Weiner Index of Some Standard Power 3 Mean Graphs

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Abstract:

The Wiener index W(G) of a connected graph G is the sum of distances of all pairs of vertices of G. In this paper we investigate the Wiener index for Hurdle graph and H-Graph .

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1. Introduction

In chemical graph theory the Wiener index (also Wiener number) was introduced by Harry Wiener. The Wiener index of a graph G = (V, E) denoted by W(G), was introduced in 1947 by chemist Harold Wiener. Wiener Index is the sum of distances between all the vertices of $G: W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u,v)$.

Definition 1.1

Let G be a power-3-mean graph. The Wiener Index W(G) of G is defined by W(G) of G is defined by

$$W(G) = \sum_{\{u,v\} \subseteq v(G)} d_G(u,v)$$

Remark 1.2

In any graph of power-3-mean graph the vertex and edge must be labelled “1”.

Remark 1.3

In any power-3-mean graph every vertex must degree ≤ 3.

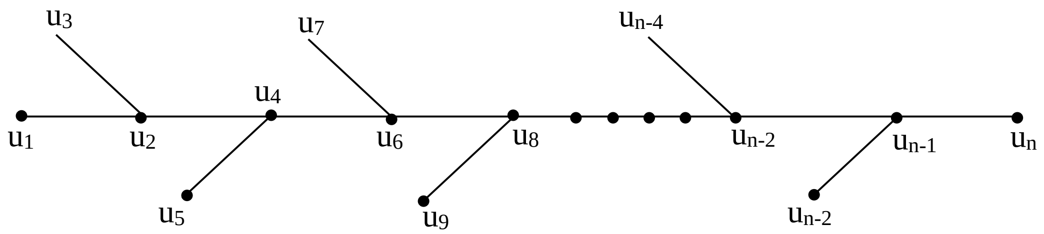
2. Main Results

Theorem 2.1

Let G = Hd_n be a Power 3 Mean graph, then the Wiener index of Hurdle graph Hd_n is $W(d_n) = (2n – 3) \times 1 + (3n – 6) \times 2 + (4n – 12) \times 3 \ldots. + 4(n – 1)$ Average distance is $\mu(\text{Hdn}) = \frac{W(\text{Hdn})}{\frac{|V(\text{Hdn})|}{2}}$

Proof

Let G = Hd_n be a Power 3 Mean graph



$$W(G)=\sum_{u,v\in V(G)} d_G(u,v)$$

= d(u_1, u_2) + d(u_1u_3) + d(u_1, u_4) … d(u_1u_n) + d(u_2, u_3) + d(u_2, u_4) + … + d(u_2, u_n) + … + d(u_{n-2}, u_{n-1}) + d(u_{n-2}, u_n) + d(u_{n-1}, u_n)

= (1 + 2 + 2 + 3 + 3 + … + (n – 2) + (n – 2) + (n – 1) + (n – 1) + (1 + 1 + 2 + 2 + 3 + 3 + … + (n – 3) + (n – 3) + (n – 2) + (n – 2) + (2 + 3 + 3 + … + (n – 2) + (n – 2) + (n – 1) + (n – 2) + (1 + 1 + 2 + 2 + …. + (n – 3) + (n – 3) + (2 + 3 + 3+ … + (n – 2) + (n – 2) + … + (1 + 1 + 2 + 2) + (2 + 3 + 3) + (1 + 1) + (2)

= (2n – 3) × 1 + (3n – 6) × 2 + (4n – 1) × 3 + … + 4(n – 1)

∴ W(G) = (2n – 3) × 1(3n – 6) × 2 + (4n – 12) × 3 + … + 4(n – 1)

Example 2.2

Wiener index of Hurdle graph Hd_6 is shown below

$$W(G)=\sum_{u,v\in V(G)} d_G(u,v)$$

= (1 + 2 + 2 + 3 + 4 + 4 + 5 + 5) + (1 + 1 + 2 + 2 + 3 + 3 +4 + 4) + (2 + 3 + 3+ + 4 + 4 + 5 + 5) + (1 + 1 + 2 + 2 + 3 + 3) + (2 + 3 + 3 + 4 + 4) + (1 + 1 + 2 + 2) + (2 + 3 + 3) + (1 + 1) + 2

