Even Path Decomposition of Geometric Mean Graphs

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Abstract

In this paper we investigate the Even Path Decomposition of Geometric Mean Graphs. We use Graph labeling technique for Decomposition of Geometric Mean Graphs.

Keywords: Geometric Mean Graphs, Even Path Decomposition, Triangular Snake graphs, Diamond Snake graphs, Hexagonal Snake graphs.

AMS Subject Classification: 05C78.

1.Introduction :

In this paper, we consider simple undirected graph without loops or multiple edges. The concept of Continuous Monotonic Decomposition was introduced by N.Gnana Dhas and J.Paulraj Joseph. The concept of Arithmetic Odd Decomposition was introduced by E.Ebin Raja Merly and N.Gnanadhas in [2]. E.Ebin Raja Merly introduced the concept of Even decomposition of a connected graph and investigated their variations. In this paper we discuss Even path decomposition (EPD) in Geometric Mean snake Graphs. Throughout this paper, P_{2i} denotes the path of size $2i$. A cycle of length t is denoted as C_t.

A decomposition $(G_1, G_2, G_3, \dots, G_n)$ of G is said to be a Linear decomposition or Arithmetic Decomposition if each G_i is connected and $|E(G_i)| = a + (i-1)d$, for all $i = 1,2,3,\dots,n$ and $a, d \in Z$. The Arithmetic Decomposition with $a = 2$ and $d = 2$ is known as Even Decomposition (ED) since the number of edges of each subgraph of G is even, we denote ED as $(G_2, G_4, G_6, \dots, G_{2n})$. A decomposition $(P_2, P_4, P_6, \dots, P_{2n})$ of a graph G is an Even Path Decomposition (EPD) if $|E(P_{2i})| = 2i$ for all $i = 1,2,3, \dots, n$. Clearly $q = n(n+1)$, the sum of first n even numbers $2,4,6, \dots, 2n$.

Definition: 1.1

A graph **G=(V,E)** with p vertices and q edges is said to be a **Geometric Mean graph** if it is possible to label the vertices **x∈V** with distinct labels **f(x)** from **1,2,…,q+1** in such a way that when each edge **e = uv** is labeled with,

$$\mathbf{f(e = uv)} = \left\lceil \sqrt{f(u)f(v)} \right\rceil \text{(OR)} \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **Geometric Mean labeling** of G.

Definition: 1.2

Let $G = (V, E)$ be a connectd simple graph of order p and size q. If $G_1, G_2, \dots, G_n$ are edge disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup \dots \cup E(G_n)$, then $(G_1, G_2, \dots, G_n)$ is said to be a Decomposition of (G).

Definition: 1.3

A Decomposition $G_1, G_2, \dots, G_n$ is said to be Continuous Monotonic Decomposition (CMD) if $|E(G_i)| = i$, for every $i = 1,2,3, \dots, n$. Clearly $q = \frac{n(n+1)}{2}$

Definition: 1.3

A Decomposition $G_1, G_2, \ldots, G_n$ is said to be Linear Decomposition (LD) or Arithmetic Decomposition if $|E(G_i)| = a + (i-1)d$, for every $i = 1,2,3, \ldots, n$ and $a, d \in Z$. Clearly $q = \frac{n}{2}[2a + (n-1)d]$.

If $a = 1$ and $d = 1$, then $q = \binom{n+1}{2}$. That is, LD is a CMD. If $a = 1$ and $d = 2$, then $q = n^2$. That is the number of edges of G is a perfect square. Here after we consider the edge disjoint sub graphs of G as $G_1, G_3, G_5, \ldots, G_{(2n-1)}$.

Theorem 1.4 Any Path is a Geometric Mean Graph.

Theorem 1.5 Any Triangular Snake is a Geometric Mean Graph.

Theorem 1.6 Any Diamond Snake is a Geometric Mean Graph.

Theorem 1.7 Any Hexagonal Snake is a Geometric Mean Graph.

