LAPLACE-CARSON TRANSFORM FOR THE PRIMITIVE OF SYSTEM OF CONVOLUTION TYPE LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS OF FIRST KIND

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ABSTRACT: In the present paper, authors determine the primitive of system of convolution type linear Volterra integro-differential equations of first kind by using Laplace-Carson transform. Four numerical problems have been considered and solved using Laplace-Carson transform for explaining the applicability of Laplace-Carson transform for determining the primitive of system of convolution type linear Volterra integro-differential equations of first kind. Results of numerical problems depict that the Laplace-Carson transform provides the primitive of system of convolution type linear Volterra integro-differential equations of first kind without doing tedious calculation work.

KEYWORDS: Volterra Integro-Differential Equation; Laplace-Carson Transform; Convolution; Inverse Laplace-Carson Transform

MATHEMATICS SUBJECT CLASSIFICATION: 44A10, 45J05, 45A05, 45D05.

INTRODUCTION: In the present scenario, integral transforms can be used for solving many interesting problems of engineering, mechanics, statistics, physics, elasticity, potential theory, plasticity, fluid dynamics, population dynamics, and aero dynamics by developing their mathematical model using ordinary differential equations, partial differential equations, integral equations, delay differential equations, integro-differential equations and their systems. System of Volterra integro-differential equations appears when we convert higher order initial value problem into integral equation. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations.

Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications.

The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51] used Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [52-53] by using Aboodh and Sadik transformations respectively.

Aggarwal and Sharma [54] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [55]. Aggarwal and Bhatnagar [56] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [57]. Aggarwal [58] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [59] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [60] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [61].

Mishra et al. [62] defined the relationship between Sumudu and some efficient integral transforms. Aggarwal [63] proposed Kamal transform of Bessel's functions. Aggarwal and other scholars [64-73] used Aboodh; Mohand; Kamal; Elzaki; Laplace-carson; Laplace; Sadik; Sawi; Sumudu and Shehu transformations for determining the solution of first kind Volterra integro-differential equation. Kumar and Aggarwal [74] considered Laplace transform and used it in solving system of linear Volterra integro-ordinary differential equations of first kind. Aggarwal et al. [75] determined the solutions of population growth and decay problems using Sumudu transform. Aggarwal et al. [76] proposed the Sawi transform of Bessel's functions with application for evaluating definite integrals. Aggarwal et al. [77] determined the primitive of second kind linear Volterra integral equation using Shehu transform.

Higazy et al. [78] used Sawi decomposition method for Volterra integral equation. Higazy et al. [79] determined the number of infected cells and concentration of viral particles in plasma during HIV-1 infections using Shehu transformation. Watugula [80] gave the Sumudu transform and solved differential equations and control engineering problems using it. Abdelilah and Hassan [83] used Kamal transform for solving partial differential equations. Kumar et al. [84] applied Mohand transform for solving linear Volterra integral equations of first kind. Applications of Mohand transform to mechanics and electrical circuit problems were given by Kumar et al. [85].

Aboodh et al. [86] solved delay differential equations by Aboodh transformation method. Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods was given by

Aboodh et al. [87]. Mohand et al. [88] determined the solution of ordinary differential equation with variable coefficients using Aboodh transform. Elzaki and Ezaki [89] used Elzaki transform and determined the solution of ordinary differential equation with variable coefficients. Elzaki and Ezaki [90] used Elzaki transform for solving partial differential equations. Shendkar and Jadhav [91] applied Elzaki transform on differential equations for their solutions. Aggarwal and Kumar [92] used Laplace transform for system of second kind linear Volterra integro-differential equations.

The main aim of this paper is to determine the primitive of system of convolution type linear Volterra integro-differential equations of first kind by using Laplace-Carson transform.

DEFINITION OF LAPLACE-CARSON TRANSFORM: The Laplace-Carson (Mahgoub) transform of the function $G(t)$ for all $t \geq 0$ is defined as [68]:

$L\{G(t)\} = p\int_0^\infty G(t)e^{-pt}dt = g(p)$, where L is Laplace-Carson transform operator. Standard properties of Laplace-Carson transform and Laplace-Carson transform of useful mathematical functions are presented in Table: 1 and Table: 2 respectively (See Table: 1 & Table: 2).

