Inverse Equitable Domination in Graphs

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Abstract : A subset $\boldsymbol{D^e}$ of a vertex set $\boldsymbol{V(G)}$of a graph $\boldsymbol{G}$ is said to be inverse equitable dominating set if for every vertex $\boldsymbol{v} \in \boldsymbol{V - D^e}$ there exists a vertex of $\boldsymbol{u} \in \boldsymbol{D^e}, \boldsymbol{s}$uch that $\boldsymbol{uv} \in \boldsymbol{E(G)}$ and $|\boldsymbol{deg(v) - deg(u)}| \leq \boldsymbol{1}$. The inverse equitable domination number of a graph $\boldsymbol{G}$ is the minimum cardinality of an inverse equitable dominating set of $\boldsymbol{G}$ and is denoted by $\boldsymbol{\gamma^{e^{-1}}(G)}$.In this paper ,we study the graph theoretic properties of $\boldsymbol{\gamma^{e^{-1}}(G)}$ and many bounds were obtained in terms of elements of $\boldsymbol{G}$ and its relationships with other domination parameters were found.

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I. Introduction:

All the graphs considered here are finite , undirected with no loops and multiple edges . A graph G consists of pair (V, E) , where V is a non-empty finite set whose elements are called vertices or nodes and E is a set of unordered pairs of distinct elements of V . The elements of E are called edges of the graph$G$. The degree of a vertex of a graph is the number of edges incident to the vertex, with loops counted twice . The degree of vertex v is denoted by $deg(v)$. $N(v)$ and $N[v]$ denote the open and closed neighborhoods of a vertex v respectively. A vertex $v \in G$ is called pendent if one of its vertices is a pendant vertex. Some of the graph theoretic terms are found in chartrand and lesnaik [4]. .

The rigorous study of dominating sets in Graph theory began around 1960. According to [7], the domination problems were studied form the 1950' s onwards, but the rate of research on domination significantly increased in the mid 1970's . In 1962, Ore [5] coined the name "dominating set " and " domination number" for the same concept .

An tremendous management of basics of domination as well the survey of numerous highly developed topics in domination and a variety of domination parameters have been defined and studied by several authors and more than 75 models of dominations are listed in the appendix of [7].Swaminath [10] introduced the theory of equitable domination in graphs. Now considering to subsequent genuine universal issues .In networks , the nodes that has almost same competence may interact with each other in a improved way. In this society the people with nearly equal status approaches very friendly. In an industries, human resources with equal powers shape the organization and forward directly with equitability between general public irrespective of their wealth, health,, strength, position etc. is the object of self ruled nation.

II. Definitions and Notations:

In this section we recall some of basic definitions in literature which will be useful for our present work.

A path P_p is a graph whose vertex set is arranged in the sort of $v_1, v_2, v_3, \dots\dots\dots.. v_{n-1}, v_n$, such that the edges are $\{v_i, v_{i-1}\}$, where $i = 1, 2, 3, \dots\dots\dots\dots.., n-2, n-1$. Equivalently a path with at least two vertices are joined and has two extreme vertices (vertices with degree 1). A cycle C_p is defined as a closed trial (A graph in which no edge is repeated) moreover other vertex are not repeated . A star is a graph which is complete bipartite graph $K_{1,p}$, a tree with one internal vertex and p number of leafs (but no internal nodes and $(P + 1)$ leaves for $P \leq 1)$.

The notation $\beta_0(G)$ is the minimum number of vertices in a maximal independent set of a vertex of G.

A spanning subgraph is, that includes each vertex of the graph considered. The sub graph H is a spanning subgraph obtained by retaining all vertices and deleting just one edge. Thus H is a subgraph and also a spanning subgraph.

A set $D \subseteq V(G)$ is said to be a domating set of G , if every vertex in $V - D$ is adjacent to some vertex in D . The minimum cardinality of vertices in such a set is called the domination number of G and donate by $\gamma(G)$. A dominating set D is called the total dominating set , if for every vertex $v \in V$, there exists a vertex $u \in D$, $u \neq v$ such that u is adjacent to v . The total domination number of G , denoted by $\gamma_t(G)$ is the minimum cardinality of total dominating set of G.