2. Main Results:

Definition: 2.1

If $a = 2$ and $d = 2$ in AD, then $q = n(n+1)$. That is, the number of edges of G is the sum of first n even numbers $2,4,6, \ldots, 2n$. Thus, we call this decomposition as Even Decomposition (ED). Since, the number of edges of each subgraphs of G is even, we denote the Even Decomposition as $G_2, G_4, \ldots.., G_{2n}$.

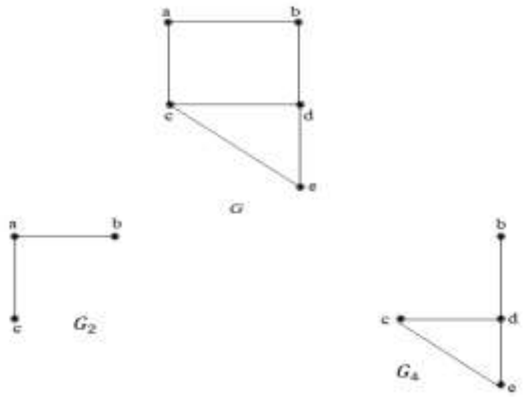
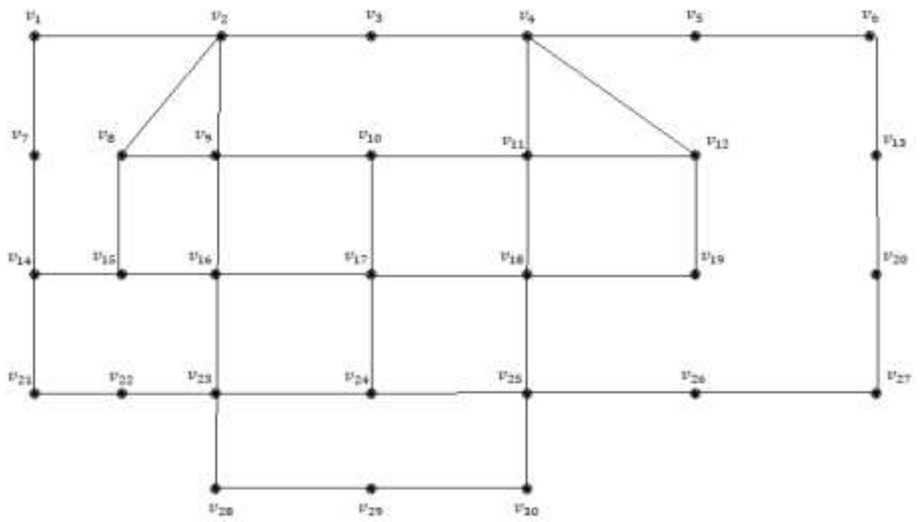


Figure : 2.1 Even Decomposition G_2, G_4 of G

Definition: 2.2

An Even Decomposition (ED) as $G_2, G_4, \ldots.., G_{2n}$ of G is said to be an Even Path Decomposition (EPD) if each G_{2i} is a path of size $2i$ and it is denoted by $P_2, P_4, \ldots.., P_{2n}$.



