Kinematics, Dynamics, and Control of a 3-RPS Robot

A Literature Survey

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Abstract: Parallel Manipulators are one of the trending re- search topics in robotics. Due to its high structural stiffness and positional accuracy, their applications in industries are rapidly increasing. An important parallel manipulator is the 3-RPS manipulator introduced by Hunt. Since there has been enormous research carried out in the study characteristics of this manipulator. In recent years, many novel applications were proposed for 3-RPS. The understanding of kinematics, dynamics, and control methodologies lays a foundation of research and development. In recent years, this manipulator has been used for many novel applications.

***Index Terms* – 3-RPS, applications, control, dynamics, kinematics, parallel manipulator.**

I. Introduction

The main application of the Industrial Robot is manipulation and usually, the links of such robots are connected in series. This type of robots is known as anthropomorphic open chain mechanisms or Serial Manipulators. Due to the open chain these robots have substantial workspace and reach [12]. However, they have poor rigidity and payload to weight ratio. In the late 1980's, a new category of manipulators was invented called as Parallel Manipulators. These were initially used as tire-testing machines and flight simulators [15]. They were able to fulfil the needs of automation industry. The closed loop structures of these robots gave high structural stiffness and enabled them to perform high speed motions. In addition, these mechanisms also reduced the positioning and orienting errors. However, there is a compromise of small workspace and a low ratio of working envelope and size with these mechanisms [18]. In the early development years, researchers focused on developing mechanisms with a higher degree of freedom. Therefore, 6-degree of freedom (DOF) spatial parallel manipulators were developed. The best example is the Gough Stewart platform. Although, in some cases, not all 6-DOF are needed. For instance, only five degrees of freedom are required to move a cutting tool to a point with any desired directions, if the cutting tool is symmetric [13]. This opens an entirely new field of research. The 3-RPS is one of the prominent mechanisms in this field. The parallel 3RPS mechanism was first introduced by Hunt [38]. The simplified mechanism is proposed for holding the spindle on a milling machine design, and as a robotic wrist by Waldron [39]. A 3-RPS manipulator is a 3-DOF parallel manipulator. It has a base platform and a similar moving platform which are connected by three identical legs. A leg consists of 3 joints- Rotary(R), Prismatic(P), Spherical(S)-out of which R and S are passive joints and P is the active joint. The rotary joint is connected to the base whereas the spherical joint is connected to the moving platform [40]. In recent years, this manipulator is used in diverse applications. Some prominent examples include the ankle rehabilitation device [33] and the biomimetic inspired snake robot [32].

II. Methodology of Survey

The goal of this paper is to provide a holistic view highlighting the importance aspects and related work of this manipulator. This is done by reviewing and classifying all the existing literature of the 3-RPS parallel manipulator. This literature survey was done from the point when the first 3-RPS parallel manipulator was proposed by Hunt [38]. All the major works were surveyed and classified in different categories for the ease of the reader. The main categories are:

- Kinematic Analysis
- Dynamic Analysis
- Control Algorithms
- Applications

III. Literature Survey

3.1 Kinematics

Kinematics is defined as the study of the robot's geometry and position without considering the effects of force and moments. It is divided into 2 major types forward and inverse kinematics. Forward Kinematic refers to the calculation of robot end effector's position based on the joint variable input. Inverse kinematic is opposite where joint variables are calculated from the robot's end effector position. There has been an intensive research on the kinematic and various methodologies to get an optimal algorithm for kinematics of 3-RPS manipulator. It can be summarized as:

1. **Lee and Shah (1988)** - In this paper the analytical method to analyse basic kinematic of 3-RPS manipulator are developed. In addition, the systematic design procedures and work envelope is described [12].
2. **Sung Kim and Tsai (2003)** - Dimensional Synthesis of 3-RPS manipulator is done. It is proofed that maximum six orientations and positions can be prescribed at will. From these six positions, there are 10 RPS chains which can be used to construct up to 120 manipulators. In addition, solutions for fewer than six positions are also given [13].
3. **Gallardo and Orozco (2007)** - Kinematic model using the screw theory and use of Sylvester dialytic elimination [14].
4. Parasuraman and Liang (2010) - In this paper the 3-RPS manipulator is designed from dimensional synthesis point of view. It is developed using Solidworks and the simulations were performed in 20sim and sim mechanics [15].
5. **Zhao and Wei (2010)** - The explicit solution model for the inverse kinematics of 3-RPS is proposed. This model is very efficient and elegant when compared to implicit one. The computation time is significantly decreased when compared to the implicit model [16].
6. **Yu and Lingtao (2010)** - This paper describes in detail the modelling of 3-RPS in MATLAB. The structural, simulation and branch models are created, and the simulation is performed using SimMechanics [17].
7. **Rad, Stan, Balan, and Lapusan (2010)** - A design of a medical 3- RPS robotic system is given. The forward kinematics is performed using the Newton-Kantorovich (N-K) method and simulation is done in MATLAB. In addition, the reachable workspace is also generated [18].
8. **Schadlbauer, Dominic R. Walter and Manfred L. Husty (2011)** - In this paper, a complete algebraic approach is adopted for the kinematics. This is done using the Study's kinematic mapping. Different operation modes and singularity are also analysed [19].
9. **Pundru and Nalluri (2013)** - Kinematic using Sylvester dialytic elimination technique [20].
10. **Babu, Raju, and Ramji (2013)** - An optimal kinematic design using multi-objective optimisation is developed. The functions used for this are Global Conditioning Index (GCI), Global stiffness Index (GSI) and Workspace volume. Compliance of the end-effector is determined using the static analysis [21].
11. **Hongli, Tiantian, and Mahemuti (2014)** - A new optimisation method called as Particle Swarm Optimisation is developed for the kinematics. This is a branch of Evolutionary Algorithms. It gives accurate solutions because of global and local searching abilities. It overcomes the major disadvantage of conventional numerical methods of initial values setting. Calculation results show that this new method is simple, convenient, and with a generality for solving the parallel manipulator forward kinematics problems [22].
12. **Nurahmi and Schadlbauer (2015) –** Kinematics using the algebraic approach of Study's kinematic mapping of the Euclidean group SE (3). The description of a special one degree of freedom called as the vertical Darboux motion (VDM) [23].
13. **Wang, Fan, Zhang, Lu, and Zhao (2017) –** In this paper, the mobility is analysed based on screw theory. In addition, the inverse kinematics, singularity and workspace are determined [24].
14. **Wang, Yu and Pei (2018)** - A new fast forward algorithm for kinematics is developed in this paper. It is based on the special geometrical condition of 3-RPS which eliminates the error produced by parasitic motion. This algorithm is very efficient, especially for real-time control and precision [25].

3.2 Dynamics

Unlike Kinematics, in dynamics the forces and moments are considered to derive the motion equations of a manipulator. Here, we assume that the links and joints are rigid. There are several ways to derive the dynamic equations. The most famous models are Lagrangian and Newton-Euler (NE) Model. Other models include Kane's method and the principle of virtual work. Like kinematics, there are two categories - direct and inverse dynamics. In direct dynamics the motion is derived based on force and moments, whereas in inverse dynamics joint forces and torques are obtained based on the motion of the manipulator. There have been voluminous publications on the dynamics of 3-RPS manipulator but focused mainly on energy-based methods. The main works can be summarized as:

