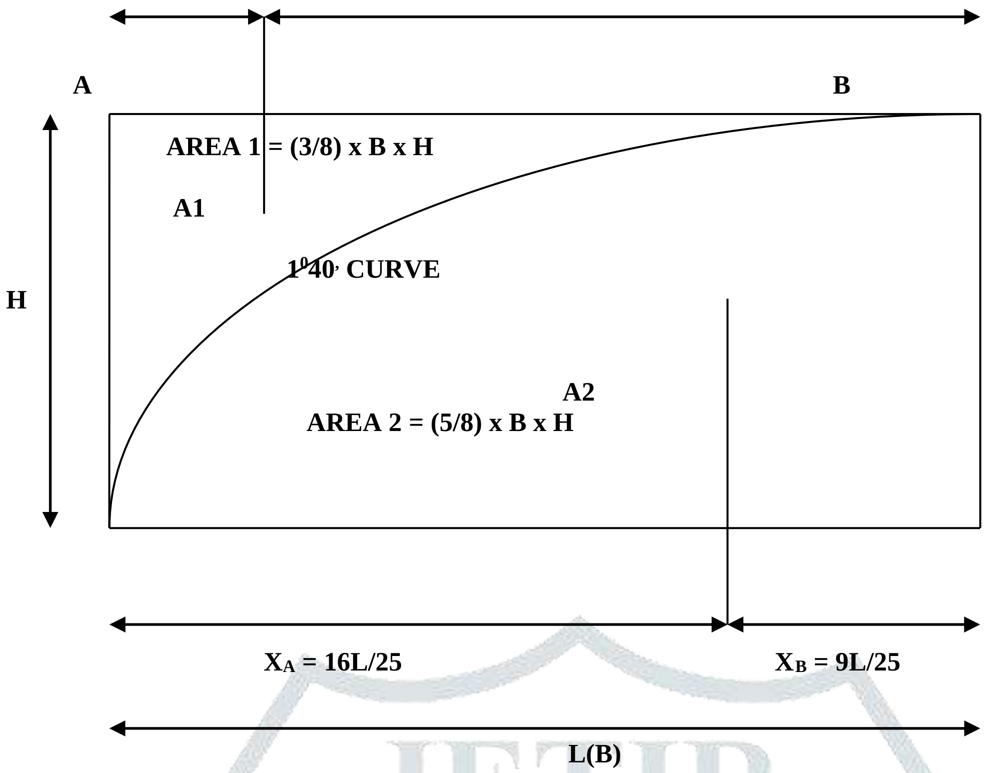
$$\frac{wL^4}{120EI} = \frac{5wL^3\ X_B}{192EI}$$

$$X_B = \frac{192}{5x120} = \mathbf{\frac{8L}{25}}$$

$$\mathbf{X_B = \frac{8L}{25}\quad for\quad \frac{L}{2}\ \ Distance}$$

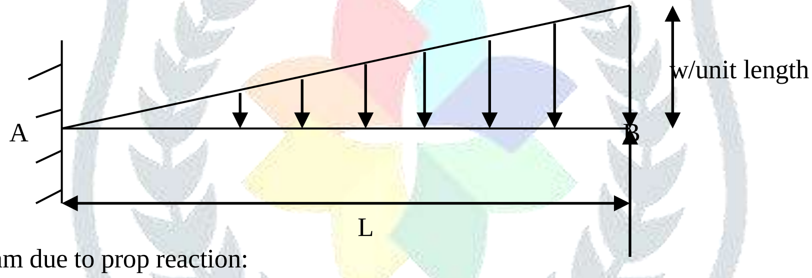
$$\mathbf{X_B = \frac{16L}{25}\quad for\quad L\ \ Distance}$$