G

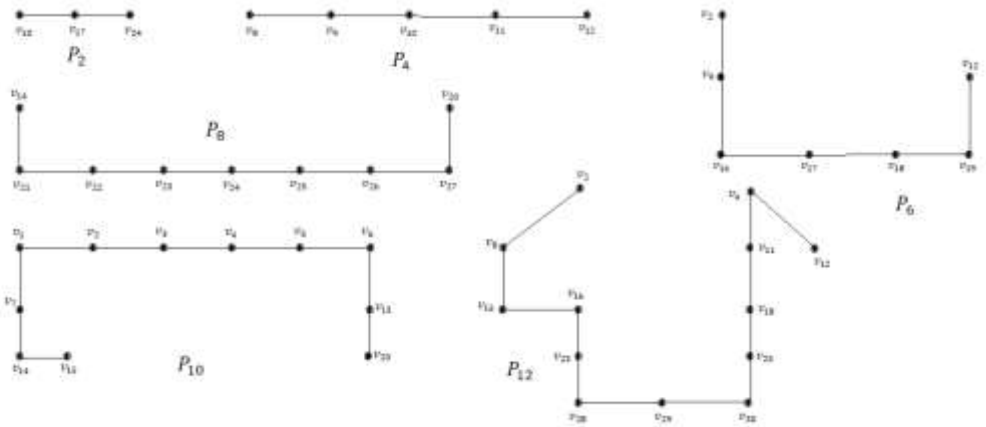


Figure : 2.2 Even Path Decomposition $P_2, P_4, P_6, P_8, P_{10}, P_{12}$ *of* G

Theorem: 2.3

Any Triangular Snake kC_3 satisfies Even Path Decomposition (EPD) of Geometric Mean Graphs.

Proof:

Let $G = kC_3$ is a graph obtained from a path $u_1u_2 \ldots . u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$. That is every edge of a path is replaced by a cycle C_3. A graph G has $2n+1$ vertices and $3n$ edges.

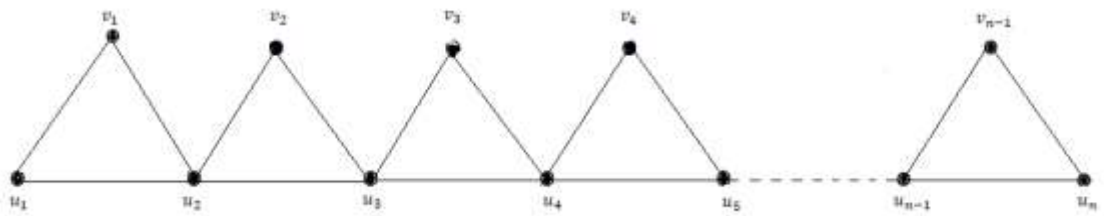


Figure: 2.3 kC_3

By Theorem 1:4, G is a Geometric Mean Graph. Now, we decompose the graph G with Even Path Decomposition (EPD). After EPD of G we get subgraphs P_i ; $2 \leq i \leq 2n$ of G. Each subgraphs have

$n+1$ vertices and n edges, also it satisfies the labeling patern of Geometric Mean Graphs. We assign labels for vertices, we get distinct edge labels for each subgraphs P_i ; $2 \leq i \leq 2n$.

By theorem 1.4 each subgraph P_i is a Geometric Mean Graph. Hence $G = \bigcup_{i=2}^{2n} P_i$. G is the union of subgraphs. Also it satisfies the condition of EPD, that is $q = n(n+1)$. Obiviously, G is a EPD of Geometric Mean Graphs.

Example: 2.4

Decomposition of Geometric Mean labeling of Triangular Snake $10C_3$ is displayed below.