= 9 × 1 + (12 × 2) + (12 × 3) + (8 × 4) + (4 × 5)

Average distance is

$$\mu(\text{Hdn}) = \frac{W(Hd_6)}{\frac{|V(Hd_6)|}{2}}$$

$$W(G) = 121$$

$$\mu(Hd_6) = \frac{W(Hd_6)}{\frac{|V(Hd_6)|}{2}}$$

$$= \frac{121}{55} \simeq 2.2$$

Theorem 2.3

Let H be a Power 3 Mean graph. Then, the Wiener index of H graph is

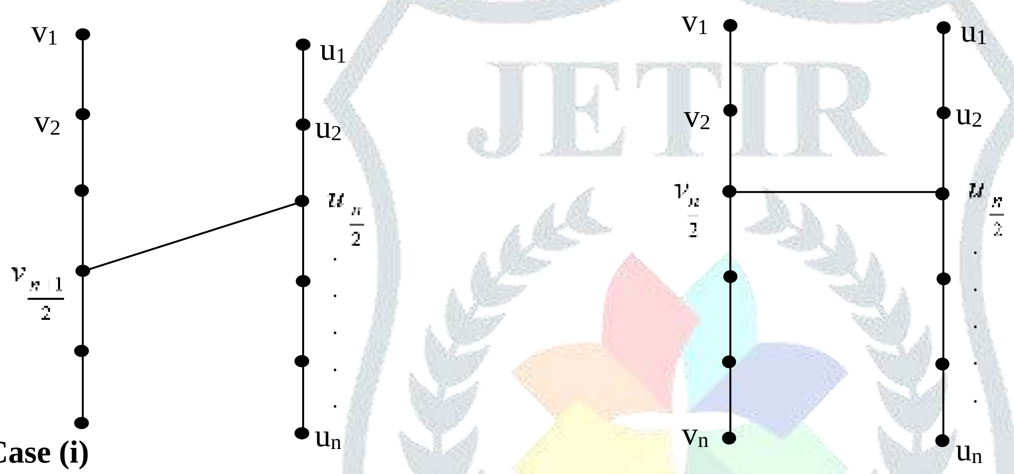
$$W(G) = \begin{cases} (2n-1)\times 1 + (2n\times 2) + (2n+2)\times 3 + (2n+4)\times 4 + \ldots + 4n & \text{if } n \text{ is odd} \\ (2n-1)\times 1 + (2n\times 2) + (2n+2)\times 3 + (2n+4)\times 4 + (2n-2)\times \\ 5 + \ldots + 1\times(n-1) & \text{if } n \text{ is even} \end{cases}$$

and Average distance

$$\mu(H) = \frac{W(\mathrm{H})}{\frac{|V(H)|}{2}}$$

Proof

Let H be a Power 3 Mean graph.



Case (i)
***n* is odd**

$$W(G) = \sum_{u,v\in V(G)} d_G(u,v)$$

$= d(u_1, u_2) + d(u_1, u_3) + \ldots + d(u_1, u_n) + d(u_2, u_3) + d(u_2, u_4) + \ldots + d(u_2, u_n) + \ldots + d(u_{n-2}, u_{n-1}) + d(u_{n-2}, u_n) + d(u_{n-1}, u_n)$

= (1 + 2 + 3 + … + n + (n – 1) + … (n – (n – 3) + … + (n – 1) + n + (n + 1) + (1 + 2 + 3 + … + (n – 1) + (n – 2) + … (n – (n – 4) + … + (n – 1) + n) + (1 + 2 + 3 + … + (n – 2) + (n – 3) + … + (n – (n – 5) + … + (n – 2) + (n – 1) + … + (n – 2) + (n – 1) + … + (1 + 2 + 3) + (1 + 2) + 1

