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On k-Root Square Mean Labeling of Graphs

Aswathy H[1], Sandhya S.S[2]

Author 1: Research Scholar

Sree Ayyappa College for Women, Chunkankadai.

Author 2: Assistant Professor, Department of Mathematics,

Sree Ayyappa College for Women, Chunkankadai.

[Affiliated to ManonmaniamSundararanar University,

Abishekapatti – Tirunelveli - 627012, Tamilnadu, India]

E mail : aswathyrh22@gmail.com[1]

sssandhya2009@gmail.com[2]

Abstract

A graph G = (V,E) with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices x∈V with distinct labels f (x) from 1,2,……….. q+1 in such a way that when each edge e = uv is labeled with f (e=uv) = $\left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G . In this paper we investigate the K- Root Square Mean labeling of some Graphs.

Key words: Labeling, Root Square Mean Labeling, Path, Twig , Triangular Ladder.

AMS Subject Classification : 05C78

Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by V(G) and the edge set is denoted by E(G). For all detailed survey of graph labelling, we refer to Galian [1]. For all other standard terminology and notations we follow Harary[2]. S.S.Sandhya, S.Somasundaram and S. Anusa introduced the concept of Root Square Mean labeling of graphs in [3]. In this paper we investigate the k- Root Square Mean labeling of some Graphs.

Definition 1.1 : A graph G = (V,E) with p vertices and q edges is said to be a **Root Square Mean graph** if it is possible to label the vertices x∈V with distinct labels f (x) from 1,2,……….. q+1 in such a way that when each edge e = uv is labeled with f (e=uv) $= \left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the edge labels are distinct. In this case, f is called a Root Square Mean labeling of G.

Definition 1.2 : A function f is called K- Root Square Mean labeling of a graph $G = (V,E)$ with p vertices and q edges if $f : V(G) \to \{k, k+1, \dots, k+q\}$ be an injective function and the induced edge labeling $f(e = uv)$ be defined by $f(e = uv)$ be defined by $f(e) = \left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$ with distinct edge labels.

Theorem 1.3 : Path P_n is a K- Root square Mean graph for all k and $n \geq 2$.

Proof: Let $V(P_n) = v_i;\ 1 \leq i \leq n$ and $E(P_n) = \{e_i = \{v_i v_{i+1}\}\ ;\ 1 \leq i \leq n-1\}$

Define a function $f: V(P_n) \to \{k, k+1, \dots, k+q\}$ by

$f(v_i) = K + i - 1\,; 1 \leq i \leq n$

The edge labels are $f(v_i v_{i+1}) = K + i - 1; 1 \leq i \leq n-1$.

The function f provides k- Root square mean labeling of the graphs.

Hence P_n is a k- Root square Mean graph.

Example 1.4: 100-Root Square Mean labeling of P_5 is give below.

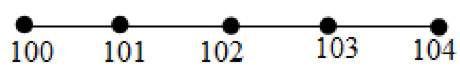

Figure 1

Theorem 1.5 : Twig graph is k- Root square mean graph for all $k$and $n \geq 3$

Proof : Let G be a Twig graph.

Let $V(G) = \{v_i\,; 1 \leq i \leq n; u_i\,; 1 \leq i \leq n-2$ and $w_i\,; 1 \leq i \leq n-1\,\}$

$E(G) = \{u_i v_i\,, v_i w_i; 1 \leq i \leq n-2\} \cup \{v_i v_{i+1}\,; 1 \leq i \leq n-1\}$

Define a function $f: V(G) \to \{k, k+2, k+3, \dots, k+q\}$ by

$f(v_i) = k\ \ f\{v_{i+2}\} = k + 1 + 3i; 1 \leq i \leq n-2$.

$f(w_i) = k + 3i\ \ ; 1 \leq i \leq n-2$.

$f(u_i) = k + 3i - 1\ ; 1 \leq i \leq n-2$.