TABLE: 1 USEFUL PROPERTIES OF LAPLACE-CARSON TRANSFORM [1-2, 57]

S.N.	Name of Property	Mathematical Form
1	Linearity	$[L\{aG_1(t) + bG_2(t)\} = aL\{G_1(t)\} + bL\{G_2(t)\}]$
2	Change of Scale	$L\{G(at)\} = g\left(\frac{p}{a}\right)$
3	Shifting	$L\{e^{at}G(t)\} = \frac{p}{(p-a)}g(p-a)$
4	First Derivative	$[L\{G'(t)\} = pg(p) - pG(0)]$
5	Second Derivative	$[L\{G''(t)\} = p^2g(p) - p^2G(0) - pG'(0)]$
6	nth Derivative	$\begin{bmatrix} L\{G^{(n)}(t)\} \\ = p^n g(p) - p^n G(0) - p^{n-1}G'(0) \\ - \cdots \ldots - pG^{(n-1)}(0) \end{bmatrix}$
7	Convolution	$\left[L\{G_1(t) * G_2(t)\} = \frac{1}{p}L\{G_1(t)\}L\{G_2(t)\}\right]$

Table: 2 LAPLACE-CARSON TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [1-2, 9, 23, 38, 40, 48]

S.N.	$G(t)$	$L\{G(t)\} = g(p)$
1	1	1
2	t	$\frac{1}{p}, p > 0$
3	t^2	$\left(\frac{2}{p^2}\right), p > 0$
4	$t^n, n\epsilon N$	$\left(\frac{n!}{p^n}\right), p > 0$
5	$t^n, n > -1$	$\frac{1}{p^n}\Gamma(n+1), p > 0$
6	e^{at}	$\frac{p}{p-a}, p > a$
7	$sinat$	$\frac{ap}{p^2+a^2}, p > 0$
8	$cosat$	$\frac{p^2}{p^2+a^2}, p > 0$
9	$sinhat$	$\frac{ap}{p^2-a^2}, p > \|a\|$
10	$coshat$	$\frac{p^2}{p^2-a^2}, p > \|a\|$
11	$J_0(t)$	$\frac{p}{\sqrt{p^2+1}}$
12	$J_1(t)$	$p - \frac{p^2}{\sqrt{p^2+1}}$
13	$erf(\sqrt{t})$	$\frac{1}{\sqrt{(1+p)}}$

INVERSE LAPLACE-CARSON TRANSFORM: If $L\{G(t)\} = g(p)$ then $G(t)$ is called the inverse Laplace-Carson transform of $g(p)$.

Mathematically, it is represented as $G(t) = L^{-1}\{g(p)\}$, where the operator L^{-1} is called the inverse Laplace-Carson transform operator. Inverse Laplace-Carson transform of useful mathematical functions are presented in Table: 3 (See Table: 3).

TABLE: 3 INVERSE LAPLACE-CARSON TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [57, 68]

S.N.	$g(p)$	$G(t) = L^{-1}\{g(p)\}$
1	1	1
2	$\frac{1}{p}$	t
3	$\frac{1}{p^2}$	$\frac{t^2}{2!}$
4	$\frac{1}{p^n}, n\epsilon N$	$\frac{t^n}{n!}$
5	$\frac{1}{p^n}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6	$\frac{p}{p-a}$	e^{at}
7	$\frac{p}{p^2+a^2}$	$\frac{sinat}{a}$
8	$\frac{p^2}{p^2+a^2}$	$cosat$
9	$\frac{p}{p^2-a^2}$	$\frac{sinhat}{a}$
10	$\frac{p^2}{p^2-a^2}$	$coshat$
11	$\frac{p}{\sqrt{p^2+1}}$	$J_0(t)$
12	$p - \frac{p^2}{\sqrt{p^2+1}}$	$J_1(t)$
13	$\frac{1}{\sqrt{(1+p)}}$	$erf\left(\sqrt{t}\right)$

LAPLACE-CARSON TRANSFORM FOR THE PRIMITIVE OF SYSTEM OF CONVOLUTION TYPE LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS OF FIRST KIND:

The general system of convolution type linear Volterra integro-differential equations of first kind is given by [74]

$$\left.\begin{array}{l} f_1(x) = \begin{Bmatrix} \int_0^x K_{11}(x-t)\, u_1{}^{(l)}(t)dt + \int_0^x K_{12}(x-t)\, u_2(t)dt \\ + \cdots . + \int_0^x K_{1n}(x-t)\, u_n(t)dt \end{Bmatrix} \\ f_2(x) = \begin{Bmatrix} \int_0^x K_{21}(x-t)\, u_1(t)dt + \int_0^x K_{22}(x-t)\, u_2{}^{(l)}(t)dt \\ + \cdots . + \int_0^x K_{2n}(x-t)\, u_n(t)dt \end{Bmatrix} \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ f_n(x) = \begin{Bmatrix} \int_0^x K_{n1}(x-t)\, u_1(t)dt + \int_0^x K_{n2}(x-t)\, u_2(t)dt \\ + \cdots . + \int_0^x K_{nn}(x-t)\, u_n{}^{(l)}(t)dt \end{Bmatrix} \end{array}\right] \tag{1}$$