1. **Lee and Shah (1988)** - With Kinematic Analysis, Lee and Shah proposed the dynamic modelling for 3-RPS using the Lagrangian approach. This paper gives a method to calculate forces required for the manipulator to follow a predetermined trajectory [1].
2. **Dasgupta and Choudhary (1999)** - This paper proposes a general strategy for Newton-Euler (NE) dynamic model for parallel manipulator in which 3-RPS manipulator was discussed. The advantages of NE model in inverse dynamic computations are highlighted [2].
3. **Pendar and Vakil (2004)** - The use of Natural Orthogonal Compliment (NOC) matrices in place of Lagrange multipliers in Lagrangian model of dynamics is discussed. Unlike Lagrangian multipliers, this method uses some low order matrix inversions, so that the model is efficient [4].
4. **Sokolov and Xirouchakis (2006)** - An alternative model that is the Principle of Virtual Work is used for dynamic modelling. The kinematics is done using the screw theory [5].
5. **Song and Li (2007)** - Modelled dynamics using principle of virtual work [7].
6. **Cui Qunfeng and Zheng Xiangzhou (2009)** - This paper presents the static analysis of 3-RPS manipulator. It calculates the different constraint forces on passive links and the moments on links. The centre of mass of the limb should be near to limb line and revolute joint should be designed with more mechanical strength [8].
7. **Zhao, Liu, and Huang (2011)** - Screw theory was used in this paper to perform the force analysis [9].

8. **Staicu Stefan (2012)** - Modelled inverse dynamics using the principle of virtual work and Lagrangian model [10]
9. **Hanzaki, Ali Rahmani and Elnaz Yoosefi (2011)** - Improved the dynamic modelling of 3-RPS using the Decoupled Natural Orthogonal Complement (DeNOC) model [11].

3.3 Control

After modelling the kinematics and dynamics, the next important stage in design of the manipulator is the control algorithm. Robots usually have a closed loop control system since the feedback helps to get the required positional accuracy. Resolved-rate control and resolved-acceleration control [26] are two general control schemes to control robots. In case of parallel manipulators, it is very necessary to design the control algorithm taking in account the singularities of the manipulator. For 3-RPS, various control algorithms have been proposed and can be summarized as:

1. **Kao, Wu and Fung (2007)**- Control model using damped velocity, called as the Damped Rate Resolve Acceleration (DRRAC). This model is asymptotically stable and plans a path not avoiding the singularities. It also improves the workspace [26].
2. **Kao and Zhan (2010)** - The efficiency of DRRAC for singularity robustness is discussed in detail [27].
3. **Arabshahi and Novinzadeh (2014)-** An impedance control algorithm is proposed based on the Inverse Dynamic Control (IDC). The relationship between force and position is adjusted using the ratio of Laplace transform of mechanical effort to Laplace transform of mechanical flow. In this paper, dynamics is performed using the Lagrangian Model [28].
4. **Li, Xiang, Chai, and Wu (2015)** - The control of parallel manipulator depends on Singularities. This paper focuses on the derivation of singularities based on geometric algebra. Twist and Wrench are represented by screw theory and outer product is used to describe their linear dependency. Reciprocity between twist and wrenches is represented using duality. Tilt and Torsion angles are used for orientation. An overall and thorough perspective of the singularity loci distribution of the 3-RPS parallel manipulator is disclosed [29].
5. **Shang, Tao, and Meng (2016)** - The prismatic joint is particularly important for 3-RPS since it is the only one actuated. Linear actuators, rack and pinion mechanism and pneumatic cylinder can be used for this purpose. This paper proposed an efficient control model for pneumatic cylinders. The errors in trajectory tracking are decreased and the algorithm is expanded to posture trajectory tracking. The validation of this model is done using the NI-CompactRio, NI-PXI, and Veristand platform. This paper gives a competitive edge to the pneumatic servo solution for the prismatic link of 3-RPS manipulator [30].

3.4 Application

Parallel Manipulators are a current research trend in the field of robotics. When compared to serial manipulators the applications of parallel manipulators are limited. However, the survey of 3-RPS manipulators reveals novel applications and capabilities of this simple mechanism. The novel applications are:

1. **Zhu Dachang, Feng Yanping, and Fang Yuefam (2005)** - Proposed a novel 3-RPS manipulator for packaging and assembly. The kinematics, jacobian and singularity analysis has been performed via the screw theory [31].
2. **Mintenbeck and Estana (2010)** - Developed a Biomimetic hyper-redundant 3-RPS manipulator inspired from snakes. The model consists of multiple 3- RPS manipulators linked mechanically. This robot can be used as a flexible tool for human surgery [32].
3. **Nurahmi, Solichin, Harnany, and Kurniawan (2017)** - Developed a novel 3-RPS manipulator which can be used as an ankle rehabilitation device to give patients passive training [33]. This device can also be used as a wearable walking assistance. The dimensional synthesis of this was done using Euler Parameterization. In another paper, different operation modes were derived for the same manipulator [34].
4. **Liu, Hu, Xu, Wang, and Du (2017)** - A Vectored Thruster for Autonomous underwater vehicles (AUVs) is proposed. This mechanism improves the performance of AUV's at zero and low forward speeds [35]. Additionally, the use of this specific mechanism improved the positional accuracy and response time. It also gave a very compact structure to the AUV.
5. **Zheng T., Zheng F., Rui, Yan, Niu, and Zhang (2019)** - A novel Three Extensible Rod (TER) mechanism based on 3-RPS structure was developed. This was developed for the design of a space large deployable paraboloid structure with power and communication integration (SSPCI) [36]. This is used to track the sun for power. The Linear extensible rod lowered the energy consumption and does require heavy speed reducers [37].

IV. Results and Discussion

The aim of this section is to review the major research ideas in every domain for the 3-RPS Parallel Manipulator. This can help to find the potential research gaps in this field.

1. Kinematics – In this domain the main ideas is to develop forward and inverse kinematics model. To develop these two major approaches – algebraic approach and screw theory approach are being formulated. In addition, dimensional

synthesis is also done for this manipulator. These works will help us to understand and validate the 3-RPS mechanism. However, there are a very few publications on the real-time algorithm for kinematics.

2. Dynamics – The study of dynamics helps to understand the effect of force and torque on a manipulator. In addition, selection of proper motors for the actuator can be done based on this model. For 3-RPS, there are two major dynamic models – Lagrangian and Principle of Virtual work are being designed. In addition, static analysis model is also formulated.
3. Control – In this domain the Damped Rate Resolve Acceleration and Inverse Dynamics Control are two major ideas. The prismatic link is only actuation for the 3-RPS, so efficient control models for this are developed.
4. Applications – Unlike serial manipulators, parallel manipulators are new in the field. Therefore, there are limited applications. The 3-RPS manipulator qualifies as positioning device and as a ankle rehabilitation device.

The **Fig. 1** plots the number of papers per domain for the 3-RPS parallel manipulator, it is evident that this robot has been a subject of research as the applications are limited. However, this presents a great opportunity to researcher to develop potential applications with a firm knowledge of kinematics, dynamics and control.

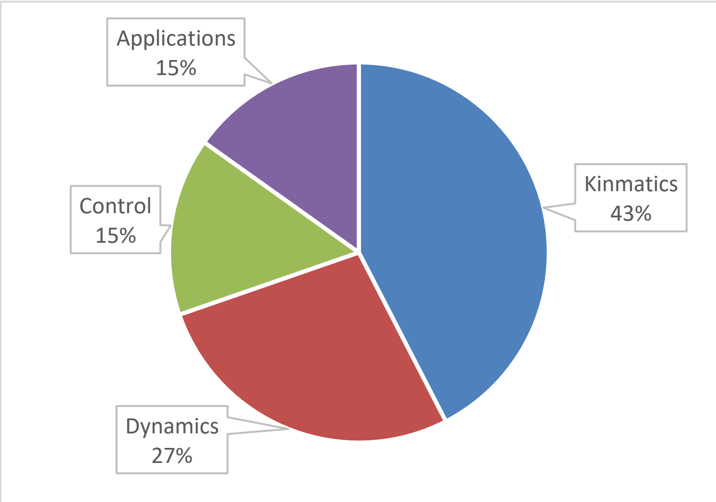

Fig.1 Number of papers per domain

V. Conclusion

It is evident from the survey that the growth of applications for 3-RPS parallel manipulator increased in the recent years. This is valid since it takes times for developing robust kinematic, dynamic and control models. In the all the above stated applications, 3-RPS manipulator gives high stiffness and positional accuracy, a major drawback of serial manipulator. The aim of this paper to provide reader the overall view of the 3-RPS is fulfilled. In years to come the applications of parallel manipulators will increase rapidly due to these characteristics.

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