JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Variety of right singular *l*-fuzzy languages

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Abstract : In this paper, we give variety description of right singular *l*-fuzzy languages. Also we provide the Eilenberg variety theorem for the class of regular right singular *l*-fuzzy languages.

***IndexTerms* -** Generalized fuzzy languages,right singular *l*-fuzzy languages, $*$-variety of *l*-fuzzy languages, Conjunctive variety of *l*-fuzzy languages,right singular semiring.

I. INTRODUCTION Theory of formal languages is one of the fundamental concept in theoretical computer science. N.Chomsty classified languages over a finite alphabet into four classes, known as Chomsty hierarchy [4]. The class of regular (recognizable) languages is one among these classes with lot of combinatorial properties. The study of regular languages using monoid was initiated by Kleene [5]. The concept of fuzzy automaton was introduced by Wee in 1967. More on recent development of algebraic theory of fuzzy automata and formal fuzzy languages can be found in the book by Mordeson and Malik [6]. The varieties of fuzzy languages were introduced by Petkovic [7]. He proved that the class of all recognizable fuzzy languages is a variety and there is a one-one correspondence between the variety of all recognizable fuzzy languages and the pseudovariety of finite monoids. Semiring recognizable languages was first studied by Polak [9]. In [10], he introduced the concept of syntactic semiring of a language and studied its properties. Also he established a one-one correspondence between the lattices of all conjunctive variety of languages and pseudovariety of finite idempotent semirings.

We introduce the notion of $*$-variety of monoid recognizable *l*-fuzzy languages in [3]. Here we obtain a mutual isomorphism between varieties of *l*-fuzzy languages and varieties of finite monoids. In [2] we introduce the notion of variety of semiring recognizable *l*-fuzzy languages. Also we obtain a one to one correspondence between varieties of semiring recognizable *l*-fuzzy languages and all pseudovarieties of finite idempotent semirings.

The aim of this paper is to provide a variety structure of right singular recognizable *l*-fuzzy languages.

2.Preliminaries

In this section we recall the basic definitions, results and notations that will be used in the sequel. All undefined terms are as in [5, 6, 8, 11]. A lattice is a partially ordered set in which every subset consisting of two element has a least upper bound and a greatest lower bound. A lattice *l* is called complemented if it is bounded and if every element in *l* has a complement. A lattice *l* is called a complete lattice if every nonempty subset of *l* has greatest lower bound and least upper bound in *l*.

Definition 2.1 (cf.[10]). *An idempotent semiring is a nonempty set S together with two binary operations + and · and two constant elements* 0 *and* 1 *such that*

i) $(S, +, 0$ *is a commutative idempotent monoid.*

ii) $(S, ., 1)$ *is a monoid.*

iii) the distributive laws $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $(a + b) \cdot c = a \cdot c + b \cdot c$ *hold for every* $a; b; c \epsilon S$.

iv) $0 \cdot a = a \cdot 0 = 0$ *for every a.*

Let A be a finite set. When we deal with languages A is called an alphabet and elements of A are called letters. A finite sequence of letters in A is called a word. The length of the word w is the number of letters of A occurring in w. $\overline{\lambda(u)}$ A word of length zero is called empty word and is denoted by ε. A^+ denotes the set of all nonempty words over an alphabet A and $A^* = A^+ \cup \{\varepsilon\}$ is a monoid under the operation concatenation, called free monoid over A. A subset of A^* is called the language L over an alphabet A.

Let $F(A^*)$denote the set of all finite subsets of. A^*. This set equipped with the operations usual union and multiplication $UV = \{ uv \mid u \in U, v \in V \}$form the free idempotent semiring over the alphabet A.

Let l be a complete complemented distributive lattice. Any function λ from A^* into l is called a l-fuzzy language over the alphabet A. The complement $\bar{\lambda}$ of a l-fuzzy language λ is defined as $\bar{\lambda}(u) = \overline{\lambda(u)}$ where $\overline{\lambda(u)}$ denotes the complement of $\lambda(u)$ in l.

For l-fuzzy languages λ_1, λ_2 over A, their join $(\vee)$ and meet $(\wedge)$ are defined by $(\lambda_1 \vee \lambda_2)(u) = \lambda_1(u) \vee \lambda_2(u)$ *and* $(\lambda_1 \wedge \lambda_2)(u) = \lambda_1(u) \wedge \lambda_2(u)$.