$$\text{with} \begin{Bmatrix} u_1{}^{(m)}(0) = a_{1m}, m = 0,1,2, \dots, l-1; \\ u_2{}^{(m)}(0) = a_{2m}, m = 0,1,2, \dots, l-1; \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots , \\ u_n{}^{(m)}(0) = a_{nm}, m = 0,1,2, \dots, l-1 \end{Bmatrix} \tag{2}$$

Operating Laplace-Carson transform on system (1) and using convolution theorem of Laplace-Carson transform, we have

$$\left.\begin{array}{l} L\{f_1(x)\} = \frac{1}{p}\begin{bmatrix} L\{K_{11}(x)\}L\{u_1{}^{(l)}(x)\} + L\{K_{12}(x)\}L\{u_2(x)\} \\ + \cdots . + L\{K_{1n}(x)\}L\{u_n(x)\} \end{bmatrix} \\ L\{f_2(x)\} = \frac{1}{p}\begin{bmatrix} L\{K_{21}(x)\}L\{u_1(x)\} + L\{K_{22}(x)\}L\{u_2{}^{(l)}(x)\} \\ + \cdots . + L\{K_{2n}(x)\}L\{u_n(x)\} \end{bmatrix} \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ L\{f_n(x)\} = \frac{1}{p}\begin{bmatrix} L\{K_{n1}(x)\}L\{u_1(x)\} + L\{K_{n2}(x)\}L\{u_2(x)\} \\ + \cdots . + L\{K_{nn}(x)\}L\{u_n{}^{(l)}(x)\} \end{bmatrix} \end{array}\right] \tag{3}$$

Using the property “Laplace-Carson transforms of derivatives” on system (3), we have

$$\left.\begin{array}{l}
L\{f_1(x)\} = \left[\begin{array}{c} \frac{1}{p}L\{K_{11}(x)\}\left\{\begin{array}{c} p^l L\{u_1(x)\} \\ -p^l u_1(0) \\ -p^{l-1}u_1{}'(0) \\ -\cdots..-pu_1{}^{(l-1)}(0) \end{array}\right\} \\ +\frac{1}{p}L\{K_{12}(x)\}L\{u_2(x)\} \\ +\cdots.+\frac{1}{p}L\{K_{1n}(x)\}L\{u_n(x)\} \end{array}\right] \\
L\{f_2(x)\} = \left[\begin{array}{c} \frac{1}{p}L\{K_{21}(x)\}L\{u_1(x)\} \\ +\frac{1}{p}L\{K_{22}(x)\}\left\{\begin{array}{c} p^l L\{u_2(x)\} \\ -p^l u_2(0) \\ -p^{l-1}u_2{}'(0) \\ -\cdots..-pu_2{}^{(l-1)}(0) \end{array}\right\} \\ +\cdots.+\frac{1}{p}L\{K_{2n}(x)\}L\{u_n(x)\} \end{array}\right] \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
L\{f_n(x)\} = \left[\begin{array}{c} \frac{1}{p}L\{K_{n1}(x)\}L\{u_1(x)\} \\ +\frac{1}{p}L\{K_{n2}(x)\}L\{u_2(x)\} \\ +\cdots.+\frac{1}{p}L\{K_{nn}(x)\}\left\{\begin{array}{c} p^l L\{u_n(x)\} \\ -p^l u_n(0) \\ -p^{l-1}u_n{}'(0) \\ -\cdots..-pu_n{}^{(l-1)}(0) \end{array}\right\} \end{array}\right]
\end{array}\right] \tag{4}$$

