



FRACTIONAL DIFFERENTIAL OPERATORS INVOLVING SPECIAL FUNCTIONS AND GENERAL CLASS OF POLYNOMIALS

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Abstract

In this paper we use fractional differential operators $D_{k,\alpha,x}^n$ to derive a certain fractional Calculus formulae for Fox's H-function by the application of fractional Calculus formulae involving a general class of polynomials.

Key words

Fractional differential operator, special-function general class of polynomials.

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INTRODUCTION AND DEFINITIONS

The fractional derivative of special function of one and more variables is important such as in the evaluation of series, [10,15] the derivation of generating function [12, chap.5] and the solution of differential equations [4,14; chap-3] motivated by these and many other avenues of applications, the fractional differential operators $D_{k,\alpha,x}^n$ and ${}_x D_x^\mu$ are much used in the theory of special function of one and more variables .

We use the fractional derivative operator defined in the following manner [14]

$$D_{k,\alpha,x}^n(x^\mu) = \prod_{r=0}^{n-1} \left[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}} \right] x^{\mu+nk} \quad \dots(1.1)$$

Where $\alpha \neq \mu+1$ and α and k are not necessarily integers

Using the following form of the binomial theorem

$$(X + \xi)^{-\lambda} = \xi^{-\lambda} \sum_{m=0}^{\infty} \frac{(\lambda)_m}{m!} \left(\frac{-x}{\xi} \right)^m$$

Raina [5] obtained a fractional differential formula for the function z^p using generalized Gauss theorem, while Ross [7] obtained the fractional integral transformation by obtained the fractional integral formula for the function $(\alpha z + \beta)^a$ using series expansion method .kalla et al [4] has derived the fractional integral transformation by orthogonal polynomials. Ali et al [1] generated the expansion of the Laguerre polynomials and Soni and Singh [12] obtained the **fractional** differential formulae involving a general class of polynomials.

The present work is an attempt in the direction of obtaining fractional calculus formula by utilizing series expression method, introduced by srivastava [9]. The name general

class of polynomials, itself indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials.

MAIN RESULT -I

$$D_{k,\alpha,x}^n [X^\mu (X + \xi)^{-\lambda} S_l^m \{X^\rho (X + \xi)^{-\sigma}\}] =$$

$$\xi^{-\lambda} X^{\mu+nk} \sum_{j=0}^{\lfloor l \rfloor} \frac{(-l)_{mj}}{j!} A_{l,j} \left(\frac{X^\rho}{\xi^\sigma}\right)^j \sum_{m=0}^{\infty} \frac{(\lambda+\sigma)_m}{m!} \frac{(-1)^m}{\xi^m} \prod_{g=0}^{n-1} \frac{\Gamma\mu + m + \rho j + gk + 1}{\Gamma\mu + m + \rho j + gk - \alpha + 1} X^m$$

Provided that $\min(k, \lambda, \rho, \sigma) > 0$ $\left|\frac{x}{\xi}\right| < 1$ and $\text{Re}(k+\rho j-\mu+1) > 0$

.....(1)

MAIN RESULT -II

$$D_{k,\alpha,x}^n [X^\mu S_n^m \{X^\rho (X + \xi)^{-\sigma}\}] H_{P,Q}^{M,N} (X^\mu)$$

$$= \sum_{m=0}^{\infty} \sum_{j=0}^{\lfloor n \rfloor} \frac{(-n)_{mj}}{j!} \frac{(-1)^m}{m!} \frac{(\sigma)_m}{\xi^m} A_{n,j} \xi^{-\sigma j} X^{\mu+\rho j+m+nk} H_{P+1,Q+1}^{M,N+1} \left[X^\mu / \begin{matrix} (-\mu - \mu s - m - \rho j, k)_{s=0,n-1} (a_j, \alpha_j)_{1,P} \\ (b_j, \beta_j)_{1,Q} (-\alpha - \mu - \mu s - m - \rho j, k)_{s=0,n-1} \end{matrix} \right]$$