Let λ_1, λ_2 be l-fuzzy languages over A. Then their left and right quotients are defined by

$$\lambda_1^{-1}\lambda_2(u) = \vee_{v \in A^*}(\lambda_2(vu) \wedge \lambda_1(v)), \ u \in A^*.$$

and

$$\lambda_2^{-1}\lambda_1(u) = \vee_{v \in A^*}(\lambda_2(uv) \wedge \lambda_1(v)), \ u \in A^*.$$

Let A and B be finite alphabets and $\varphi: A^* \to B^*$ be a homomorphism. Let λ be a l-fuzzy language over B. The inverse of λ under φ is a l-fuzzy language $\lambda\varphi^{-1}$ over A defined by $\lambda\varphi^{-1}(u) = \lambda(\varphi(u)), u \in A^*$.

Let $c \in l$ then the scalar product $c \cdot \lambda$ of the l-fuzzy language λ is defined as $c \cdot \lambda(u) = c \wedge \lambda(u)$.

Let λ be a l-fuzzy language over A. The c-cut of λ is the crisp language $\lambda_c$defined by $\lambda_c = \{u \in A^* \mid \lambda(u) \geq c\}$

A family of recognizable l-fuzzy languages is a $*$-variety of l-fuzzy languages, if it is closed under joins, meets, complements, scalar products, quotients, inverse homomorphic images and cuts.

Let λ be a l-fuzzy language over A. The function $\lambda_{min}: F(A^*) \to l$ defined by $\lambda_{min}(U) = \wedge_{u \in U} \lambda(u), U \in F(A^*)$

is called the generalized fuzzy language determined by λ. If $|U| = 1$ *then* $\lambda_{min}(u) = \lambda(u)$. So we can view $\lambda_{\min:}$ as the generalization of λ.

Let λ_{1min} *and* λ_{2min} be generalized fuzzy languages determined by λ_1 and λ_2 respectively. Then their meet, left quotient and right quotient are defined as follows

$$(\lambda_{1min} \wedge \lambda_{2min})(U) = \lambda_{1min}(U) \wedge \lambda_{2min}(U).$$
$$\lambda_{1min}^{-1}\lambda_{2min}(U) = \vee_{v \in A^*}(\lambda_{2min}(vU) \wedge \lambda_{1min}(v)),$$
$$\lambda_{2min}^{-1}\lambda_{1min}(U) = \vee_{v \in A^*}(\lambda_{2min}(Uv) \wedge \lambda_1(v)), \quad \text{for } U \in F(A^*)$$

Theorem: 2.2. Let λ_1, λ_2 be l-fuzzy languages over A. Then

(i). $\lambda_1 \wedge \lambda_2))_{min} = \lambda_{1min} \wedge \lambda_{2min}$

(2) $(\lambda_1^{-1}\lambda_2)_{min} = \lambda_{1min}^{-1}\lambda_{2min}$,

(3) $(\lambda_2^{-1}\lambda_1)_{min} = \lambda_{2min}^{-1}\lambda_{1min}$,

Theorem 2.3. *Let A and B be finite alphabets* φ *from* $F(A^*)$ to $F(B^*)$ *be a semiring homomorphism. If* λ *is a l-fuzzy language over B, then* $(\lambda\varphi^{-1})_{\min} = \lambda_{min}\varphi^{-1}$

Definition 2.4. *Let λ be a l-fuzzy language over A. We say that λ is recognized by an idempotent semiring S, if there exist a semiring homomorphism $\beta: F(A^*) \to S$ and a l-fuzzy ordered ideal γ of S such that $\lambda_{min} = \gamma\beta$. We also say that the idempotent semiring S recognizes λ by a homomorphism $\beta: F(A^*) \to S$.*

Definition 2.5. *Let lC be a family of l-fuzzy languages and lC_{min} be the family of associated generalized l-fuzzy languages. We say that lC is a conjunctive variety if lC_{min} is closed under finite meet, quotients and inverse homomorphic images*

For a conjunctive variety of *l*-fuzzy languages $lF(A^*)$, ,let lF^s be the family of finite idempotent semirings defined by $lF^s = \{Syn_{\lambda_{min}}: \lambda \in lF(A^*)\}$. For a pseudo variety of idempotent semirings S , letS^f be the family of *l*-fuzzy languages defined by $S^f = \{\lambda: \lambda \in lF(A^*) for\ some\ A\ and Syn_{\lambda_{min}}: \in S\}$.