Using equation (2) in system (4), we get

$$\left.\begin{array}{l}
L\{f_1(x)\} = \frac{1}{p}\left[\begin{array}{c} L\{K_{11}(x)\}\left\{\begin{array}{c} p^l L\{u_1(x)\} \\ -p^l a_{10} \\ -p^{l-1}a_{11} \\ -\cdots..-pa_{1(l-1)} \end{array}\right\} \\ +L\{K_{12}(x)\}L\{u_2(x)\} \\ +\cdots.+L\{K_{1n}(x)\}L\{u_n(x)\} \end{array}\right] \\
L\{f_2(x)\} = \frac{1}{p}\left[\begin{array}{c} L\{K_{21}(x)\}L\{u_1(x)\} \\ +L\{K_{22}(x)\}\left\{\begin{array}{c} p^l L\{u_2(x)\} \\ -p^l a_{20} \\ -p^{l-1}a_{21} \\ -\cdots..-pa_{2(l-1)} \end{array}\right\} \\ +\cdots.+L\{K_{2n}(x)\}L\{u_n(x)\} \end{array}\right] \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
L\{f_n(x)\} = \frac{1}{p}\left[\begin{array}{c} L\{K_{n1}(x)\}L\{u_1(x)\} \\ +L\{K_{n2}(x)\}L\{u_2(x)\} \\ +\cdots.+L\{K_{nn}(x)\}\left\{\begin{array}{c} p^l L\{u_n(x)\} \\ -p^l a_{n0} \\ -p^{l-1}a_{n1} \\ -\cdots..-pa_{n(l-1)} \end{array}\right\} \end{array}\right]
\end{array}\right] \tag{5}$$

After simplification system (5), we have

$$\left.\begin{array}{l}
\begin{bmatrix} (p^l L\{K_{11}(x)\})L\{u_1(x)\} \\ +L\{K_{12}(x)\}L\{u_2(x)\} \\ +\cdots\ldots + L\{K_{1n}(x)\}\{u_n(x)\} \end{bmatrix} = \begin{bmatrix} pL\{f_1(x)\} \\ +\begin{Bmatrix} p^l a_{10} + p^{l-1}a_{11} \\ +\cdots. + pa_{1(l-1)} \end{Bmatrix} L\{K_{11}(x)\} \end{bmatrix} \\
\begin{bmatrix} L\{K_{21}(x)\}L\{u_1(x)\} \\ +(p^l L\{K_{22}(x)\})L\{u_2(x)\} \\ +\cdots\ldots + L\{K_{2n}(x)\}L\{u_n(x)\} \end{bmatrix} = \begin{bmatrix} pL\{f_2(x)\} \\ +\begin{Bmatrix} p^l a_{20} + p^{l-1}a_{21} \\ +\cdots.. + pa_{2(l-1)} \end{Bmatrix} L\{K_{22}(x)\} \end{bmatrix} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\begin{bmatrix} L\{K_{n1}(x)\}L\{u_1(x)\} \\ +L\{K_{n2}(x)\}L\{u_2(x)\} \\ +\cdots\ldots + (p^l L\{K_{nn}(x)\})L\{u_n(x)\} \end{bmatrix} = \begin{bmatrix} pL\{f_n(x)\} \\ +\begin{Bmatrix} p^l a_{n0} + p^{l-1}a_{n1} \\ +\cdots.. + pa_{n(l-1)} \end{Bmatrix} L\{K_{nn}(x)\} \end{bmatrix}
\end{array}\right] \tag{6}$$

The solution of system (6) is given as

$$L\{u_1(x)\} = \frac{\begin{vmatrix}
\begin{bmatrix} pL\{f_1(x)\} \\ +\begin{Bmatrix} p^l a_{10} \\ +p^{l-1}a_{11} \\ +\cdots. \\ +pa_{1(l-1)} \end{Bmatrix} L\{K_{11}(x)\} \end{bmatrix} & L\{K_{12}(x)\} & \ldots\ldots & L\{K_{1n}(x)\} \\
\begin{bmatrix} pL\{f_2(x)\} \\ +\begin{Bmatrix} p^l a_{20} \\ +p^{l-1}a_{21} \\ +\cdots.. \\ +pa_{2(l-1)} \end{Bmatrix} L\{K_{22}(x)\} \end{bmatrix} & (p^l L\{K_{22}(x)\}) & \ldots. & L\{K_{2n}(x)\} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots.. & \ldots\ldots\ldots.. & \ldots\ldots.. \\
\begin{bmatrix} pL\{f_n(x)\} \\ +\begin{Bmatrix} p^l a_{n0} \\ +p^{l-1}a_{n1} \\ +\cdots.. \\ +pa_{n(l-1)} \end{Bmatrix} L\{K_{nn}(x)\} \end{bmatrix} & L\{K_{n2}(x)\} & \ldots. & (p^l L\{K_{nn}(x)\})
\end{vmatrix}}{\begin{vmatrix}
(p^l L\{K_{11}(x)\}) & L\{K_{12}(x)\} & \ldots\ldots\ldots.. & L\{K_{1n}(x)\} \\
L\{K_{21}(x)\} & (p^l L\{K_{22}(x)\}) & \ldots.. & L\{K_{2n}(x)\} \\
\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots\ldots.. & \ldots\ldots\ldots.. & \ldots\ldots\ldots\ldots\ldots.. \\
L\{K_{n1}(x)\} & L\{K_{n2}(x)\} & \ldots\ldots\ldots.. & (p^l L\{K_{nn}(x)\})
\end{vmatrix}}$$

