More Results on Laplacian Matrix of Power 3 Mean Graphs

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Abstract:

In this paper, We investigate the Laplacian matrix for some standard graphs such as Triangular Balloon graph, $P_n \odot K_{1,3}$, $P_n \odot K_3$,

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Keywords: Power 3 Mean Graph, Triangular Balloon graph, $P_n \odot K_{1,3}$, $P_n \odot K_3$,

1. Introduction :

In the mathematical field of graph theory, the Laplacian matrix is also called the graph Laplacian. The Laplacian matrix can be used to find many useful properties of a graph. For all detailed survey of graph Labelling we refer J.A. Gallian [2] and Harary [1]. The concept of Harmonic Mean labelling was introduced by S. Somasundram and S.S. Sandhya and further S.S. Sandhya and S. Sreeji introduced Power 3 mean labelling of graphs. Motivated by the work done by several authors we introduce a new concept called Laplacian Matrix of Power 3 Mean Graphs.

Definition 1.1

Let G be a power-3 mean graph. The Laplacian matrix L_G is given by L(G) = D(G) - A(G). Where D(G) is the Diagonal matrix of the vertex degrees and A(G) is the Adjacency matrix.

Remark 1.2

In this Laplacian matrix of Power 3 mean graphs there is no column and row with all elements are zero.

Remark 1.3

Laplacian matrix of power 3 mean graphs has no isolated vertex.

2. Main Results

Theorem 2.1

Laplacian matrix of $G = T_m B_n$ is $L(T_m B_m) = D(T_m B_n) - A(T_m B_n)$

Proof

Let $G = T_m B_n$ be a Power 3 Mean graph

Now we consider for $G = T_m B_n$ with 11 vertices.



								11		-			
		г2	0	0	0	0	0	0	0	0	0	01	
			2	0	0	0	0	0	0	0	0	0	
		lõ	0	2	0	0	Ŭ.	0	0	0	0	ŏ	
		0	0	0	2	0	0	0	0	0	0	0	
		0	0	0	0	4	0	0	0	0	0	0	
$D(T_{11}B_{11})$	=	0	0	0	0	0	2	0	0	0	0	0	4 1
		0	0	0	0	0	0	4	0	0	0	0	54 V
		0	0	0	0	0	0	0	2	0	0	0	\mathcal{L}
		0	0	0	0	0	0	0	0	4	0	0	
		0	0	0	0	0	0	0	0	0	2	0	
		LO	0	0	0	0	0	0	0	0	0	2	
					100								
		гO	1	0	0	1	0	0	0	0	0	01	
			0	1	0	0	0	0	0	0	0	0	
			1	0	1	0	0	0	0	0	0		
			0	1	0	1	0	0	0	0	0		
			0	0	1	0	1	1	0	0	0	0	
$A(T_{11}B_{11})$	=		0	0	0	1	0	1	0	0	0	ő	
	_	lõ	0	0	0	1	1	0	1	1	0	ŏ	
		l	0	0	0	0	0	1	0	1	0	ŏ	
		lõ	0	0	0	0	0	1	1	0	1	1	
		0	0	0	0	0	0	0	0	1	0	1	
		L	0	0	0	0	0	0	0	1	1	<u>[</u>]	

Laplacian matrix

 $L(T_nB_n) = D(T_nB_n) - A(T_nB_n)$

 $L(T_{11}B_{11}) = D(T_{11}B_{11}) - A(T_{11}B_{11})$

	<u>2</u>	-1	0	0	-1	0	0	0	0	0	ך 0
	-1	2	-1	0	0	0	0	0	0	0	0
	0	-1	2	-1	0	0	0	0	0	0	0
	0	0	-1	2	-1	0	0	0	0	0	0
	-1	0	0	-1	4	-1	-1	0	0	0	0
=	0	0	0	0	-1	2	-1	0	0	0	0
	0	0	0	0	-1	-1	4	-1	-1	0	0
	0	0	0	0	0	0	-1	2	-1	0	0
	0	0	0	0	0	0	-1	-1	4	-1	-1
	0	0	0	0	0	0	0	0	-1	2	-1
	LΟ	0	0	0	0	0	0	0	-1	-1	2