Let D be a dominating set in G , if $V(G) - D$ contains another dominating set D^{-1} , then D^{-1} is called an inverse dominating set with respect to D . The minimum cardinality of vertices in such a set is called an inverse domination number in G and is denoted by $\gamma^{-1}(G)$.

For following figures we observe :

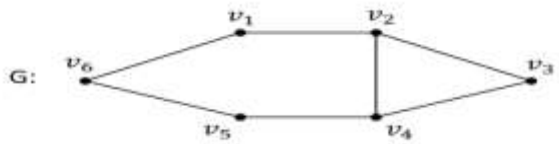

Fig 1 (a)

D_1= {3, 6}, D_2= { 1, 3, 5}, D_3= {1, 3, 6} and minimal inverse equitable dominating set of G is {3, 6}. Thus $\gamma^{e^{-1}}(G) = |D_1| = 2$.

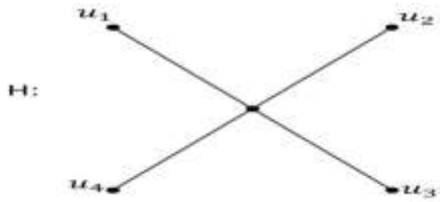

Fig 1 (b)

In fig 1(b), $\gamma^{e^{-1}} - (G)$ set is $\{ u_1, u_2, u_3, u_4 \}$ .Thus $\gamma^{e^{-1}}(H) = 4$.

Fig 1 (c)

In fig 1(c), $\gamma^{e^{-1}}$ − set is $\{u_1, u_4, u_7\}$. Thus $\gamma^{e^{-1}}(K) = 3$.

We have the following observation from above fig 1 (a), fig1 (b), fig 1(c).

Observation [1]: For any graph G, all the end vertices are considered in inverse equitable dominating set.

III. Results:

Initially, we give the inverse equitable domination number for some standard graphs, which are straight forward in the following theorem.

1. For any star $K_{1,p}.p$ with $p \geq 2$ vertices ,

$$\gamma^{e^{-1}}(K_{1,p}) = p.$$

2. For any path P_p with $p \geq 2$ vertices,

$$\gamma^{e^{-1}}(P_p) = \begin{cases} \left\lceil \frac{p-1}{2} \right\rceil & if\ p = even \\ \left\lceil \frac{p-1}{3} \right\rceil + 1 & if\ p = odd \end{cases}$$

3. For any cycle C_p with $p \geq 3$ vertices ,

$$\gamma^{e^{-1}} = \left\lceil \frac{p}{3} \right\rceil.$$

4. For any wheel W_p with $p \geq 4$ vertices,

$$\gamma^{e^{-1}}(W_p) = \left\lceil \frac{p-1}{3} \right\rceil.$$

In the following theorem we establish the relation between inverse equitable domination number and maximum independence number of G.

Theorem 1: For any (p,q) graph G, $\gamma^{e^{-1}}(G) \leq \beta_o(G)$.

Proof: Suppose V$=\{v_1, v_2, v_3, \dots \dots \dots, v_n\}$ is vertex set of G and let $D^e = \{v'_1, v'_2, v'_3, \dots \dots \dots \dots \dots \dots, v'_{i-1}, v'_i\}$; $\forall\, i \leq n$ be the subset of $V(G)$, thus for all vertex $v'_i \in V - D^e$,there exists a vertex $u'_i \in D^e$, such that $u'_i v'_i \in E(G)$ and $|deg(v_i) - deg\,(u_i)| \leq 1$, then D^e is a inverse equitable dominating set gives $|D| = \gamma^{e^{-1}}(G) = |D^e|$. Now let B be the maximal independent set with some of vertices in $\{V - D\}$.

Now we consider the following two cases.

Case1: Suppose $V - D - B = \emptyset$ then $V - D = B$ forms independent inverse equitable dominating set of G.