Then the induced edge labels are

$f(v_i v_{i+1}) = k + 3i - 3$; $1 \leq i \leq n - 1$.

$f(v_{i+1} w_i) = k + 3i - 1$; $1 \leq i \leq n - 2$.

$f(v_{i+1} u_i) = k + 3i - 2$; $1 \leq i \leq n - 2$.

The function f provides k- Root square Mean labeling of the graph.

Hence twig is a k- Root Square mean graph.

Example 1.6 : 200-Root square mean labeling of twig graph is shown below.

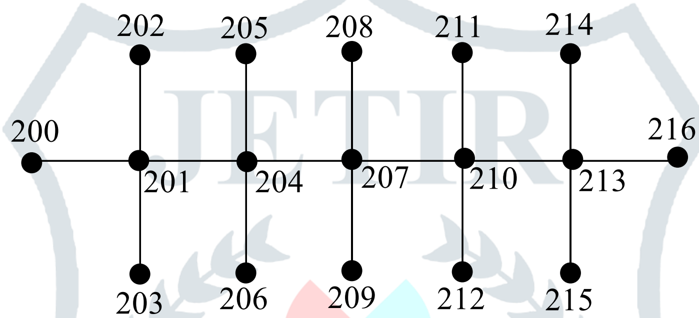

Figure 2

Theorem 1.7 :

The Triangular Ladder TL_n is k- Root Square mean graph for all k and $n \geq 2$.

Proof : Let TL_n be a Triangular ladder. The graph TL_n contains two paths of length n such as $v_1 v_2 \dots v_2$ and $u_1 u_2 \dots u_n$ join $v_i u_{i+1}; 1 \leq i \leq n - 1$

The edge set as $\{u_i u_{i+1}, v_i v_{i+1}, v_i u_{i+1}; 1 \leq i \leq n - 1\} \cup \{u_i v_i; 1 \leq i \leq n\}$

TL_n has 4n-3 edges

The labeling pattern of TL_n is

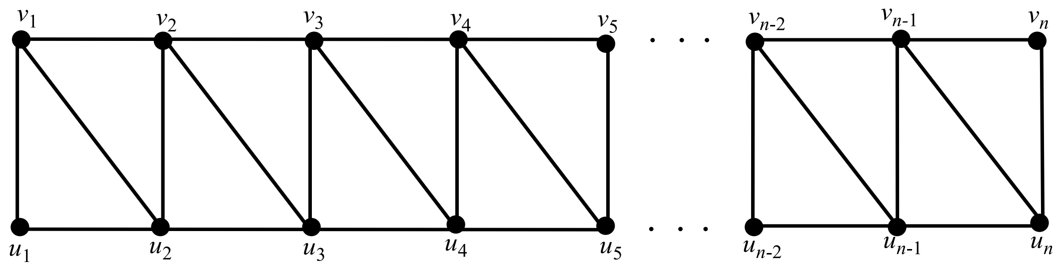

Figure 3

Define a function $f: V(TL_n) \to \{K, K+1, \dots, K+q\}$ by

$f(v_{i+1}) = k + 4i + 1 \;; 1 \le i \le n-1$.

$f(u_1) = k$

$f(u_{i+1}) = k + 4i - 1 \;; 1 \le i \le n-1$

The induced edge labels are

$f(u_i u_{i+1}) = k + 4i - 3 \;; 1 \le i \le n-1$

$f(v_i v_{i+1}) = k + 4i - 1 \;; 1 \le i \le n-1$.

$f(u_i v_i) = k + 4i - 4 \quad ; 1 \le i \le n$

$f(v_i u_{i+1}) = k + 4i - 2 \;; 1 \le i \le n-1$.

The function f provides k- Root square mean labeling of the graphs.

Hence TL_n is a k- Root square Mean graph.

Example 1.8 : 500 - Root square mean labeling of TL_5 is shown below.