Now we can Prove n Vertices

		<u>۲</u> 2	-1	0	0	-1	0	0	0	0	0	ך 0
	-1	2	-1	0	0	0	0	0	0	0	0	
	-1	2	-1	0	0	0	0	0	0	0	0	
	0	-1	2	-1	0	0	0	0	0	0	0	
		-1	0	0	-1	4	-1	-1	0	0	0	0
		0	0	0	0	-1	2	-1	0	0	0	0
$L(I_{11}B_{11})$	=	0	0	0	0	-1	-1	4	-1	-1	0	0
		0	0	0	0	0	0	-1	2	-1	0	0
		0	0	0	0	0	0	-1	-1	4	-1	-1
		:	: 1		ć : -	:	:	:	No	:	:	- :
	0	0	0	0	0	0	0	0	-1	2	-1	
		Lo	0	0	0	0	0	0	0	-1	-1	2

Theorem 2.2

Laplacian matrix of $G = P_n \odot K_{1,3}$ is $L(P_n \odot K_{1,3}) = D(P_n \odot K_{1,3}) - A(P_n \odot K_{1,3})$

Proof

Let $G = P_n \odot K_{1,3}$ be a Power 3 Mean graph.

Now we consider for $G = P_n \odot K_{1,3}$ with 12 vertices.





Laplacian Matrix

$$L(P_3 \odot K_{1,3}) = D(P_3 \odot K_{1,3}) - A(P_3 \odot K_{1,3})$$

-	_	_	
-	_	_	

	- 4	-1	-1	-1	-1	0	0	0	0	0	ן 0
	-1	1	0	0	0	0	0	0	0	0	0
	-1	0	1	0	0	0	0	0	0	0	0
	-1	0	0	1	0	0	0	0	0	0	0
	-1	0	0	0	5	-1	-1	-1	-1	0	0
=	0	0	0	0	-1	1	0	0	0	0	0
	0	0	0	0	-1	0	1	0	0	0	0
	0	0	0	0	-1	0	0	1	1	0	0
	0	0	0	0	-1	0	0	4	-1	-1	-1
	0	0	0	0	0	0	0	-1	1	0	1
I	0	0	0	0	0	0	0	-1	0	0	-1
Similarl	y we a	can pro	ve n	vertice	s 🔍						
		$L(P_n$	⊙ <i>K</i> _{1,3}) =		. 11	6		ds.		
1	- 4	-1	-1	-1	-1	0	0	0	0	0	ן 0
	- 4 1	-1 1	$^{-1}_{0}$	$-1 \\ 0$	-1 0	0 0	0 0	0 0	0 0	0 0	0 0
	- 4 1 1	-1 1 1	-1 0 0	$-1 \\ 0 \\ 0$	-1 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	- 4 1 1	-1 1 1 0	-1 0 0 1	$-1 \\ 0 \\ 0 \\ 0 \\ 0$	-1 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0
	- 4 1 1 -1 -1	-1 1 1 0 0	-1 0 0 1 0	$-1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	-1 0 0 5	0 0 0 0 -1	0 0 0 0 -1	0 0 0 0 -1	0 0 0 0 -1	0 0 0 0 0	0 0 0 0
	4 -1 -1 -1 -1 0	$-1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0$	$-1\\0\\1\\0\\0$	$-1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	-1 0 0 5 -1	0 0 0 0 -1 1	0 0 0 -1 0	0 0 0 0 -1 0	0 0 0 -1 0	0 0 0 0 0 0	0 0 0 0 0 0
	4 -1 -1 -1 -1 0 0	$-1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ $	$-1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0$	$ \begin{array}{c} -1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \end{array} $	0 0 0 -1 1 0	0 0 0 -1 0 1	0 0 0 -1 0 0	0 0 0 -1 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0
		$ \begin{array}{c} -1 \\ 1 \\ 0 \\ $	$-1\\0\\1\\0\\0\\0\\0\\0$	$ \begin{array}{c} -1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \\ -1 \\ -1 \end{array} $	0 0 0 -1 1 0 0	0 0 0 -1 0 1 0	0 0 0 -1 0 0 1	0 0 0 -1 0 0 0	0 0 0 0 0 0 0 0	0 0 0 0 0 0 0
	4 -1 -1 -1 0 0 0 :	$ \begin{array}{c} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{array} $	-1 0 1 0 0 0 0 :	-1 0 0 0 0 0 0 0 0 0	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \\ -1 \\ \vdots \end{array} $	0 0 0 -1 1 0 0 :	0 0 0 -1 0 1 0 :	0 0 0 -1 0 0 1 :	0 0 0 -1 0 0 0 :	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0
		$ \begin{array}{c} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{array} $	-1 0 1 0 0 0 0 : 0	$ \begin{array}{c} -1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \\ -1 \\ \vdots \\ -1 \end{array} $	0 0 0 -1 1 0 0 : 0	0 0 0 -1 0 1 0 : 0	0 0 0 -1 0 0 1 : 4	0 0 0 -1 0 0 0 : -1	0 0 0 0 0 0 0 0 : -1	0 0 0 0 0 0 0 0 0 : -1
	$ \begin{array}{c} 4 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} -1 \\ 1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \\ -1 \\ \cdots \\ -1 \\ 0 \\ \end{array} $	0 0 0 -1 1 0 0 : 0	0 0 0 -1 0 1 0 : 0 0	0 0 0 -1 0 0 1 : 4 -1	0 0 0 -1 0 0 : -1 1	0 0 0 0 0 0 0 0 : -1 0	0 0 0 0 0 0 0 0 : -1 0
	$ \begin{array}{c} 4 \\ -1 \\ -1 \\ -1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} $	$ \begin{array}{c} -1 \\ 1 \\ 0 \\ $	$ \begin{array}{r} -1 \\ 0 \\ 1 \\ 0 \\ $	-1 0 0 0 0 0	$ \begin{array}{r} -1 \\ 0 \\ 0 \\ 0 \\ 5 \\ -1 \\ -1 \\ -1 \\ \cdots \\ -1 \\ 0 \\ 0 \end{array} $	0 0 0 -1 1 0 0 : 0 0 0	0 0 0 -1 0 1 0 : 0 0 0	$ \begin{array}{c} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \\ \vdots \\ 4 \\ -1 \\ -1 \end{array} $	0 0 0 -1 0 0 0 : -1 1 0	0 0 0 0 0 0 0 0 : -1 0 1	0 0 0 0 0 0 0 0 : -1 0 0