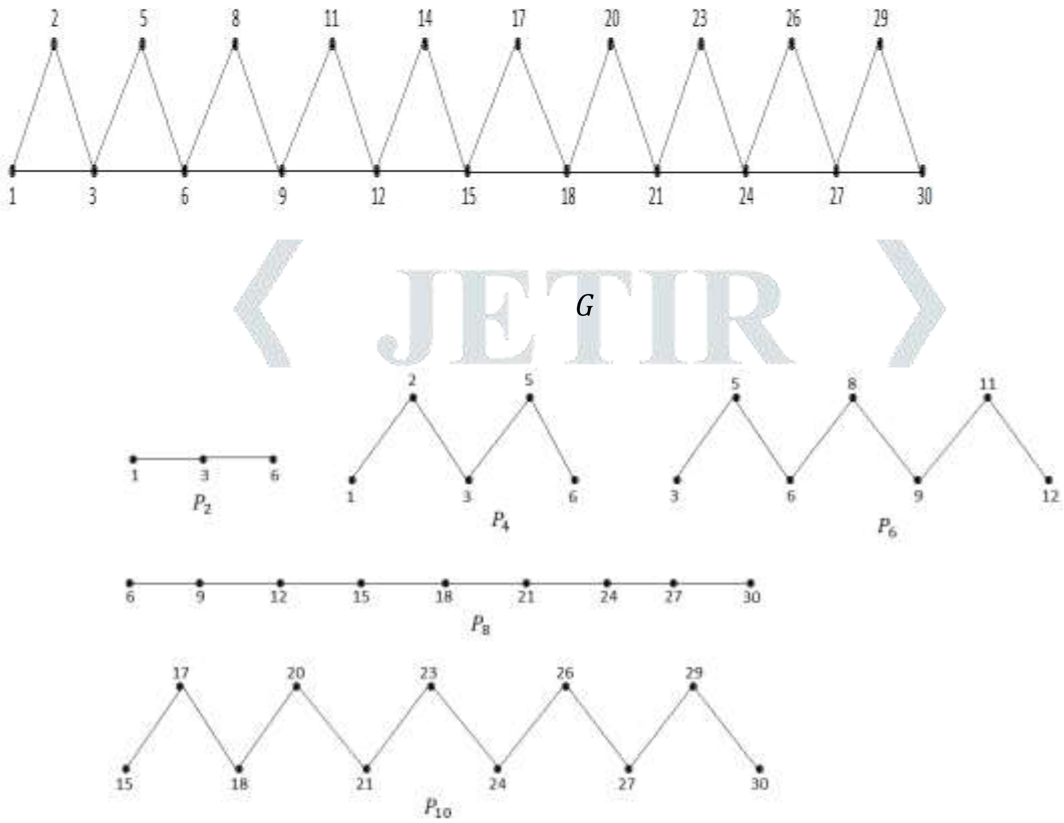


Figure : 2.4 Even Path Decomposition $P_2, P_4, P_6, P_8, P_{10}$ *of* G

In the example, Number of Subgraphs, $n = 5$, Number of edges, $q = 30$

Therefore, $q = n(n+1) = 5(5+1) = 30$. It satisfies EPD.

Definition: 2.5

A kC_t – cyclic snake graph has been defined as a connected graph in which all the blocks are isomorphic to the cycle C_t and the block-cut point graph is a Path P, where P is the path of minimum length that contains all the cut vertices of a kC_t cyclic snake graphs.

A kC_t cyclic snake graph has $(t-1)k+1$ vertices and tk edges where k is the number of blocks in the cyclic snake graph. If t is even, then the kC_t cyclic snake graph is said to be Even cyclic snake graph.

Let the path $v_0w_1v_1w_2v_2 \ldots \ldots w_kv_k$ of minimum length that contains all the cut vertices of kC_t cyclic snake graph is considered as the base path. Here $d(v_iv_{i+1}) = 2$; $0 \leq i \leq k-1$.

Definition: 2.6

A Diamond Snake graph is obtained from a path $v_0 v_1 \ldots . v_k$ by joining vertices v_i and v_{i+1} to two new vertices u_{i+1} and w_{i+1} for $0 \leq i \leq k-1$. That is every edge of a path $v_0 v_1 v_2 \ldots \ldots v_k$ of size k is replaced by a cycle C_4 and $d(v_i v_{i+1}) = 2$.

Theorem : 2.7

Diamond Snake Graph kC_4 admits EPD of Geometric Mean graphs.

Proof:

Let $G = kC_4$ is a graph obtained from a path $v_0 v_1 v_2 \ldots \ldots v_k$ by joining vertices v_i and v_{i+1} to two new vertices u_{i+1} and w_{i+1} for $0 \leq i \leq k-1$. That is every edge of a path $v_0 v_1 v_2 \ldots \ldots v_k$ of size k is replaced by a cycle C_4 and $d(v_i v_{i+1}) = 2$.

A Diamond Snake graph has $3k+1$ vertices and $4k$ edges where k is the number of blocks in kC_4. Let $V = \{v_0 w_1 v_1 w_2 v_2 \ldots \ldots w_k v_k\}$ be the set of vertices of kC_4.