X_A = 4L/15 **X_B = 11L/15**

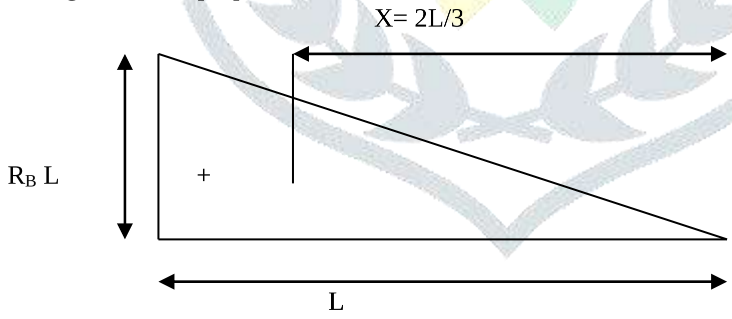


III ANALYSIS

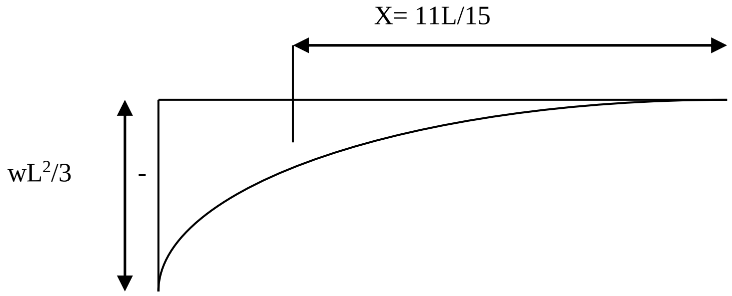
Analysis of propped cantilever beam carries uniformly varying load zero at fixed end and maximum at propped end by using Mohr's theorem-ii



Bending moment diagram due to prop reaction:



Bending moment diagram due to cantilever beam due to loading:



Applying Mohrs Theorem – II

Area of (+) B.M.D = (1/2) x L x $LR_B = R_BL^2/2$

$X_{B1} = (2L/3)$

Area of (-) B.M.D = (3/8) x L x $(wL^2)/3 = -wL^3/8$
$X_{B2} = (11L/15)$

1/EI (AREA OF (+) BMD) X_{B1} = 1/EI (AREA OF (-) BMD) X_{B2}

$$(R_B L^2/2) \times (2L/3) = -(wL^3/8) \times (11L/15)$$

$$\mathbf{R_B = -11wL/40}$$

R_A:

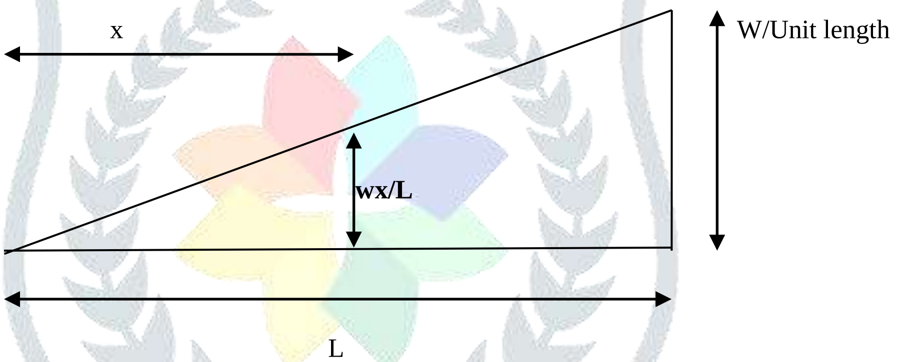
$R_A + R_B$ = Total Load

$R_A + (11wL/40) = (wL/2)$ **= 9wL/40**

SFD:
SF@ A = 9wL/40
SF@ B = (wL/2) – (9wL/40) =-11wL/40

BMD:
BM@B=0KNm
BM@A= R_B x L – $(wL^2/3) = -7wL^2/120$
Maximum bending moment:

S.F=0

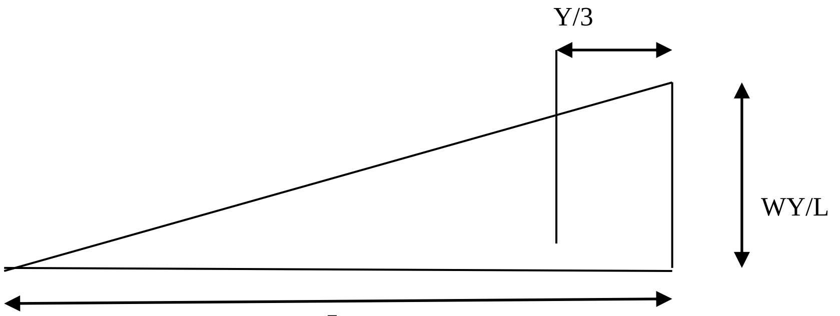


$$\mathbf{R_A - (1/2) \times X \times (wX/L) = 0}$$
$$(9wL/10) - (wX^2/2L) = 0$$

$$\mathbf{X} = \frac{L\sqrt{18}}{\sqrt{40}} \text{ from A}$$

Maximum B.M = $R_A \times \left(\frac{L\sqrt{18}}{\sqrt{40}}\right) - \left((1/2) \times \left(\frac{L\sqrt{18}}{\sqrt{40}}\right) \times \left(w\left(\frac{L\sqrt{18}}{\sqrt{40}}\right)/L\right) \times (1/3) \times \left(\frac{L\sqrt{18}}{\sqrt{40}}\right)\right) = \frac{wL^2}{20}\left(\frac{27\sqrt{2}}{4\sqrt{10}} - \frac{3\sqrt{18}}{\sqrt{40}}\right)$