Theorem 2.6. *The mappings $lF \to lF^s$ and $S \to S^f$ are mutually inverse lattice isomorphisms between the lattices of all conjunctive varieties of recognizable l-fuzzy languages and all pseudovarieties of finite idempotent semirings.*

3 Right Singular *l*-Fuzzy Languages

Definition 3.1. *A l-fuzzy laguage $\lambda : A^+ \to l$ is said to be right singular if it satisfies the condition $\lambda(puvq) = \lambda(pvq)$ for all p, q, u, v inA^+.*

The class of right singular *l*-fuzzy languages on A^+ is denoted by RlFL(A^+).

Example 3.2. *Let* l = ({c},{d},{c,d}, ∅, ∪, ∩) *be a complete distributive lattice*

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and A = { a, b}; g. Let $\lambda : A^+ \to l$ be defined by

$$\lambda(u) = \begin{cases} \{c\} & if\ u \in aA^+a \\ \{d\} & \text{if } u \in bA^+b \\ \phi & otherwise \end{cases}$$

;

Then λ is a right singular l-fuzzy language.

4 $*$-variety of right singular *l*-fuzzy languages

Lemma.4.1. *Let* $\lambda \in$ RlFL(A^+), *then* $\bar{\lambda} \in$ RlFL(A^+).

Proof. Since $\lambda \in$ RlFL(A^+).we have $\lambda(puvq) = \lambda(pvq)$ for all $p, q, u, v \in A^+$: So

$$\begin{aligned} \bar{\lambda}(puvq) &= \overline{\lambda(puvq)} \\ &= \overline{\lambda(pvq)} \\ &= \bar{\lambda}(pvq) \end{aligned}$$

for all $p,q,u,v \in A_+$.Thus $\bar{\lambda} \in$ RlFL(A^+).Hence RlFL(A^+).is closed under complements.

Lemma 4.2. RlFL(A^+).*is closed under scalar multiplication.*

Proof. Let $\lambda \in$ RlFL(A^+), and c∈*l*, then

$$\begin{aligned} (c \cdot \lambda)(puvq) &= c \wedge (\lambda(puvq)) \\ &= c \wedge \lambda(pvq) \\ &= (c \cdot \lambda)(pvq), \end{aligned}$$

for all $p, q, u, v \in A^+$. Thus $c \cdot \lambda \in$RlFL(A^+). Hence RlFL(A^+). is closed und scalar multiplication .

The following result shows that RlFL(A^+).is closed under join and meet.

Lemma 4.3. *Let $\lambda_1, \lambda_2 \in$* RlFL(A^+).*Then , $\lambda_1 \vee \lambda_2$ and $\lambda_1 \wedge \lambda_2$ are in* RlFL(A^+).

Proof. Since $\lambda_1, \lambda_2 \in$ RlFL(A^+)., we have $\lambda_1(puvq) = \lambda_1(pvq)$ and $\lambda_2(puvq) = \lambda_2(pvq)$ for all $p, q, u, v \in A^+$. So

$$\begin{aligned} \lambda_1 \vee \lambda_2(puvq) &= \lambda_1(puvq) \vee \lambda_2(puvq) \\ &= \lambda_1(pvq) \vee \lambda_2(pvq) \\ &= \lambda_1 \vee \lambda_2(pvq), \text{ for all } p, q, u, v \in A^+. \end{aligned}$$

Thus $\lambda_1 \wedge \lambda_2 \in$RlFL($A^+$). Since $\lambda_1 \wedge \lambda_2 = \overline{\overline{\lambda_1} \vee \overline{\lambda_2}}$, *we* have $\lambda_1 \wedge \lambda_2 \in$ RlFL(A^+).

Lemma.4.4. *Let λ be a right singular l-fuzzy language on A^+. B be a finite alphabet and $\varphi: B^+ \to A^+$ be a homomorphism. Then $\lambda\varphi^{-1}$ is a right singular l-fuzzy language over B where $\lambda\varphi^{-1}(u) = \lambda(\varphi(u))$ for all $u \in B^+$.*

Proof. Since $\lambda \in$ RlFL(A^+).we have $\lambda(puvq) = \lambda(pvq)$ for all $p, q, u, v \in A^+$. So

$$\begin{aligned}\lambda\varphi^{-1}(rxys) &= \lambda(\varphi(rxys)) \\ &= \lambda(\varphi(r)\varphi(x)\,\varphi(y)\varphi(s)) \\ &= \lambda(\varphi(r)\varphi(y)\varphi(s)) \\ &= \lambda(\varphi(rys)) \\ &= \lambda\varphi^{-1}(rys) \text{ for all r,s,x,y} \in B^+\end{aligned}$$

Thus $\lambda\varphi^{-1}$ is a right singular *l*-fuzzy language over *B*.