$$L\{u_2(x)\} = \frac{\begin{vmatrix} (p^l L\{K_{11}(x)\}) & \left[\begin{matrix} pL\{f_1(x)\} \\ + \left\{\begin{matrix} p^l a_{10} \\ +p^{l-1} a_{11} \\ +\cdots. \\ +pa_{1(l-1)} \end{matrix}\right\} L\{K_{11}(x)\} \end{matrix}\right] & \dots & L\{K_{1n}(x)\} \\ L\{K_{21}(x)\} & \left[\begin{matrix} pL\{f_2(x)\} \\ + \left\{\begin{matrix} p^l a_{20} \\ +p^{l-1} a_{21} \\ +\cdots.. \\ +pa_{2(l-1)} \end{matrix}\right\} L\{K_{22}(x)\} \end{matrix}\right] & \dots. & L\{K_{2n}(x)\} \\ \dots\dots\dots\dots\dots\dots\dots.. & \dots\dots\dots\dots & \dots\dots\dots\dots\dots.. & \dots\dots\dots\dots\dots\dots.. \\ L\{K_{n1}(x)\} & \left[\begin{matrix} pL\{f_n(x)\} \\ + \left\{\begin{matrix} p^l a_{n0} \\ +p^{l-1} a_{n1} \\ +\cdots.. \\ +pa_{n(l-1)} \end{matrix}\right\} L\{K_{nn}(x)\} \end{matrix}\right] & \dots. & (p^l L\{K_{nn}(x)\}) \end{vmatrix}}{\begin{vmatrix} (p^l L\{K_{11}(x)\}) & L\{K_{12}(x)\} & \dots\dots\dots.. & L\{K_{1n}(x)\} \\ L\{K_{21}(x)\} & (p^l L\{K_{22}(x)\}) & \dots.. & L\{K_{2n}(x)\} \\ \dots\dots\dots\dots & \dots\dots\dots\dots\dots.. & \dots\dots\dots.. & \dots\dots\dots\dots\dots.. \\ L\{K_{n1}(x)\} & L\{K_{n2}(x)\} & \dots\dots\dots.. & (p^l L\{K_{nn}(x)\}) \end{vmatrix}}$$

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$$L\{u_n(x)\} = \frac{\begin{vmatrix} (p^l L\{K_{11}(x)\}) & L\{K_{12}(x)\} & \dots & \left[\begin{matrix} pL\{f_1(x)\} \\ + \left\{\begin{matrix} p^l a_{10} \\ +p^{l-1} a_{11} \\ +\cdots. \\ +pa_{1(l-1)} \end{matrix}\right\} L\{K_{11}(x)\} \end{matrix}\right] \\ L\{K_{21}(x)\} & (p^l L\{K_{22}(x)\}) & \dots. & \left[\begin{matrix} pL\{f_2(x)\} \\ + \left\{\begin{matrix} p^l a_{20} \\ +p^{l-1} a_{21} \\ +\cdots.. \\ +pa_{2(l-1)} \end{matrix}\right\} L\{K_{22}(x)\} \end{matrix}\right] \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \dots\dots\dots\dots\dots.. & \dots\dots\dots.. & \dots\dots\dots\dots\dots.. \\ L\{K_{n1}(x)\} & L\{K_{n2}(x)\} & \dots\dots & \left[\begin{matrix} pL\{f_n(x)\} \\ + \left\{\begin{matrix} p^l a_{n0} \\ +p^{l-1} a_{n1} \\ +\cdots.. \\ +pa_{n(l-1)} \end{matrix}\right\} L\{K_{nn}(x)\} \end{matrix}\right] \end{vmatrix}}{\begin{vmatrix} (p^l L\{K_{11}(x)\}) & L\{K_{12}(x)\} & \dots\dots\dots.. & L\{K_{1n}(x)\} \\ L\{K_{21}(x)\} & (p^l L\{K_{22}(x)\}) & \dots.. & L\{K_{2n}(x)\} \\ \dots\dots\dots\dots & \dots\dots\dots\dots\dots.. & \dots\dots\dots.. & \dots\dots\dots\dots\dots.. \\ L\{K_{n1}(x)\} & L\{K_{n2}(x)\} & \dots\dots\dots.. & (p^l L\{K_{nn}(x)\}) \end{vmatrix}}$$

After simplification of above equations, we have the values of $L\{u_1(x)\}, L\{u_2(x)\}, \dots., L\{u_n(x)\}$. After taking the inverse Laplace-Carson transforms on these values, we get the required values of $u_1(x), u_2(x), \dots., u_n(x)$.