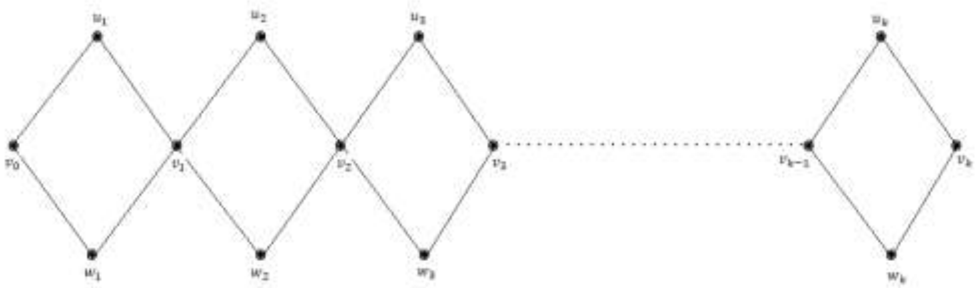
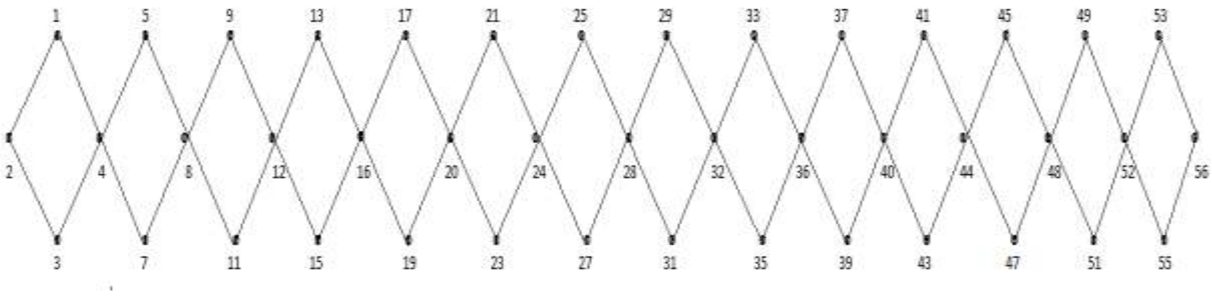


Figure : 2.6 $G = kC_4$.

By Theorem 1.5 G is a Geometric Mean Graph. Now, we decompose the graph with EPD satisfies the condition $q = n(n+1)$. After decomposition of G we get subgraphs P_i ; $2 \leq i \leq 2n$ of G. Each subgraph have $n+1$ vertices and n edges. Also each subgraphs satisfies the labeling pattern of Geometric Mean graphs. We assign labels for vertices we get distinct edge labels for each subgraphs P_i ; $2 \leq i \leq 2n$.

By theorem 1.4 each subgraph P_i is a Geometric Mean Graphs. Hence, $G = \bigcup_{i=2}^{2n} P_i$, each subgraph of G is a Geometric Mean graph. Therefore, G is the union of Geometric Mean graph and also satisfies the condition of EPD. Obiviously, G is a decomposition of Geometric Mean graphs.

Example : 2.8 Decomposition of Geometric Mean labeling of Triangular Snake $14C_4$ is displayed below.