From the above lemma it follows that RlFL(A^+) is closed under the inverse homomorphic images.

Lemma.4.5. *Let* $\lambda_1, \lambda_2 \in$ RlFL(A^+).

Then $(i)\lambda_1{}^{-1}\lambda_2 \in$ RlFL(A^+).

(ii) $\lambda_2{}^{-1}\lambda_1 \in$ RlFL(A^+).

Proof. **(i)** Since $\lambda_1, \lambda_2 \in$ RlFL(A^+).we have $\lambda_1(puvq) = \lambda_1(pvq)$ and $\lambda_2(puvq) = \lambda_2(pvq)$ for all $p, q, u, v \in A^+$ *. So*

$$\begin{aligned}\lambda_1{}^{-1}\lambda_2\,(puvq) &= \vee_{w \in A^+} \{\lambda_2(wpuvq) \wedge \lambda_1(w)\} \\ &= \vee_{w \in A^+} \{\lambda_2((wp)uvq) \wedge \lambda_1(w)\} \\ &= \vee_{w \in A^+} \{\lambda_2((wp)vq) \wedge \lambda_1(w)\} \\ &= \vee_{w \in A^+} \{\lambda_2(wpvq) \wedge \lambda_1(w)\} \\ &= \lambda_1{}^{-1}\lambda_2\,(pvq)\end{aligned}$$

for all $p, q, u, v \in A^+$ .Thus$\lambda_1{}^{-1}\lambda_2 \in$ RlFL(A^+).

(ii) Similarly if $\lambda_1, \lambda_2 \in$ RlFL(A^+) then $\lambda_2{}^{-1}\lambda_1 \in$ RlFL(A^+). Thus RlFL(A^+) is closed under left and right quotient .

Lemma 4.6. *Let* $\lambda \in$ RlFL(A^+) *and* $\lambda_c = \{u \in A^+ \mid \lambda(u) \geq c\}$ *for all* $c \in l$. *Then* RlFL(A^+).*is closed under the c-cut. (ie*$\chi_{\lambda_c} \in$ RlFL(A^+) *for all* $c \in l$*).*

Proof. Since $\lambda \in$ RlFL(A^+).we have $\lambda(puvq) = \lambda(pvq)$ for all for all $p, q, u, v \in A^+$ .So

$$puvq \in \lambda_c \Leftrightarrow c \leq \lambda(puvq) = \lambda(pvq) \Leftrightarrow pvq \in \lambda_c$$

for all for all $p, q, u, v \in A.^+$ *H*ence RlFL(A^+).is closed under *c*-cut.

Theorem.4.7. RlFL(A^+) *is a* $*$*-variety of l-fuzzy languages.*

Proof. By Lemmas 4.1, 4.2, 4.3, 4.4,4.5 and 4.6, RlFL(A^+).is a $*$-variety of *l*-fuzzy languages.

5.Conjunctive variety of right singular *l*-fuzzy languages

Let $\lambda \in$ RlFL(A^+). and λ_{min} be the generalized fuzzy language determined by λ. Then from the definition of λ_{min}, we have

$$\begin{aligned}\lambda_{min}(\mathrm{pUVq}) &= \wedge_{uv \in UV}\, \lambda(puvq) \\ &= \wedge_{v \in V}\, \lambda(pvq) \\ &= \lambda_{min}(\mathrm{pVq})\end{aligned}$$

for all $p, q \in A^+$ and $U, V \in F(A^*)$.

Thus λ is right singular if and only if the generalized fuzzy language determined by λ satisfies the condition

$$\lambda_{min}(\mathrm{pUVq}) = \lambda_{min}(\mathrm{pVq})$$

for all $p, q \in A^+$ and $U, V \in F(A^*)$.

The following result shows that *RlFL*$_{min}(A^+)$. is closed under the operation meet.

Lemma 5.1. *If* λ_1 *and* λ_2 *are in* RlFL(A^+).*then* $(\lambda_1 \wedge \lambda_2))_{min}$*belongs to RlFL*$_{min}(A^+)$.

Proof. Let $\lambda_1, \lambda_2 \in$ RlFL(A^+). Then $\lambda_{1min}(pUVq) = \lambda_{1min}(pVq)$ and $\lambda_{2min}(pUVq) = \lambda_{2min}(pVq)$, for all $p, q \in A^+$ and $U, V \in F(A^+)$.