NUMERICAL PROBLEMS: In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following system of convolution type linear Volterra integro-differential equations of first kind

$$\left.\begin{aligned} \int_0^x u_1'(t)dt + \int_0^x u_2(t)dt = 0 \\ -\int_0^x u_1(t)dt + \int_0^x u_2'(t)dt = 0 \end{aligned}\right\} \tag{7}$$

with $u_1(0) = 1, u_2(0) = 0$ (8)

Operating Laplace-Carson transform on system (7) and using convolution theorem of Laplace-Carson transform, we have

$$\left.\begin{aligned} \frac{1}{p}L\{1\}L\{u_1'(x)\} + \frac{1}{p}L\{1\}L\{u_2(x)\} = 0 \\ -\frac{1}{p}L\{1\}L\{u_1(x)\} + \frac{1}{p}L\{1\}L\{u_2'(x)\} = 0 \end{aligned}\right\} \tag{9}$$

Using the property "Laplace-Carson transforms of derivatives" on system (9), we have

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - pu_1(0)] + \frac{1}{p}L\{u_2(x)\} = 0 \\ -\frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - pu_2(0)] = 0 \end{aligned}\right\} \tag{10}$$

Using equation (8) in system (10), we get

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - p] + \frac{1}{p}L\{u_2(x)\} = 0 \\ -\frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - p.0] = 0 \end{aligned}\right\} \tag{11}$$

After simplification system (11), we have

$$\left.\begin{aligned} L\{u_1(x)\} + \frac{1}{p}L\{u_2(x)\} = 1 \\ -\frac{1}{p}L\{u_1(x)\} + L\{u_2(x)\} = 0 \end{aligned}\right\} \tag{12}$$

The solution of system (12) is given by

$$\left.\begin{aligned} L\{u_1(x)\} = \frac{\begin{vmatrix} 1 & \frac{1}{p} \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ -\frac{1}{p} & 1 \end{vmatrix}} = \frac{p^2}{p^2+1} \\ L\{u_2(x)\} = \frac{\begin{vmatrix} 1 & 1 \\ -\frac{1}{p} & 0 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ -\frac{1}{p} & 1 \end{vmatrix}} = \frac{p}{p^2+1} \end{aligned}\right\} \tag{13}$$

Operating inverse Laplace-Carson transforms on system (13), we get the required solution of system (7) with (8) as

$$\left.\begin{aligned} u_1(x) &= L^{-1}\left\{\frac{p^2}{p^2+1}\right\} = cosx \\ u_2(x) &= L^{-1}\left\{\frac{p}{p^2+1}\right\} = sinx \end{aligned}\right\}.$$

Problem: 2 Consider the following system of convolution type linear Volterra integro-differential equations of first kind

$$\left.\begin{aligned} \int_0^x u_1{}'(t)dt + \int_0^x u_2(t)dt = \frac{x^2}{2} \\ -\int_0^x u_1(t)dt + \int_0^x u_2{}'(t)dt = 0 \end{aligned}\right\} \quad (14)$$

with $u_1(0) = 1, u_2(0) = 0$ (15)

Operating Laplace-Carson transform on system (14) and using convolution theorem of Laplace-Carson transform, we have

$$\left.\begin{aligned} \frac{1}{p}L\{1\}L\{u_1{}'(x)\} + \frac{1}{p}L\{1\}L\{u_2(x)\} = \frac{1}{2}L\{x^2\} \\ -\frac{1}{p}L\{1\}L\{u_1(x)\} + \frac{1}{p}L\{1\}L\{u_2{}'(x)\} = 0 \end{aligned}\right\} \quad (16)$$

Using the property "Laplace-Carson transforms of derivatives" on system (16), we have

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - pu_1(0)] + \frac{1}{p}L\{u_2(x)\} = \frac{1}{2}\left(\frac{2}{p^2}\right) \\ -\frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - pu_2(0)] = 0 \end{aligned}\right\} \quad (17)$$

Using equation (15) in system (17), we get

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - p] + \frac{1}{p}L\{u_2(x)\} = \frac{1}{p^2} \\ -\frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - p.0] = 0 \end{aligned}\right\} \quad (18)$$

