



# AN INVESTIGATION OF WAVE PROPAGATION AND MULTIPLE SCATTERING AND TRANSPORT THEORY IN RANDOM MEDIA

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**Abstract:** This paper describes an Investigation of Wave Propagation and Scattering in Random Media. The investigation of wave propagation in a random medium has two distinct aspects. A medium has to be considered as a continuum. The properties of the medium are characterized by their dielectric permittivity. The second relates to the scattering of randomly distributed discrete scatter waves. The second important class under consideration includes advanced backscattering and the associated effects of dual-way through turbulent environments, including increased intensity fluctuations, partial reversal of the wavefront in turbulent media, super focusing, magic-cap effects, and variety of others effect. Included in the analysis are the characteristics of intensity, wave fluctuations, pulse propagation and scattering, coherence bandwidth, and coherent time propagation of communication channels through random media. Recent advances in remote-sensing techniques include the use of inversion techniques to deal with inverse problems.

Keywords: Effect, Media, Waves, Scatters, Properties etc.

## Introduction

Remote sensing problems are of interest in the problem of wave scattering from a random volume. For example, a layer of ice above the ground constitutes an inhomogeneous medium where the location, size, shape, and orientation of ice particles are statistical parameters. At the same time a forest medium can be considered as a collection of randomly distributed dielectric particles with different geometries, sizes, and dilatrix. As the electromagnetic wave enters such media, there is wave scattering, attenuation, and depolarization. The importance of these phenomena depends on both the physical and electrical properties of the medium, as well as the characteristics of incident waves such as waveform, incident angle, and polarization. Nowadays advanced high resolution polymeters and interferometric synthetic aperture radars are used to efficiently gather the radar response of terrestrial targets for a wide variety of remote sensing applications. To interpret remotely sensed data, a theoretical model capable of predicting the electromagnetic scattering behavior of random media is needed. Several theoretical approaches are available for the interpretation of remote sensing measurements. However, it should be noted that due to the complex nature of the scattering problem, precise solutions cannot be found for most practical situations. Therefore, various types of simplifications of the physical parameters and approximations of the equations must be applied to find the solution. Available models for random media can be classified into three general groups, semi-empirical models approximate models, and numerical / Monte Carlo based models. In the discrete case, it is assumed that the particles are sparse in a random medium and the principle of single scattering is applied to evaluate the backscatter energy [7, 8]. In the coherent approach the first mean field is obtained using Foti's [9] approximation, which defines a uniform medium for the scattering region, and then the statistics of the scattered field to see the scattered mean in the

equivalent medium Form, and applying the single scattering principle [10, 11]. To simplify the mathematics involved in this technique, particles are usually thought of as simple geometric shapes such as cylinders, disks, or spheres. This technique is referred to in the literature as the [12] deformed-Born approximation. Radiological transfer (RT) theory can also be applied to random media composed of discrete scatter. In this case, the extinction and phase function of the random medium, two fundamental quantities of the RT equation, are obtained from the single scattering properties of the component particles [13, 14, 15] and then solve the transport equations under appropriate boundary conditions. is done . We first discuss how the Renaissance technique applied the half-incomplete temperature random medium and then we generalize the solution to a multiplayer random medium. A similar approach is applied to the random medium to obtain an expression for the function of the mean diadic Green's medium referred to as Dyson's equation for the mean-field.

**Properties of Electromagnetic Waves in Random Media**

In the beginning. Consider a fully polarized and monochromatic plane wave traveling in the direction. As this wave enters a random medium, its amplitude, phase, and polarization fluctuate irregularly with time and position. If the frequency spectrum of wave fluctuations is limited to a narrow bandwidth (quasi-monochromatic with half frequency), the wave's electric field vector can be represented

$$E_0(r, t) = Re[E(r, t)exp(-i\omega t)]$$

Throughout this study, the primary time dependence  $exp(-i\omega t)$  is assumed. In general, the field E is complex, and due to its narrow bandwidth, is a slowly varying function of time. In accord with the vector nature of the wave, the Cartesian components of  $E(r, t)$  are

$$E_x = U_x(r, t)exp[i\phi_x(r, t)]$$

$$E_y = U_y(r, t)exp[i\phi_y(r, t)]$$

$$E_z = 0,$$

Where,  $U_x$ ,  $U_y$  and  $\phi_x$ ,  $\phi_y$  are also slowly varying functions of time.