Theorem 2.3

Laplacian matrix of G = $P_n \odot K_3$ is L($P_n \odot K_3$) = D($P_n \odot K_3$) – A($P_n \odot K_3$)

Proof

Let $G = P_n \odot K_3$ be a Power 3 Mean graph

Now we consider for G = with 9 vertices



Laplacian matrix of G

 $L(P_n \odot K_3) = D(P_n \odot K_3) - A(P_n \odot K_3)$

Г З	-1	-1	-1	0	0	0	0	ן 0
-1	2	-1	0	0	0	0	0	0
-1	-1	2	0	0	0	0	0	0
-1	0	0	4	-1	-1	-1	0	0
0	0	0	-1	2	-1	0	0	0
0	0	0	-1	-1	2	0	0	0
0	0	0	-1	0	0	3	-1	-1
0	0	0	0	0	0	-1	2	-1
Γ0	0	0	0	0	0	-1	-1	2
	$\begin{bmatrix} 3 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 \\ -1 & 2 \\ -1 & -1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 &$	$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 4 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0$	$\begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 &$	$\begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

Similarly in general we can prove for n vertices

		г 3	-1	-1	-1	0	0	0	0	ך 0
$I(D \cap K)$		-1	2	-1	0	0	0	0	0	0
		-1	-1	2	0	0	0	0	0	0
		-1	0	0	4	-1	-1	-1	0	0
	_	0	0	0	-1	2	-1	0	0	0
$L(r_n O K_3)$	_	0	0	0	-1	-1	2	0	0	0
		1	:	:	:	:	:	:	:	:
		0	0	0	-1	0	0	3	-1	-1
		0	0	0	0	0	0	-1	2	-1
		LO	0	0	0	0	0	-1	-1	2

3. Conclusion

The study of Laplacian matrix can be used in many machine learning applications. It is very interesting to derive Laplacian matrix representation of power-3 mean graphs. In this paper we investigated the Laplacian matrix for Triangular Balloon graph $P_n \odot K_{1,3}$, $P_n \odot K_3$, which are power-3 mean graphs.

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