Provided that $\min(k, \lambda, \rho, \sigma) > 0$ $\left|\frac{x}{\xi}\right| < 1$ and $\text{Re}(k+\rho j-\mu+1) > 0$

.....(2)

Proof:-Result I For the proof of this result we shall utilize following definition introduced by srivastava [9] or general class of polynomials

$$S_n^m (X) = \sum_{j=0}^{\lfloor n \rfloor} \frac{(-n)_{mj}}{j!} A_{l,j} X^j \tag{2.2}$$

Where m is an arbitrary positive integer and the coefficient ($A_{l,j} > 0$) are arbitrary constant real or complex

Expressing the general class of polynomials $S_n^m (x)$ occuring on its left hand side in the series from given (2.2) the left hand side of (2.1) {say \oplus } takes the following form

$$\oplus = D_{k,\alpha,x}^n \left[X^\mu (X + \xi)^{-\lambda} \sum_{j=0}^{\lfloor n \rfloor} \frac{(-n)_{mj}}{j!} A_{l,j} X^{\rho j} \{(X + \xi)^{-\sigma j}\} \right]$$

Using the following form of the Binomial theorem

$$(X + \xi)^{-\lambda} = \xi^{-\lambda} \sum_{m=0}^{\infty} \frac{(\lambda)_m}{m!} \left(\frac{-x}{\xi}\right)^m \tag{2.3}$$

In the above expression we have

$$\oplus = \xi^{-\lambda} \sum_{j=0}^{\lfloor l \rfloor} \frac{(-l)_{mj}}{j!} A_{l,j} \xi^{-\sigma j} \sum_{m=0}^{\infty} \frac{(\lambda+\sigma)_m}{m!} \frac{(-1)^m}{\xi^m} D_{k,\alpha,x}^n (X^{k+\rho j+m})$$

We use the fractional derivative operator defined in the following manner [15]

$$D_{k,\alpha,x}^n (x^\mu) = \prod_{r=0}^{n-1} \left[\frac{\sqrt{\mu + rk + 1}}{\sqrt{\mu + rk - \alpha + 1}} \right] x^{\mu+nk}$$

Where $\alpha \neq \mu+1$ and α and k are not necessarily integers

and after simplification we get required result (2.1)

Proof:- Result II First Taking as method in proof I and then using by mellin Barnes type contour integral for H- function for one variable and then simplification we get required result (2.2)

Special case I :- As special case of our main result if we take $\sigma=0$ and $\lambda=0$ we deduce the Then the formula (2.1) we have

$$D_{k,\alpha,x}^n (X^\mu S_n^m X^\rho) = \sum_{j=0}^{\lfloor \frac{n}{m} \rfloor} \frac{(-n)_{mj}}{j!} A_{n,j} \prod_{g=0}^{n-1} \frac{\Gamma_{\mu+\rho j+gk+1}}{\Gamma_{\mu+\rho j+gk-\alpha+1}} X^{\mu+\rho j+nk} \quad (3.1)$$

Special case :-II if we take $\sigma=0$

$$D_{k,\alpha,x}^n (X^\mu S_n^m X^\rho H_{P,Q}^{M,N}(X^\mu)) = (-1)^m X^{\mu+m+nk} \xi^{-\lambda} \sum_{j=0}^{\lfloor \frac{n}{m} \rfloor} \frac{(-n)_{mj}}{j!} A_{n,j} X^{\rho j} H_{P+1,Q+1}^{M,N+1} \left[\begin{matrix} (-\mu - \mu s - m - \rho j, k)_{s=0,n-1} (a_j, \alpha_j)_{1,P} \\ X^\mu / (b_j, \beta_j)_{1,q} (-\alpha - \mu - \mu s - m - \rho j, k)_{s=0,n-1} \end{matrix} \right] \quad (3.2)$$

if we take $\lambda = 0$ in (3.2) while this is independent from λ i.e. there is no change in (3.2)

3. Conclusion

In this paper we get fractional differential operator formulae involving special function and general class of polynomials. and their special cases.

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5. Refrence

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