By the definition of $\wedge$, We have

$$\begin{aligned}(\lambda_{1min} \wedge \lambda_{2min})(\mathrm{pUVq}) &= \lambda_{1min}(pUVq) \wedge \lambda_{2min}(\mathrm{pUVq}) \\ &= \lambda_{1min}(pVq) \wedge \lambda_{2min}(\mathrm{pVq}) \\ &= (\lambda_{1min} \wedge \lambda_{2min})(\mathrm{pVq})\end{aligned}$$

for all $p, q \in A^+$ and $U, V \in F(A^*)$. Thus $(\lambda_{1min} \wedge \lambda_{2min}) \in$ *RlFL*$_{min}(A^+)$.).Since$(\lambda_1 \wedge \lambda_2))_{min} = \lambda_{1min} \wedge \lambda_{2min}$, $(\lambda_1 \wedge \lambda_2))_{min}$belongs to *RlF L*$_{min}((A^+)$.

The following result shows that *RlF* $L_{min}(A^+)$.is closed under quotients.

Lemma 5.2. *If* λ_1, λ_2 $\in$ RlFL(A^+).*then* $({\lambda_1}^{-1}\lambda_2)_{min}$ *and* $({\lambda_2}^{-1}\lambda_1)_{min}$ *are in* *RlFL*$_{min}(A^+)$.

Proof. Let λ_1, λ_2 $\in$ RlFL(A^+) then $\lambda_{1min}(pUVq) = \lambda_{1min}(pVq)$ and $\lambda_{2min}(pUVq) = \lambda_{2min}(pVq)$, for all p, q $\in A^+$ and $U, V \in F(A^+)$. By the definition of left quotient, we have

$$\begin{aligned}{\lambda_{1min}}^{-1}\lambda_{2min}(pUVq) &= \vee_{w\in A^+}(\lambda_{2min}(w(pUVq)) \wedge \lambda_{1min}(w)) \\ &= \vee_{w\in A^+}(\lambda_{2min}(wp(UV)q)) \wedge \lambda_{1min}(w)) \\ &= \vee_{w\in A^+}(\lambda_{2min}(wp(V)q)) \wedge \lambda_{1min}(w)) \\ &= \vee_{w\in A^+}(\lambda_{2min}(w(pVq)) \wedge \lambda_{1min}(w)) \\ &= {\lambda_{1min}}^{-1}\lambda_{2min}(pVq)\end{aligned}$$

for all p, $q \in A^+$ and $U, V \in F(A^+)$. Thus ${\lambda_{1min}}^{-1}\lambda_{2min} \in$ *RlF* $L_{min}(A^+)$. Similarly we can prove that ${\lambda_{2min}}^{-1}\lambda_{1min}$ belongs to *RlFL*$_{min}(A^+)$. Since $({\lambda_1}^{-1}\lambda_2)_{min} = {\lambda_{1min}}^{-1}\lambda_{2min}$ *and* $({\lambda_2}^{-1}\lambda_1)_{min} = {\lambda_{2min}}^{-1}\lambda_{1min}$, we have $({\lambda_1}^{-1}\lambda_2)_{min}$ *and* $({\lambda_2}^{-1}\lambda_1)_{min}$ *belong to RlF* $L_{min}(A^+)$.

Lemma 5.3. *Let A, B be finite alphabets,* $\varphi: F(A^+) \to F(B^+)$.) *be a homomorphism and* $\lambda \in$ *RlFL*(B^+). *Then*$(\lambda\varphi^{-1})_{min} \in$ *RlFL*$_{min}(A^+)$.).

Proof. We have

$$\begin{aligned}(\lambda_{min}\varphi^{-1})(\mathrm{pUVq}) &= \lambda_{min}\varphi(pUVq) \\ &= \lambda_{min}(\varphi(p)\varphi(U)\varphi(V)\varphi(q)) \\ &= \lambda_{min}(\varphi(p)\varphi(V)\varphi(q)) \\ &= \lambda_{min}\varphi(pVq) \\ &= (\lambda_{min}\varphi^{-1})(\mathrm{pVq})\end{aligned}$$

for all p, $q \in A^+$ and $U, V \in F(A^+)$. Thus $\lambda_{min}\varphi^{-1}$ $\in$ *RlFL*$_{min}(A^+)$. *Since* $\lambda_{min}\varphi^{-1} = (\lambda\varphi^{-1})_{min}$, $(\lambda\varphi^{-1})_{min} \in$ *RlFL*$_{min}(A^+)$.