Hence $\gamma^{e^{-1}}(G) \leq |V - D|$

$= |B|$

Hence $\gamma^{e^{-1}}(G) \leq \beta_o(G)$.

Case 2: Suppose $V - D - B \neq \emptyset$, then vertex $u_i \in$ (V − D − B) is adjacent to any of the vertex $v_k \in B$. If each vertex in D is adjacent to at least one vertex in B, then B is also a inverse equitable γ - set of G, or else let a consider the another set $D' \subset D$ as a set of vertices in D, whereas no vertex u_i; $\forall\, i \leq n$ of D' is adjacent to the vertice of B. Since D^e is a minimal inverse equitable γ-set, moreover $B \subseteq D' \subset D \in D^e$; $\forall\, v_i - D \subseteq B$ and $u_i \in D$. Hence the result follows as

$|D^e| \leq |B|$.

$\gamma^{e^{-1}}(G) \leq \beta_o(G)$.

The following theorem gives the relationship between the inverse equitable domination number and domination number of G.

Theorem 2: If every induced subgraph $\{N(v)\}$ is complete graph of order at least two, for all v belongs to the minimal dominating set then $\gamma^{e^{-1}}(G) \leq \gamma(G)$.

Proof: Let $D^e = \{u_1, u_2, u_3, \dots \dots u_n\}$be least inverse equitable dominating set of G, for which $D^e \subset V(G)$; $\forall\, v_i \in V(G) - D^e$there exists the vertices $u_i \in D^e$ incident with v_i with $|deg(v_i) - deg(u_i)| \leq 1$. Thus $|D^e| \leq \gamma^{e^{-1}}(G)$. Now let $D = \{u'_i\}$; $\forall\, i < n$ being minimal dominating set of G. Let $\{v'_1, v'_2, v'_3 \dots \dots v'_n\}$ be the vertices which are adjacent $\{u'_1, u'_2, u'_3, \dots \dots \dots, u'_n\} \in D$ respectively. Thus $|D| = \gamma(G)$. Now since for each $u_i \in D$, we have $v_i \in V - D$ and $u_i v_i \in E(G)$ and $deg v_i \leq deg u_i$ and also $u_i \subseteq V - D$. Then $N(u_i) \subset N(v_i) \leq \gamma(G)$.

Hence $|D^e| \leq |D|$

$\gamma^{e^{-1}}(G) \leq \gamma(G)$.

Consequently the result follows.

In the next theorem we relate the inverse equitable domination number in terms of vertices and edges of G.

Theorem 3: For any connected (p,q) graph $G, q \leq 2p - \gamma^{e^{-1}}(G)$

Proof: Let $D^e \subseteq V(G)$ and for all $v_i \in V - D^e$ and $u_i \in D^e$ for which $|deg(v_i) - deg(u_i)| \leq 1$ and $u_i v_i \in E(G)$.

Then $|E(G)| \leq |v(G)| + |v(G) - D^e|$

$\leq 2\,|v(G)| - |D^e|$

$q \leq 2p - \gamma^{e^{-1}}(G)$.

Equality holds if $G \cong K_{1,p}$ withm $p \geq 3$ vertices in which $p = q - 1$. Let $V' = \{v'_1, v'_2, v'_3, \dots \dots \dots \dots \dots \dots \dots \dots \dots, v'_{n-1}, v'_n\}$; $\forall\, i \leq n$ is end vertex set and suppose $u \in D$ is non-end vertex adjoining to each vertices of V'-set , such that $v' \in D$ and every vertex $v_i \in v' - D$ and $uv_i \in E(G)$, such that $|deg(v_i) - deg(u_i)| \leq 1$. Thus also by conclusion of observation [1], the equality holds good for $G \cong K_{1,p}$.

The following theorem relates the inverse equitable domination and domination number in terms of vertices of G.

Theorem 4: For any (p,q) graph $G, \gamma^{e^{-1}}(G) + \gamma(G) \leq p$.