POINT OF INFLEXION



B.M=0

$$R_A \times Y - (1/2) \times Y \times (wY/L) \times (Y/3) = 0$$

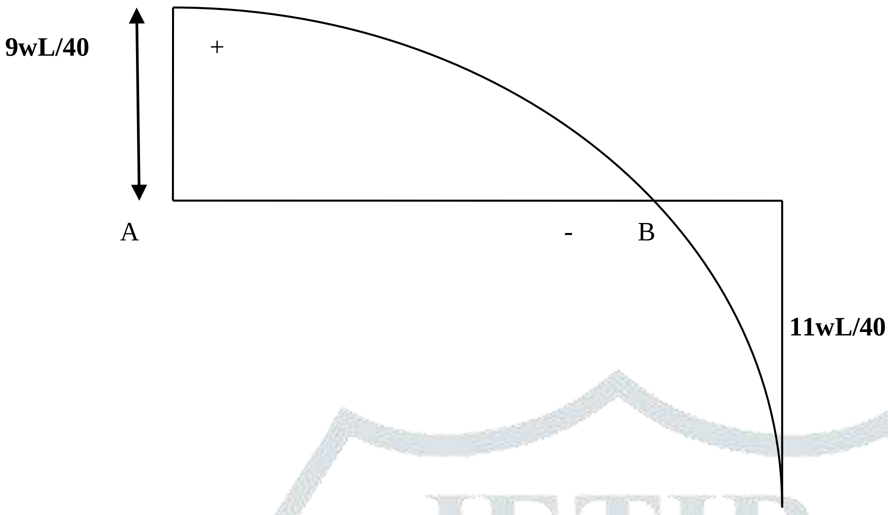
$$(9wLY/40) - (wY^3/6L) - (7wL^2/120) = 0$$

Take w=unit load and l=unit length

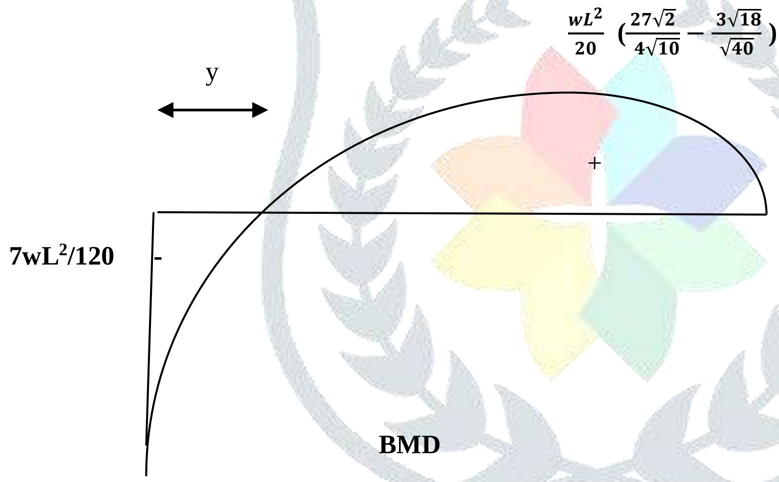
$$\mathbf{(9Y/40) – (Y^3/6) – (7/120) = 0}$$
$$\mathbf{(Y^3/6) - (9Y/40) + (7/120) = 0}$$

In the form of cubical equation

$$\mathbf{ax^3+bx^2+cx+d=0}$$
Y =0.2745L Says 0.275L from A



SFD



BMD

Analysis of fixed beam carries uniformly varying load maximum at centre by using Mohr's theorem-i

1/EI(AREA OF POSITIVE BMD) = 1/EI(AREA OF NEGATIVE BMD)

Area of positive B.M.D = 5/8 x L x $wL^2/12$ =$5wL^3/96$ (1° 40'curve = (5/8) x b x h)
Area of negative B.M.D = L x M_A (rectangular shape = b x h)

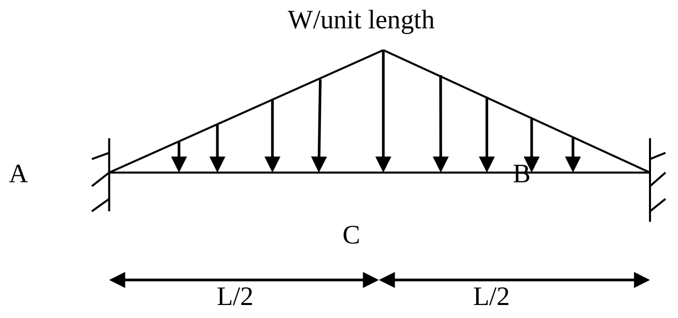
M_A = M_B (moment same at the ends because it's symmetrically loaded beam)

