More Results on Laplacian Matrix of Power 3 Mean Graphs

Sarasree.S[1], Dr. S.S. Sandhya[2]
Author1 : Research Scholar, Reg. No.:20112093182002
Sree Ayyappa College for Women, Chunkankadai.
Author 2 : Assistant Professor, Department of Mathematics,
Sree Ayyappa College for Women, Chunkankadai.
[Affiliated to Manonmaniam Sundararanar University,
Abishekapatti – Tirunelveli - 627012, Tamilnadu, India]

Abstract:

In this paper, We investigate the Laplacian matrix for some standard graphs such as Triangular Balloon graph, $P_n \odot K_{1,3}$, $P_n \odot K_3$,

AMS Subject Classification: 05C78

Keywords: Power 3 Mean Graph, Triangular Balloon graph, $P_n \odot K_{1,3}$, $P_n \odot K_3$,

1. Introduction :

In the mathematical field of graph theory, the Laplacian matrix is also called the graph Laplacian. The Laplacian matrix can be used to find many useful properties of a graph. For all detailed survey of graph Labelling we refer J.A. Gallian [2] and Harary [1]. The concept of Harmonic Mean labelling was introduced by S. Somasundram and S.S. Sandhya and further S.S. Sandhya and S. Sreeji introduced Power 3 mean labelling of graphs. Motivated by the work done by several authors we introduce a new concept called Laplacian Matrix of Power 3 Mean Graphs.

Definition 1.1

Let G be a power-3 mean graph. The Laplacian matrix L_G is given by L(G) = D(G) – A(G). Where D(G) is the Diagonal matrix of the vertex degrees and A(G) is the Adjacency matrix.

Remark 1.2

In this Laplacian matrix of Power 3 mean graphs there is no column and row with all elements are zero.

Remark 1.3

Laplacian matrix of power 3 mean graphs has no isolated vertex.

2. Main Results

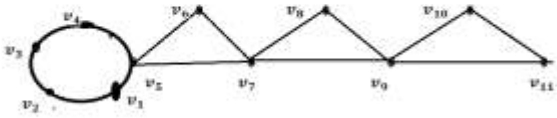
Theorem 2.1

Laplacian matrix of G = T_mB_n is $L(T_mB_m) = D(T_mB_n) - A(T_mB_n)$

Proof

Let G = T_mB_n be a Power 3 Mean graph

Now we consider for $G = T_mB_n$ with 11 vertices.



$$D(T_{11}B_{11}) = \begin{bmatrix} 2&0&0&0&0&0&0&0&0&0&0\\ 0&2&0&0&0&0&0&0&0&0&0\\ 0&0&2&0&0&0&0&0&0&0&0\\ 0&0&0&2&0&0&0&0&0&0&0\\ 0&0&0&0&4&0&0&0&0&0&0\\ 0&0&0&0&0&2&0&0&0&0&0\\ 0&0&0&0&0&0&4&0&0&0&0\\ 0&0&0&0&0&0&0&2&0&0&0\\ 0&0&0&0&0&0&0&0&4&0&0\\ 0&0&0&0&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&0&0&0&2 \end{bmatrix}$$

$$A(T_{11}B_{11}) = \begin{bmatrix} 0&1&0&0&1&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0&0\\ 1&0&0&1&0&1&1&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0&0\\ 0&0&0&0&1&1&0&1&1&0&0\\ 0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&1&1&0&1&1\\ 0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&1&0 \end{bmatrix}$$

Laplacian matrix

$$L(T_nB_n) = D(T_nB_n) - A(T_nB_n)$$

$L(T_{11}B_{11}) = D(T_{11}B_{11}) - A(T_{11}B_{11})$

$$= \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 4 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

Now we can Prove n Vertices

$$L(T_{11}B_{11}) = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 4 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