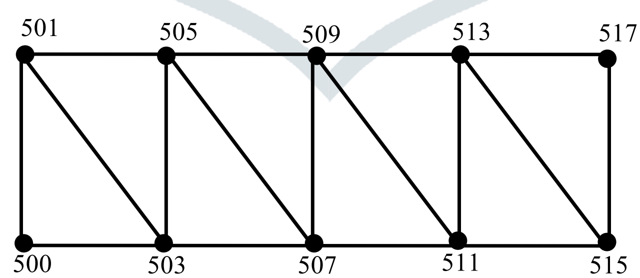

Figure 4

Theorem 1.9: $L_n \odot K_1$ is k- Root square mean graph for all k and $n \ge 2$

Proof: Labeling pattern of $L_n \odot K_1$ is given below

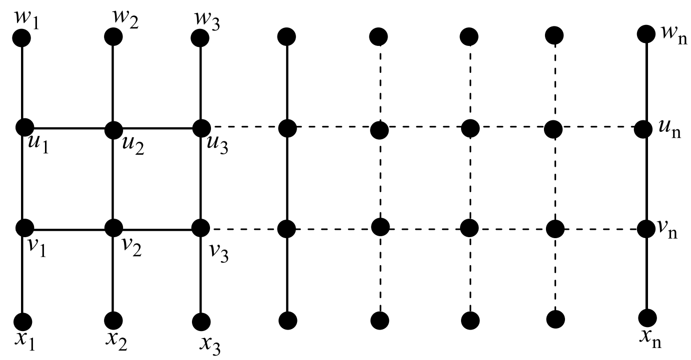

Figure 5

Let $V(L_n \odot K_1) = \{u_i, v_i, x_i, w_i; 1 \leq i \leq n-1\}$ and

$E(L_n \odot K_1) = \{u_i v_i, u_i w_i, v_i x_i; 1 \leq i \leq n-1\} \cup \{u_i u_{i+1}, v_i v_{i+1}; 1 \leq i \leq n-1\}$

Define a function $f: V(L_n \odot K_1) = \{k, k+1, \dots, k+q\}$ by

$f(u_i) = k + 5i - 4 \quad ; 1 \leq i \leq n$

$f(v_i) = k + 5i - 3 \quad ; 1 \leq i \leq n$.

$f(w_1) = k$

$f(w_{i+1}) = k + 5i \quad ; 1 \leq i \leq n-1$.

$f(x_i) = k + 5i - 2 \quad ; 1 \leq i \leq n$.

Then the edge labels are

$f(u_i u_{i+1}) = k + 5i - 2; 1 \leq i \leq n-1$

$f(v_i v_{i+1}) = k + 5i - 1; 1 \leq i \leq n-1$

$f(u_i v_i) = k + 5i - 4; 1 \leq i \leq n$

$f(u_i w_i) = k + 5i - 5; 1 \leq i \leq n$

$f(u_i x_i) = k + 5i - 3; 1 \leq i \leq n$

The function f provides k - Root square mean labeling .

Hence $L_n \odot K_1$ is a k- Root square mean graph.

Example 1.10 : 600 -Root square mean labeling of $L_n \odot K_1$ is shown below

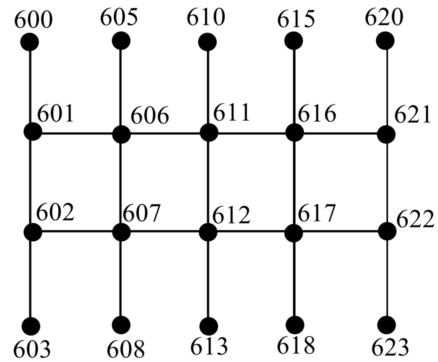

Figure 6

Remarks 1.11: If $n > k + 5,\ k_{n,n}$ is not a k-Root square mean graph.

Remarks 1.12: If $n > k + 3,\ k_{n,n}$ is not a k-Root square mean graph.

Remarks 1.13 : K_{n-e} is a k-Root square mean graph if $n = k + 3$ & $k + 4$.

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