After simplification system (18), we have

$$\begin{aligned} L\{u_1(x)\} + \frac{1}{p}L\{u_2(x)\} = 1 + \frac{1}{p^2} \\ -\frac{1}{p}L\{u_1(x)\} + L\{u_2(x)\} = 0 \end{aligned} \quad (19)$$

The solution of system (19) is given by

$$\left.\begin{aligned} L\{u_1(x)\} &= \frac{\begin{vmatrix} \left(1+\frac{1}{p^2}\right) & \frac{1}{p} \\ 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ -\frac{1}{p} & 1 \end{vmatrix}} = 1 \\ L\{u_2(x)\} &= \frac{\begin{vmatrix} 1 & \left(1+\frac{1}{p^2}\right) \\ -\frac{1}{p} & 0 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ -\frac{1}{p} & 1 \end{vmatrix}} = \frac{1}{p} \end{aligned}\right\} \quad (20)$$

Operating inverse Laplace-Carson transforms on system (20), we get the required solution of system (14) with (15) as

$$\left.\begin{aligned} u_1(x) &= L^{-1}\{1\} = 1 \\ u_2(x) &= L^{-1}\left\{\frac{1}{p}\right\} = x \end{aligned}\right\}.$$

Problem: 3 Consider the following system of convolution type linear Volterra integro-differential equations of first kind

$$\left.\begin{aligned} \int_0^x u_1{}'(t)dt + \int_0^x u_2(t)dt = 2sinx \\ \int_0^x u_1(t)dt + \int_0^x u_2{}'(t)dt = 0 \end{aligned}\right\} \quad (21)$$

with $u_1(0) = 0, u_2(0) = 1$ (22)

Operating Laplace-Carson transform on system (21) and using convolution theorem of Laplace-Carson transform, we have

$$\left.\begin{aligned} \frac{1}{p}L\{1\}L\{u_1{}'(x)\} + \frac{1}{p}L\{1\}L\{u_2(x)\} = 2L\{sinx\} \\ \frac{1}{p}L\{1\}L\{u_1(x)\} + \frac{1}{p}L\{1\}L\{u_2{}'(x)\} = 0 \end{aligned}\right\} \quad (23)$$

Using the property “Laplace-Carson transforms of derivatives” on system (23), we have

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - pu_1(0)] + \frac{1}{p}L\{u_2(x)\} = 2\left(\frac{p}{p^2+1}\right) \\ \frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - pu_2(0)] = 0 \end{aligned}\right\} \quad (24)$$

Using equation (22) in system (24), we get

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\} - p.0] + \frac{1}{p}L\{u_2(x)\} = 2\left(\frac{p}{p^2+1}\right) \\ \frac{1}{p}L\{u_1(x)\} + \frac{1}{p}[pL\{u_2(x)\} - p] = 0 \end{aligned}\right\} \quad (25)$$

After simplification system (25), we have

$$\left.\begin{array}{r} L\{u_1(x)\}+\frac{1}{p}L\{u_2(x)\}=2\left(\frac{p}{p^2+1}\right) \\ \frac{1}{p}L\{u_1(x)\}+L\{u_2(x)\}=1 \end{array}\right\} \tag{26}$$

The solution of system (26) is given by

$$\left.\begin{array}{l} L\{u_1(x)\}=\dfrac{\begin{vmatrix} \left(\frac{2p}{p^2+1}\right) & \frac{1}{p} \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ \frac{1}{p} & 1 \end{vmatrix}}=\frac{p}{p^2+1} \\ L\{u_2(x)\}=\dfrac{\begin{vmatrix} 1 & \left(\frac{2p}{p^2+1}\right) \\ \frac{1}{p} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{1}{p} \\ \frac{1}{p} & 1 \end{vmatrix}}=\frac{p^2}{p^2+1} \end{array}\right\} \tag{27}$$

Operating inverse Laplace-Carson transforms on system (27), we get the required solution of system (21) with (22) as

$$\left.\begin{array}{l} u_1(x)=L^{-1}\left\{\frac{p}{p^2+1}\right\}=sinx \\ u_2(x)=L^{-1}\left\{\frac{p^2}{p^2+1}\right\}=cosx \end{array}\right\}.$$

Problem: 4 Consider the following system of convolution type linear Volterra integro-differential equations of first kind

$$\left.\begin{array}{r} \int_0^x u_1'(t)dt-\int_0^x u_3(t)dt=0 \\ \int_0^x u_2'(t)dt+\int_0^x u_3(t)dt=0 \\ \int_0^x u_1(t)dt+\int_0^x u_2(t)dt+\int_0^x u_3'(t)dt=0 \end{array}\right\} \tag{28}$$

with $u_1(0)=0, u_2(0)=1, u_3(0)=0$ (29)