$G = 14\, C_4$

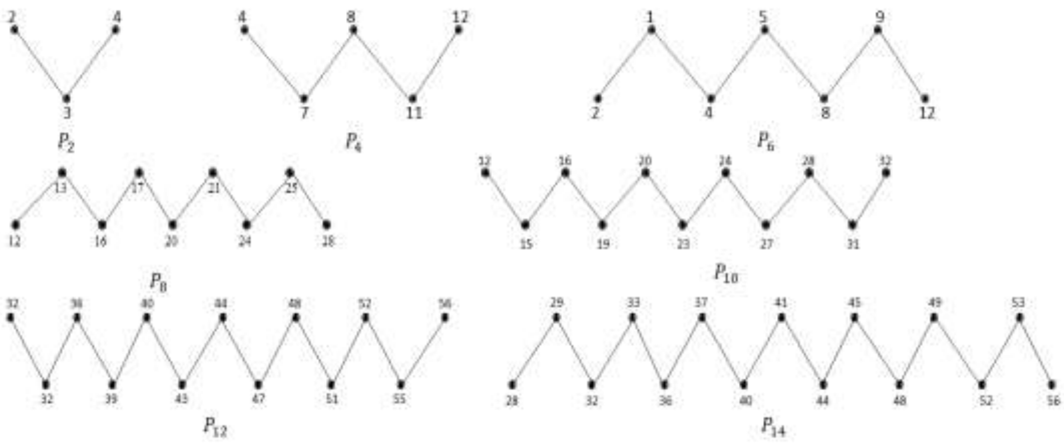


Figure : 2.7 Even Path Decomposition $P_2, P_4, P_6, P_8, P_{10}, P_{12}, P_{14}$ *of* G

In the example, Number of Subgraphs, $n = 7$, Number of edges, $q = 56$

Therefore, $q = n(n+1) = 7(7+1) = 56$. It satisfies EPD.

Definition: 2.9

Hexagonal Snake Graph kC_6 has been defined as a connected graph in which all the blocks are isomorphic to the cycle C_6 and the block cut point graph is a Path P, where P is the path of minimum length that contains all the cut verticeas of a kC_6 snake.

A kC_6 snake graph has $5k+1$ vertices and $6k$ edges, where k is the number of blocks of a Hexagonal snake graph. That is every edge of a path $v_0v_1v_2 \dots \dots v_k$ of size k is replaced by a cycle C_6 and $d(v_iv_{i+1}) = 2$.

Theorem: 2.10

Any Hexagonal Snake graph kC_6 is a Decomposition of Power-3 Mean graphs.

Proof:

Let $G = kC_6$ is a connected graph in which all the blocks are isomorphic to C_6.

Let G is a cyclic snake graph has $5k+1$ vertices and $6k$ edges, where k is the number of blocks of a Hexagonal snake.

Let the vertex set of the graph $G = \begin{Bmatrix} v_i \ ; \ 0 \le i \le k-1 \\ w_i \ ; \ 1 \le i \le k \\ u_i{}^j \ ; \ 1 \le i \le k \ ; \ 1 \le j \le 3 \end{Bmatrix}$