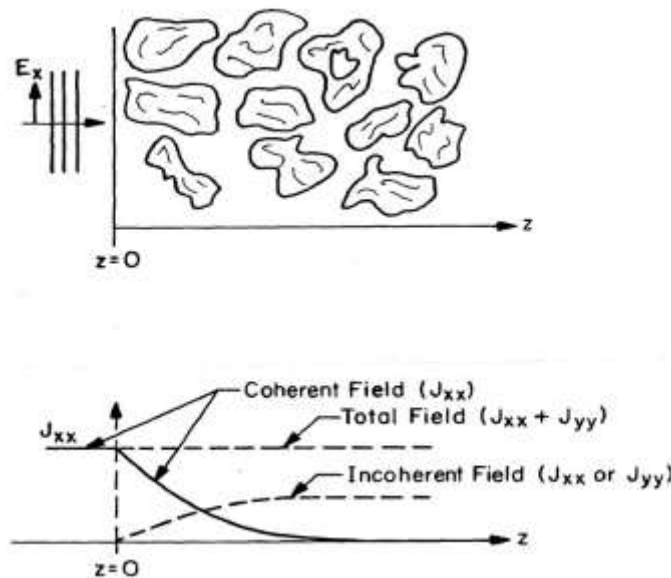


Figure 1.1: Polarized plane wave incident upon a random medium (z > 0) (General behavior)

**A Relationship between Multiple Scattering and Transport Theory**

Let E(r) be random scalar field, with a monochromatic time reliance  $exp(-i t)$ . Results for the electromagnetic field might be easily obtained by simplification. The capacity most often required in the analytic theory is the second-order correlation function of the wave field, which is given by

$$\Gamma(\underline{r}_1, \underline{r}_2) = \langle E(\underline{r}_1) E^*(\underline{r}_2) \rangle \quad (1)$$

In (C.1) the asterisk denotes the complex conjugate and the sharp brackets denote an ensemble average. In transport theory the quantity sought is the specific intensity  $j(r,s)$  of the wavefield (radiance) in a given direction  $\hat{s}$  at the point in space  $r$ . The quantity  $J(r,s)$  is given by

$$J(\underline{r}, \hat{s}) = \langle E(\underline{r}, \hat{s}) E^*(\underline{r}, \hat{s}) \rangle \quad (2)$$

The derivations in a relationship between  $\Gamma(\underline{r}_1, \underline{r}_2)$  and  $J(\underline{r}, \hat{s})$ . Note that in measurements, the magnitude would be

$\Gamma(\underline{r}_1, \underline{r}_2)$  determined by two omni-directional voltage receivers situated at the points  $\underline{r}_1$  and  $\underline{r}_2$ , whereas

$J(\underline{r}, \hat{s})$  would be determined by a very narrow-beam intensity receiver pointed in the direction  $s$  and located at  $r$ .

The energy density  $I(r)$  of the wave field at a point  $r$  can be obtained from either (C.1) or (C.2):

$$I(\underline{r}) = \Gamma(\underline{r}, \underline{r}) = \langle E(\underline{r}) E^*(\underline{r}) \rangle \quad (3)$$

And

$$I(\underline{r}) = \int_{4\pi} J(\underline{r}, \hat{s}) d\omega$$



(4)

In (4)  $d\omega = \sin\theta d\theta d\varphi$  is the unit solid angle.