Operating Laplace-Carson transform on system (28) and using convolution theorem of Laplace-Carson transform, we have

$$\left.\begin{array}{r} \frac{1}{p}L\{1\}L\{u_1'(x)\}-\frac{1}{p}L\{1\}L\{u_3(x)\}=0 \\ \frac{1}{p}L\{1\}L\{u_2'(x)\}+\frac{1}{p}L\{1\}L\{u_3(x)\}=0 \\ \frac{1}{p}L\{1\}L\{u_1(x)\}+\frac{1}{p}L\{1\}L\{u_2(x)\}+\frac{1}{p}L\{1\}L\{u_3'(x)\}=0 \end{array}\right\}$$

Using the property “Laplace-Carson transforms of derivatives” on above system, we have

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\}-pu_1(0)]-\frac{1}{p}L\{u_3(x)\}=0 \\ \frac{1}{p}[pL\{u_2(x)\}-pu_2(0)]+\frac{1}{p}L\{u_3(x)\}=0 \\ \frac{1}{p}L\{u_1(x)\}+\frac{1}{p}L\{u_2(x)\}+\frac{1}{p}[pL\{u_3(x)\}-pu_3(0)]=0 \end{aligned}\right\} \quad (30)$$

Using equation (29) in system (30), we get

$$\left.\begin{aligned} \frac{1}{p}[pL\{u_1(x)\}-p.0]-\frac{1}{p}L\{u_3(x)\}=0 \\ \frac{1}{p}[pL\{u_2(x)\}-p]+\frac{1}{p}L\{u_3(x)\}=0 \\ \frac{1}{p}L\{u_1(x)\}+\frac{1}{p}L\{u_2(x)\}+\frac{1}{p}[pL\{u_3(x)\}-p.0]=0 \end{aligned}\right\} \quad (31)$$

After simplification system (31), we have

$$\left.\begin{aligned} L\{u_1(x)\}-\frac{1}{p}L\{u_3(x)\}=0 \\ L\{u_2(x)\}+\frac{1}{p}L\{u_3(x)\}=1 \\ \frac{1}{p}L\{u_1(x)\}+\frac{1}{p}L\{u_2(x)\}+L\{u_3(x)\}=0 \end{aligned}\right\} \quad (32)$$

The solution of system (32) is given by

$$L\{u_1(x)\}=\frac{\begin{vmatrix} 0 & 0 & -\frac{1}{p} \\ 1 & 1 & \frac{1}{p} \\ 0 & \frac{1}{p} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -\frac{1}{p} \\ 0 & 1 & \frac{1}{p} \\ \frac{1}{p} & \frac{1}{p} & 1 \end{vmatrix}}=-\frac{1}{p^2} \quad (33)$$

$$L\{u_2(x)\}=\frac{\begin{vmatrix} 1 & 0 & -\frac{1}{p} \\ 0 & 1 & \frac{1}{p} \\ \frac{1}{p} & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -\frac{1}{p} \\ 0 & 1 & \frac{1}{p} \\ \frac{1}{p} & \frac{1}{p} & 1 \end{vmatrix}}=1+\frac{1}{p^2} \quad (34)$$

$$L\{u_3(x)\} = \frac{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \frac{1}{p} & \frac{1}{p} & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -\frac{1}{p} \\ 0 & 1 & \frac{1}{p} \\ \frac{1}{p} & \frac{1}{p} & 1 \end{vmatrix}} = -\frac{1}{p} \tag{35}$$

Operating inverse Laplace-Carson transforms on equations (33), (34) and (35), we get the required solution of system (28) with (29) as

$$\left.\begin{aligned} u_1(x) &= -L^{-1}\left\{\frac{1}{p^2}\right\} = -\frac{x^2}{2} \\ u_2(x) &= L^{-1}\left\{1+\frac{1}{p^2}\right\} = L^{-1}\{1\} + L^{-1}\left\{\frac{1}{p^2}\right\} = 1+\frac{x^2}{2} \\ u_3(x) &= -L^{-1}\left\{\frac{1}{p}\right\} = -x \end{aligned}\right\}.$$

CONCLUSIONS: In this paper, authors successfully determined the primitive of system of convolution type linear Volterra integro-differential equations of first kind by using Laplace-Carson transform and complete methodology is explained by considering four numerical problems. The results of numerical problems show that the Laplace-Carson transform is very effective and useful integral transform for determining the primitive of system of convolution type linear Volterra integro-differential equations of first kind.

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