= (2n – 1) × 1 + 2n × 2 + (2n + 2) × 3 + (2n + 4) × 4 + … + 4n

$\therefore n \text{ is odd}$

∴ W(G) = (2n – 1) × 1 + (2n × 2) + (2n + 2) × 3 + (2n + 4) × 4 + … + 4n if n is odd

Average distance is

$$\mu(H) = \frac{W(H)}{\frac{|V(H)|}{2}} \quad \therefore n \text{ is odd}$$

$$\mu(G) = \frac{((2n-1)\times 1 + 2n \times 2 + (2n+2)\times 3) + (2n+4)\times 4 + \ldots + 4n}{\frac{|V(G)|}{2}}$$

Case (ii)
n is even

$$W(G) = \sum_{u,v \in V(G)} d_G(u,v)$$

$= d(u_1u_n) + d(u_1u_3) + \ldots + d(u_1u_n) + d(u_2, u_3) + d(u_2, u_4) + \ldots + d(u_2, u_n) + \ldots + d(u_{n-2}, u_{n-1}) + d(u_{n-2}, u_n) + d(u_{n-1}, u_n)$

$= (1 + 2 + 3 + \ldots + n + (n-1) + \ldots + (n-(n-3) + \ldots + (n-1) + n + (n+1)) + (1 + 2 + 3 + \ldots + (n-1) + (n-2) + \ldots + (n-(n-4) + \ldots + (n-1) + n) + (1 + 2 + 3 + \ldots + (n-2) + (n-3) + \ldots + (n-(n-5) + \ldots + (n-2) + (n-1) + \ldots + (1 + 2 + 3) + (1 + 2) + 1$

$= (2n-1) \times 1 + (2n \times 2) \times 3 + (2n+4) \times 4 + (2n-2) \times 5 + \ldots + 1 \times (n+1)$

W(G) $= (2n-1) \times 1 + 2n \times 2 + (2n+2) \times 3 + (2n+4) \times 4 + (2n-2) \times 5 + \ldots + 1 \times (n+1)$ n is even

Average distance is

$$\mu(H) = \frac{W(H)}{\frac{|V(H)|}{2}}$$

$$\mu(G) = \frac{(2n-1)\times 1 + 2n \times 2 + (2n+2)\times 3 + (2n+4)\times 4 + (2n-2)\times 5 + \ldots + 1 \times (n+1)}{\frac{|V(H)|}{2}}$$

$$\therefore n \text{ is even}$$

Example 2.4

Wiener index of Power 3 Mean graph of H_6 and H_7 are

$$W(G) = W(H_6) = \sum_{u,v \in V(G)} d_G(u,v)$$

$= (1 + 2 + 3 + 4 + 3 + 2 + 1) + (1 + 2 + 3 + 4 + 3 + 2) + (1 + 2 + 3 + 4 + 3) + (1 + 2 + 3 + 4) + (1 + 2 + 3) + (1 + 2) + 1$

$= (8 \times 1) + (8 \times 2) + (8 \times 3) + (4 \times 4)$

Average distance is

$$\mu(H_6) = \frac{W(H_6)}{\binom{|V(H_6)|}{2}}$$

$$= \frac{64}{28} \simeq 2.2$$

$$W(G) = W(H_7) = \sum_{u,v \in V(G)} d_G(u,v)$$

$= (1 + 2 + 3 + 4 + 4 + 3 + 2 + 1) + (1 + 2 + 3 + 4 + 4 + 3 + 2) + (1 + 2 + 3 + 4 + 4 + 3) + (1 + 2 + 3 + 4 + 4) + (1 + 2 + 3) + (1 + 2) + 1$

$= (9 \times 1) + (9 \times 2) + (9 \times 3) + (9 \times 4)$

Average distance is

$$\mu(H_7) = \frac{W(H_7)}{\binom{|V(H_7)|}{2}}$$

$$= \frac{90}{36}$$

$$\simeq 2.5$$

Conclusion

Wiener index is used to find the sum of the lengths of the shortest paths between all pairs of vertices. It is also used to find the chemical bonding and non-hydrogen atoms of the molecule.

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