Equating

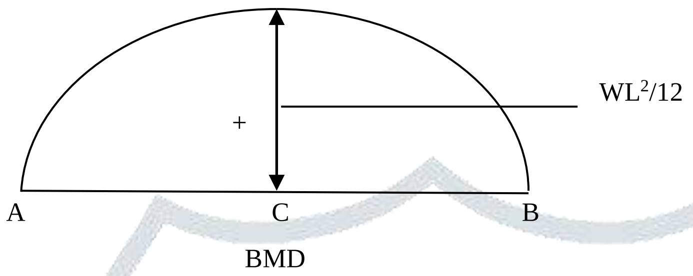
$$5wL^3/96 = - L \times M_A$$
$$\mathbf{M_A = M_B = - 5wL^2/96}$$

Maximum positive bending moment = $wL^2/12$ –5 $wL^2/96$= **$+wL^2/32$**

Symmetrically loaded beam, Reaction at A and B are same = total load/2 = (wL/2)/2 **= wL/4**

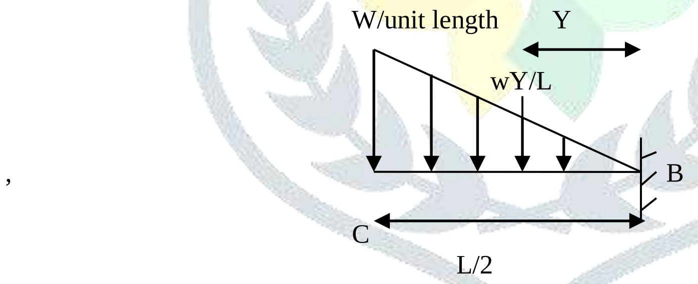


Free bending moment diagram



Fixed end bending moment diagram

Point of inflexion:



B.M = 0

R_B x Y - $5wL^2/96$- ((1/2) x Y x (2WY/L) x (1/3) x Y)= 0

(wL/4) x Y - $5wL^2/96$- ((1/2) x Y x (2Wy/L) x (1/3) x Y)= 0

Y/4 -Y^3L- 5L/96= 0

Y^3L – Y/4 +5L/96 = 0

By using cubical equation
Take w=unit load and l=unit length

$ay^3 + by^2 + cy + d = 0$

a = L , b=0, c =-0.25 and d =0.05208L

Y = 0.291L from A and B

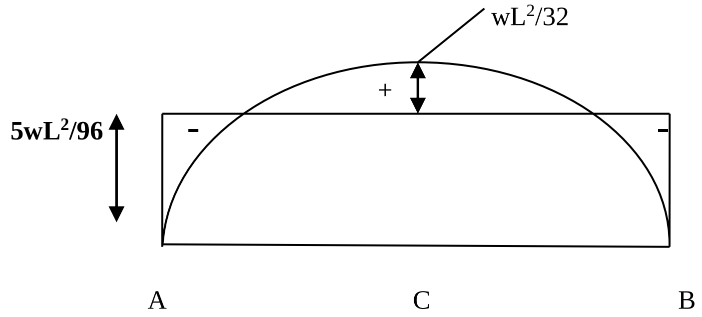
Maximum deflection at centre:

Applying Mohr's theorem –ii

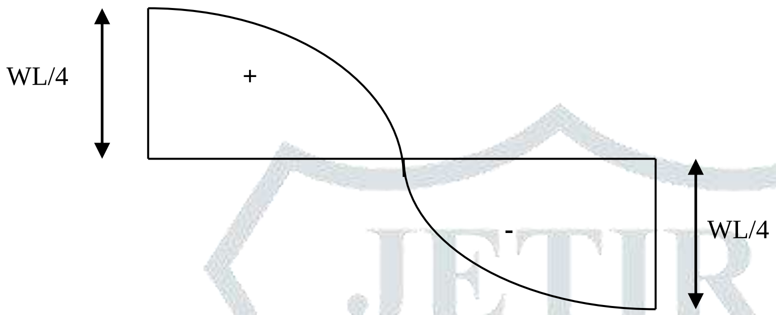
y_{max} = 1/EI (AREA OF (+) BMD)$_{CB}$ x X_B - 1/EI (AREA OF (-) BMD)$_{CB}$ x X_B

(AREA OF (+) BMD)$_{CB}$ x X_B =(5/8) x (L/2) x (wL^2/12) x (8L/25) = wL^4/120
(AREA OF (-) BMD)$_{CB}$ x X_B = L/2 x $5wL^2$/96 x L/4 = $5wL^4$/768

$$y_{max} = (wL^4/120EI) - (5wL^4/768EI)$$
$$= 7wL^4/3840EI$$



FINAL BMD



SFD

IV CONCLUSION:

Propped cantilever beam carries uniformly varying load zero at fixed end and maximum at propped end and also fixed beam carries uniformly varying load maximum at centre analyzing by using Mohr's theorem having a tool of direct area method.

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