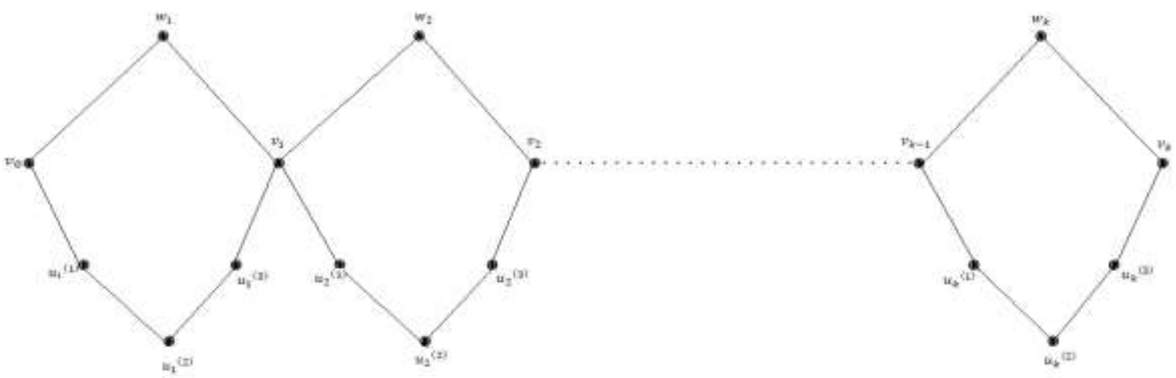


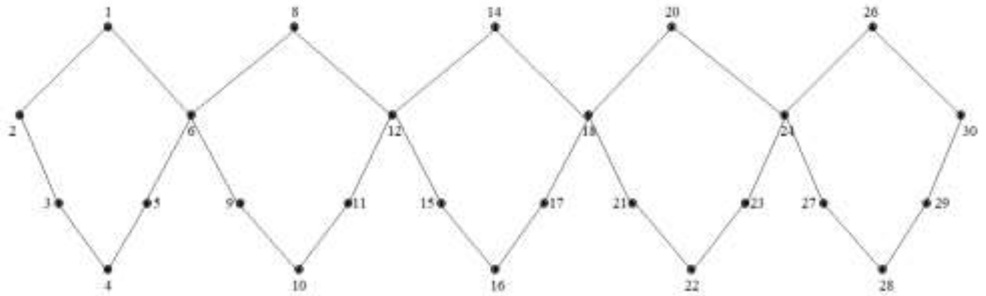
Figure : 2.8 kC_6

By Theorem 1.6, G is a Geometric Mean Graph Now we decompose the graph kC_6 with EPD satisfies $q = n(n+1)$.

After decomposition of kC_6 we get edge disjoint subgraphs P_i ; $2 \leq i \leq 2n$ of G. Each subgraphs have $n+1$ vertices and n edges. Also we label the vertices of each subgraphs we get distinct edge labels. Therefore, it satisfies Geometric Mean labeling.

By theorem 1.4 each subgraph P_i is a Geometric Mean Graphs. Hence $G = \bigcup_{i=2}^{2n} P_i$. Each subgraph is a Geometric Mean graph. Therefore, G is the union of Geometric Mean graphs and also satisfies the EPD. Obiviously, G is a Decomposition of Geometric Mean graphs.

Example: 2.11 Decomposition of Geometric Mean labeling of Triangular Snake $5C_6$ is displayed below.



$G = 5C_6$

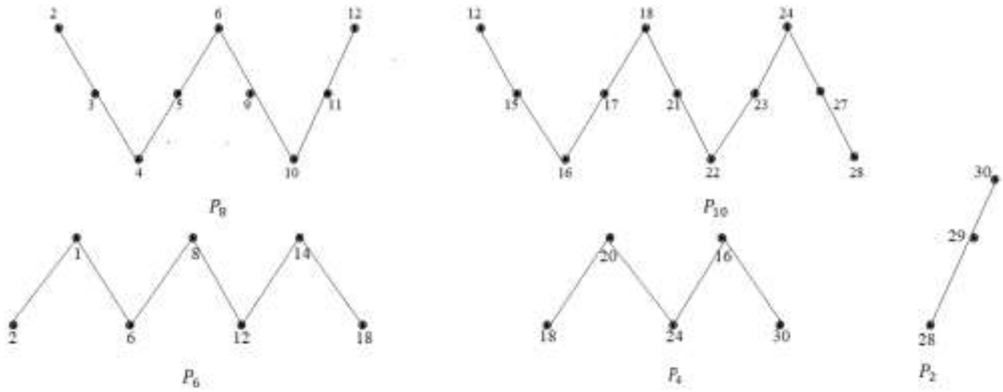


Figure: 2.8 Even Path Decomposition $P_2, P_4, P_6, P_8, P_{10}, P_{12}, P_{14}$ *of* G

In the example, Number of subgraphs $n = 5$; Number of edges $q = 30$

$$\therefore \quad q = n(n+1)$$

$$= 5(5+1) = 30$$

$\therefore$ $5C_6$ satisfies the condition of Even Path Decomposition.

3. Conclusion:

The study of labeled graph and their decomposition is important due to its diversified applications. All graphs are not Geometric Mean graphs. It is very interesting to investigate the decomposition of graphs that admits Geometric Mean Labeling. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other Geometric Mean Graphs.

4. Acknowledgement:

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5. References:

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