Now, at any point in space,  $E(r)$  can be decomposed into an equivalent set of plane wave according to the equation

$$E(\underline{r}) = \int_{-\infty}^{+\infty} E(\underline{k}) e^{+i\underline{k}\cdot\underline{r}} d\underline{k} \tag{5}$$

In (C.5)  $E(k)$  is simply the three-dimensional Fourier transform of  $E(r)$ . For completeness, we include the following inverse transform relationship:

$$E(\underline{k}) = \frac{1}{2\pi^3} \int_{-\infty}^{+\infty} E(\underline{r}) e^{-i\underline{k}\cdot\underline{r}_d} d\underline{r} \tag{6}$$

The integrations in (5) and (6) involve the whole three dimensional space represented by the domain of the field quantities. In this representation, the correlation function  $I$  becomes

$$\Gamma(\underline{r}_1, \underline{r}_2) = \iint_{-\infty}^{+\infty} \langle E(\underline{k}_1) E^*(\underline{k}_2) \rangle e^{+i(\underline{k}_1 \cdot \underline{r}_1 - \underline{k}_2 \cdot \underline{r}_2)} d\underline{k}_1 d\underline{k}_2 \tag{7}$$

change variables from  $(\underline{r}_1, \underline{r}_2)$  to  $(\underline{r}, \underline{r}_d)$  and from  $(\underline{k}_1, \underline{k}_2)$  to  $(\underline{k}, \underline{k}_d)$  by means of the formulas

$$\begin{aligned} \underline{r} &= \frac{1}{2} (\underline{r}_1 + \underline{r}_2) & , & & \underline{r}_d &= \underline{r}_1 - \underline{r}_2 \\ \underline{k} &= \frac{1}{2} (\underline{k}_1 + \underline{k}_2) & , & & \underline{k}_d &= \underline{k}_1 - \underline{k}_2 \end{aligned} \tag{8}$$

these new variables, the correlation function given by (7) becomes  $\Gamma(\underline{r}, \underline{r}_d)$  given by

$$\begin{aligned} \Gamma(\underline{r}, \underline{r}_d) &= \iint_{-\infty}^{+\infty} \langle E(\underline{k} + \frac{1}{2} \underline{k}_d) E^*(\underline{k} - \frac{1}{2} \underline{k}_d) \rangle e^{+i(\underline{k}_d \cdot \underline{r} + \underline{k} \cdot \underline{r}_d)} d\underline{k} d\underline{k}_d \\ \Gamma(\underline{r}, \underline{r}_d) &= \int_{-\infty}^{+\infty} J(\underline{r}, \underline{k}) e^{+i\underline{k}_d \cdot \underline{r}_d} d\underline{k} \end{aligned} \tag{11}$$

From (11) it can be identified that the energy density given by (0.3) is obtained by setting  $\underline{r}_d = 0$ . Thus, we have

$$I(\underline{r}) = \Gamma(\underline{r}, \underline{r}_d = 0) = \int_{-\infty}^{+\infty} J(\underline{r}, \underline{k}) d\underline{k} \tag{12}$$

Further more, when it is noted that  $\underline{k}$  has both magnitude and direction, i.e.,  $\underline{k} = k \hat{s}$  the vector wave number integration ( $d\underline{k}$ ) in (12) can be broken up into an angular integration ( $d\hat{s}$ ) and wavenumber integration ( $dk$ ) yielding

$$I(\underline{r}) = \int_{4\pi} \int_0^\infty J(\underline{r}, k \hat{s}) k^2 dk d\hat{s} \tag{13}$$

Comparing (13) and (14), we see that the specific intensity of the field  $J(\underline{r}, \hat{s})$  is simply related to the plane wave expansion field according to the equation

$$J(\underline{r}, \hat{s}) = \int_0^\infty J(\underline{r}, k \hat{s}) k^2 dk \tag{14}$$

It should be noted that  $J(r, s)$  is a directly measurable quantity whereas  $J(r, k)$  is not.

where  $\delta(\underline{k}_d)$  is the Dirac-delta function which is zero except at  $\underline{k}_d = 0$ . From (C.10) we have

$$J(\underline{r}, \underline{k}) \equiv J(\underline{k}) \leq E\left(\frac{1}{2}\underline{k}\right)E^*\left(\underline{k} - \frac{1}{2}\underline{k}_d\right) > e^{\frac{+ik_d r_d k_d}{2}} \tag{16}$$

$$\Gamma(\underline{r}_d) = \int_{-\infty}^{\infty} J(\underline{k}) e^{\frac{+ik_d r_d k_d}{2}} d\underline{k}$$

So far we have not entirely reached our goal of providing a relationship between  $J(\underline{r}, \hat{s})$  and  $\Gamma(\underline{r}, \hat{s})$ . However, we have shown that  $J(\underline{r}, \underline{k})$  is a much more fundamental and general quantity than  $J(\underline{r}, \hat{s})$ .