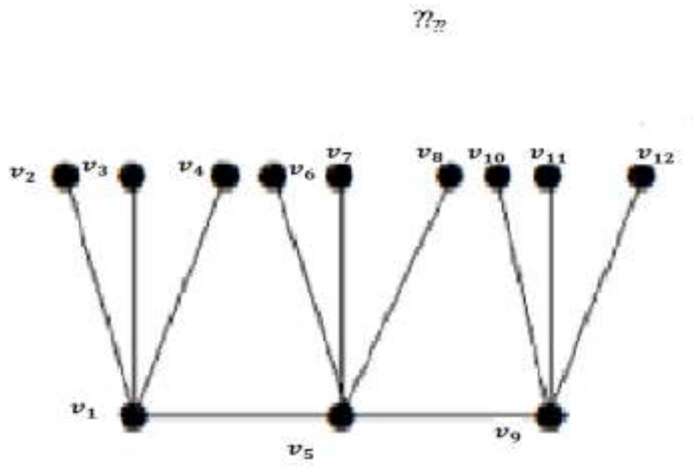
Theorem 2.2

Laplacian matrix of G = $P_n \odot K_{1,3}$ is $L(P_n \odot K_{1,3}) = D(P_n \odot K_{1,3}) - A(P_n \odot K_{1,3})$

Proof

Let G = $P_n \odot K_{1,3}$ be a Power 3 Mean graph.

Now we consider for G = $P_n \odot K_{1,3}$ with 12 vertices.



$$D(P_3 \odot K_{1,3}) = \begin{bmatrix} 4&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&5&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&4&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$A(P_3 \odot K_{1,3}) = \begin{bmatrix} 0&1&1&1&1&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&1&1&1&1&0&0\\ 0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&1&0&1&1&0&0\\ 0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&1&1&0&1&1\\ 0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&1&1&0 \end{bmatrix}$$

Laplacian Matrix

$$L(P_3 \odot K_{1,3}) = D(P_3 \odot K_{1,3}) - A(P_3 \odot K_{1,3})$$

$$= \begin{bmatrix} 4 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 5 & -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Similarly we can prove n vertices

$$L(P_n \odot K_{1,3}) =$$

$$\begin{bmatrix} 4 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 5 & -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

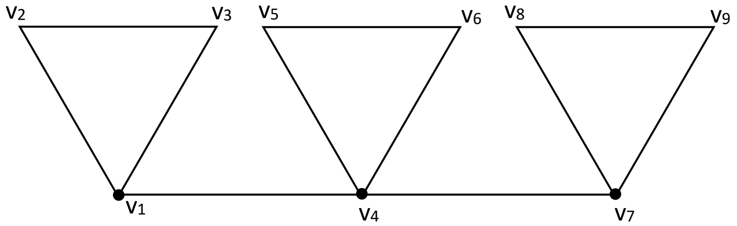
Theorem 2.3

Laplacian matrix of G $= P_n \odot K_3$ is $L(P_n \odot K_3) = D(P_n \odot K_3) - A(P_n \odot K_3)$

Proof

Let G = $P_n \odot K_3$ be a Power 3 Mean graph

Now we consider for G = with 9 vertices



$$D(P_3 \odot K_3) = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$A(P_3 \odot K_3) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Laplacian matrix of G

$L(P_n \odot K_3) = D(P_n \odot K_3) - A(P_n \odot K_3)$

$$= \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

Similarly in general we can prove for n vertices

$$L(P_n \odot K_3) = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

3. Conclusion

The study of Laplacian matrix can be used in many machine learning applications. It is very interesting to derive Laplacian matrix representation of power-3 mean graphs. In this paper we investigated the Laplacian matrix for Triangular Balloon graph $P_n \odot K_{1,3}, P_n \odot K_3,$ which are power-3 mean graphs.

Acknowledgement

The authors are thankful to reference for their valuable comments and suggestions.

References

[1]. F. Harary, *Graph Theory*, Narosa publishing House, New Delhi, 1988.

[2]. J.A. Gallian, A Dynamic Survey of Graph Labelling, *The Electronic Journal of Computations.*

[3] Sarasree.S and S.S.Sandhya “Power 3 Mean Labeling of Cycle Related Graphs”, International Journal of analytical and experimental model Analysis, Vol. XIII,Issue :IIFebruary -2021ISSN NO:0886-9367.

[4] Sarasree.S and S.S.Sandhya Wiener Index of Power 3 Mean Graphs ,communicated to Journal of Physics .

[5] Sarasree.S and S.S.Sandhya Laplacian Matrix of Power 3 mean Graphs Communicated to Materials Today Proceedings, Elsevier