Proof: We come across with subsequent two cases:

Case 1: If $G \neq T$. let P presents vertex set in G, gives $|P| = V(G)$. By the theorem [2] we can directly conclude that $\gamma^{e^{-1}}(G) \subset \gamma(G) \leq p$.

Which gives the required result.

Case 2: If $G = T$. When all the non-end vertices of T are adjacent to at least two end vertices, then that set is taken as a minimum γ - set. Let u be any intermediate vertex adjacent to at least one end vertex $v \in V - D$ and $uv \in E(T)$, satisfying the condition of inverse equitability dominating set. Thus also by theorem [2] and observation 3.1 . Hence we conclude that $\gamma^{e^{-1}}(T) + \gamma(T) \leq p$.

Now we develop the relation of inverse equitable domination number with its spanning subgraph of G.

Theorem 5: For any (p, q) graph G, $\gamma^{e^{-1}}(G) \leq \gamma^{e^{-1}}(H)$,where H is a spanning sub graph of H.

Proof: Let graph G is connected and let H is the spanning subgraph of H. Suppose D^e is inverse equitable γ - set of H, subsequently as well D^e is a inverse equitable dominating set for which $|D^e| = \gamma^{e^{-1}}(G)$ with every vertex $v \in V - D^e \in H$ and also $V(H) \subseteq V(G)$ and thus the result is straight forward.

$$\gamma^{e^{-1}}(G) \subseteq \gamma^{e^{-1}}(H)$$

$$\gamma^{e^{-1}}(G) \leq \gamma^{e^{-1}}(H).$$

Hence the result follows.

In the next theorem we relate the inverse equitable domination number in terms of diameter G.

Theorem 6: For any connected (p, q) graph G, $\gamma^{e^{-1}}(G) \geq \left\lceil \frac{diam(G)+1}{3} \right\rceil$.

Proof: Let D^e be a inverse equitable dominating set of G. Let us take a random path of distance which is $diam(G)$ that contributes not more than two edges from the induced subgraph $\langle N[v] \rangle$;$\forall\, v \in D^e$. In addition as D^econsists of all the end vertices that includes at most $\gamma^{e^{-1}}(G) - 1$ edges connecting the neighborhood of $v \in V - D^e$. For each $u \in D^e$ and $|deg(v) - deg(u)| \leq 1$.

Hence we have

$$diam(G) \leq 2\gamma^{e^{-1}}(G) + \gamma^{e^{-1}}(G) - 1$$

$$diam(G) \leq 3\gamma^{e^{-1}}(G) - 1.$$

Theorem 7: For any (p, q) graph G, $\gamma^{e^{-1}}(G) \leq \gamma_t(G) + \left\lceil \frac{p}{2} \right\rceil$.

Proof: Let $D = \{v_k\}$; $\forall\, v_k \in V(G); k \leq n$ be the dominating set and $V' = V(G) - D$, suppose $H \subseteq V'$be the minimum vertex set that are adjacent to D, so $(D\ U\ H)$ forms a total dominating set of G. Moreover D^e is thought to be inverse equitable $\gamma -$ set of G, then it is clear that ,some of the v_k that belongs to $(D\ U\ H) \subseteq V(G)$ forms a inverse equitable dominating set with almost all $uv \in E(G)$, where u & $v \in V(G)$ or either are incident to $N(v)$ or $N(u)$ respectively. Thus clearly

$$\gamma^{e^{-1}}(G) \leq |D \cup H| + \left\lceil \frac{p}{2} \right\rceil$$

$$\gamma^{e^{-1}}(G) \leq \gamma_t(G) + \left\lceil \frac{p}{2} \right\rceil.$$

Consequently result follows.

Conclusion:

The concept of Inverse equitable domination is interesting as it confirms the relation between vertices within the dominating set .while the concept of equitable domination is important as it depends upon the degree of dominating vertices. The above work is based on the concept of inverse equitable dominating sets in graph. This domination model possesses the blends of inverse domination as well as inverse equitable domination in graphs. We have derived some general results on the concept of inverse equitable domination in graph.

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