From the above lemma it follows that *RlF* $L_{min}(A^+)$is closed under inverse homomorphic images.

Theorem>5.4. *RlFL*(A^+) *is a conjunctive variety of l-fuzzy languages.*

Proof. By Lemma 5.1 and 5.2, *RlFL*$_{min}(A^+)$.is closed under meet and quotients. By Lemma 5.3, *RlFL*$_{min}(A^+)$ is closed under inverse homomorphic images. Hence *RlFL*(A^+) is a conjunctive variety of *l*-fuzzy languages.

A semiring $(S, +, \cdot)$ is right singular under multiplication if $x \cdot y = y$ for all $x; y$ in S.

Theorem 5.5 *Let S be a finite right singular semiring under multiplication recognizing the l-fuzzy language* λ. *Then* $\lambda \in$RlFL(A^+).

Proof. Since S is a finite right singular semiring recognizing the *l*-fuzzy language λ over the alphabet A, there exist a semiring homomorphism $\beta : F(A^+) \to S$ and a *l*-fuzzy ordered ideal γ of S such that $\lambda_{min} = \gamma\beta$. Since S is right singular semiring, we have

$$\begin{aligned}\text{Type equation here.}\lambda_{min}(\mathrm{pUVq}) &= \gamma\beta(pUVq) \\ &== \gamma(\beta(pUVq)) \\ &= \gamma(\beta(p)\beta(U)\beta(V)\beta(q)) \\ &= \gamma(\beta(p)\beta(V)\beta(q)) \\ &= \gamma(\beta(pVq)) \\ &= \gamma\beta(pVq) \\ &= .\lambda_{min}(\mathrm{pVq})\end{aligned}$$

for all p, $q \in A^+$ and $U, V \in F(A^+)$. Thus $\lambda \in$RlFL(A^+)

Theorem 5.6. *A l-fuzzy language* $\lambda : A^+ \to l$ *is right singular if and only if its syntactic semiring Syn(*λ_{min}*) is right singular under multiplication.*

Proof. Assume that the *l*-fuzzy language λ is right singular. Then $\lambda(puvq) = \lambda(pvq)$ for all $u,v,p,q \in A^+$. The generalized fuzzy language λ_{min} determined by λ satisfies the condition for all $p,q \in A^+$ and U, V $\in F(A^+)$, $\lambda_{min}(pUVq) = \lambda_{min}(pVq)$. That is, for all p, $q \in A^+$ and $U,V \in F(A^+)$, $\lambda_{\mathrm{min}}(\cup_{w\in UV}(pwq)) = \lambda_{\mathrm{min}}(\cup_{v\in V}(pvq))$. Thus $UV \sim_{min}$ V. Hence $[UV]_{\sim min} = [V]_{\sim min}$. That is, $[U]_{\sim min}[V]_{\sim min} .= [V]_{\sim min}$. Thus the syntactic semiring Syn(λ_{min}) of λ is right singular under multiplication.

Conversely assume that the syntactic semiring Syn(λ_{min}) of λ is right singular under multiplication. *So* $[U]_{\sim min}[V]_{\sim min} = [V]_{\sim min}$ for all $[U]_{\sim min}, [V]_{\sim min} \in Syn(\lambda_{min})$. That is, , $[UV]_{\sim min} = [V]_{\sim min}$ .Thus $UV \sim_{min} V$. Hence for all all $p, q \in A^+$ and $U, V \in F(A^+)$, $\lambda_{\min}(\cup_{w \in UV}(pwq)) = \lambda_{\min}(\cup_{v \in V}(pvq))$. *That is*, $\lambda_{min}(pUVq) = \lambda_{min}(pVq)$, for all $p, q \in A^+$ and $U, V \in F(A^+)$. Therefore λ is a rightsingular *l*-fuzzy language.

Theorem 5.7. *There exists a one to one correspondense between* RlFL(A^+).*and the pseudovariety* $F^s = \{Syn(\lambda_{min}) : \lambda \in \text{RlFL}(A^+)\}$ *of rightsingular semiring.*

Proof. Let $S \in F^s$. Then by theorem5.5, $\lambda \in RlFL(B^+)$.

Conversely if $\lambda \in RlFL(B^+)$.Then by theorem5.6 and theorem2.6 $Syn(\lambda_{min}) \in F^s$

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