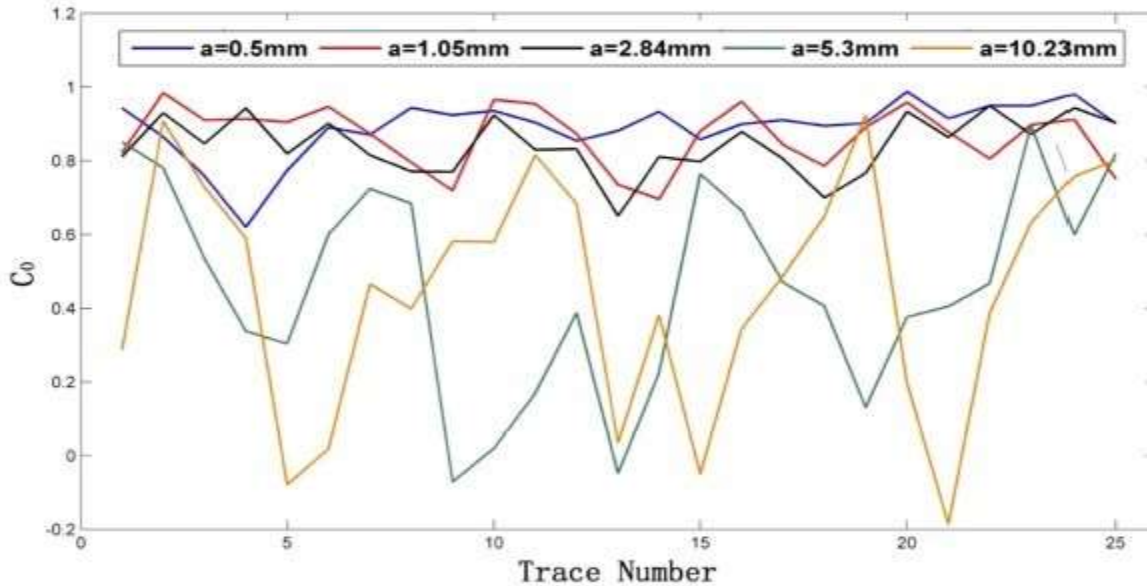


Figure 1.2: The correlation coefficient observed waveforms for different heterogeneity scales: If the plane waves with different wavenumbers are independent, then we have

$$E\left(\frac{1}{2}\underline{k} + \frac{1}{2}\underline{k}_d\right)E^*\left(\underline{k} - \frac{1}{2}\underline{k}_d\right) = E(\underline{k})E^*(\underline{k}) > \delta(\underline{k}_d)$$

which is independent of the position vector  $\underline{r}$ . This result yields the following expression for the correlation function

$$(17)$$

Thus, the second-order correlation function depends only upon the difference between the position vector  $\underline{r}_1$  and  $\underline{r}_2$ . If we take  $\langle E(\underline{r}) \rangle = 0$ , then the above equation (17) is precisely a statement of statistical homogeneity of the wave field  $E(\underline{r})$ . The significance of this example is that (16) is the often-used condition specified in transport theory for adding "independent" waves. From this example, it is clear that such a condition restricts the specific intensity to a form independent of position. Collet, Foley, and Wolf reached a similar conclusion about transport theory, but came short of providing an alternative formulation as done here with  $J(\underline{r}, \underline{k})$ . From (17) the energy density of the wave field can be found:

$$I(\underline{r}) = I(0) = \int_{-\infty}^{+\infty} J(\underline{k}) d\underline{k} \tag{18}$$

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