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Fractional Integration of Product of Generalized Galue Type Struve Function and $\bar{H}$ Function

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ABSTRACT

The aim of this research paper is to study the generalized Marichev- Saigo- Maeda fractional integral operators. We will establish two theorems which give the images of the product of generalized Galue type Struve function and $\bar{H}$ function under integral operators in terms of $\bar{H}$ function. Some useful special cases are also obtained from our main theorems.

KeyWords: Marichev-Saigo-Maeda fractional integral operators, Galue type Struve function, $\bar{H}$ function.

Introduction

Generalized Fractional Integral Operators

Let $\alpha,\beta,\eta \in \mathbb{C}$, x > 0 and $\mathrm{Re}(\alpha) > 0$; then the generalized fractional integration operators associated with Gauss Hypergeometric function, Saigo [3] are defined by the following equations;

$$\left(I_{0,+}^{\alpha,\beta,\eta} f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \times {}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right) f(t)\,dt\,, \tag{1}$$

$$= \frac{d^n}{dx^n}\left(I_{0+}^{\alpha+n,\beta-n,\eta-n} f\right)(x); \quad \left(\mathrm{Re}(\alpha)\le 0;\ n=[\mathrm{Re}(-\alpha]+1\right) \tag{2}$$

and

$$\left(I_{-}^{\alpha,\beta,\eta} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta} \times {}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{x}{t}\right) f(t)\,dt \tag{3}$$

$$= (-1)^n \frac{d^n}{dx^n}\left(I_{-}^{\alpha+n,\beta-n,\eta} f\right)(x); \quad \left(\mathrm{Re}(\alpha)\le 0;\ n=[\mathrm{Re}(-\alpha]+1\right) \tag{4}$$

The generalized fractional integration operators of arbitrary order involving Appell function $F_3(.)$ also known as Horn function [20] in the kernel have been introduced by Marichev [4] and later extended and studied by Saigo and Maeda [5], in the following forms;

Let $\alpha,\alpha',\beta,\beta',\gamma\in\mathbb{C}$, $x>0$ and $\mathrm{Re}(\gamma)>0$, then;

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\frac{x^{-\alpha}}{\Gamma(\gamma)}\int_0^x(x-t)^{\gamma-1}t^{-\alpha'}\times F_3\left(\alpha,\alpha',\beta,\beta',\gamma\ ;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt \tag{5}$$

and

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x)=\frac{x^{-\alpha'}}{\Gamma(\gamma)}\int_x^\infty(t-x)^{\gamma-1}t^{-\alpha}\times F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)dt \tag{6}$$

Further from Saigo and Maeda [5] we also have the following two results;

(i) Let $\alpha,\alpha',\beta,\beta',\gamma,\rho\in\mathbb{C}$ be such that, $\mathrm{Re}(\gamma)>0$ and $\mathrm{Re}(\rho)>\max[0,\mathrm{Re}(\alpha+\alpha'+\beta-\gamma)\ ,\ \mathrm{Re}(\alpha'-\beta')]$, then;

$$\left(I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\right)(x)=\Gamma\begin{bmatrix}\rho,\rho+\gamma-\alpha-\alpha'-\beta,\rho+\beta'-\alpha'\\ \rho+\beta',\rho+\gamma-\alpha-\alpha',\rho+\gamma-\alpha'-\beta\end{bmatrix}x^{\rho-\alpha-\alpha'+\gamma-1} \tag{7}$$

where,

$$\Gamma\begin{bmatrix}a,b,c\\ x,y,z\end{bmatrix}=\frac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(x)\Gamma(y)\Gamma(z)}$$

(ii) Let $\alpha,\alpha',\beta,\beta',\gamma,\rho\in\mathbb{C}$ be such that, $\mathrm{Re}(\gamma)>0$ and

$\mathrm{Re}(\rho)<\ 1+\min\ [\ \mathrm{Re}(-\beta),\ \mathrm{Re}(\alpha+\alpha'-\gamma),\ \mathrm{Re}(\alpha+\beta'-\gamma)]$, then;

$$\left(I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\right)(x)=\Gamma\begin{bmatrix}1-\rho-\gamma+\alpha+\alpha',1-\rho+\alpha+\beta'-\gamma,1-\rho-\beta\\ 1-\rho,1-\rho+\alpha+\alpha'+\beta'-\gamma,1-\rho+\alpha-\beta\end{bmatrix}x^{\rho-\alpha-\alpha'+\gamma-1} \tag{8}$$

Generalized Hypergeometric function:

The series definition of hypergeometric function ${}_2F_1(.)$ is given by;

$${}_2F_1(a,b;c;\ x)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}x^n\ , \tag{9}$$

where c is neither zero nor a negative integer. The series (9) is absolutely convergent within the circle of convergence $|x|<1$, on the circle of convergence the series is absolutely convergent if, Re (c-a-b) > 0.

Also, if Re (c-a-b) > 0, Re (c) > Re (b) >0, then;

$${}_2F_1(a,b;c;\ 1)\ =\ \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\ ,$$

The generalized hypergeometric function ${}_pF_q$ defined by Rainville [7], is given by;

$${}_pF_q\begin{bmatrix}a_1,a_2,......,a_p;\\ b_1,b_2,.......,b_q;\end{bmatrix}z\Bigg]=\sum_{m=0}^{\infty}\frac{(a_1)_m,........,(a_p)_m}{(b_1)_m,........,(b_q)_m}\cdot\frac{z^m}{m!} \tag{10}$$

where p and q are positive integers or zero, $\alpha_1,.........,\alpha_p$, and $\beta_1,.........,\beta_q$ take complex values, provided that no zeros appear in the denominator, i.e., $(\beta_j \in \mathbb{C} / Z_0^-;\ j= 1,.........,q)$.

H-function:

The H-function introduced by C. Fox [6], in terms of Mellin-Barnes type of contour integral is defined as follows;

$$H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[z \middle| \begin{matrix} (a_1,\alpha_1),.......,(a_p,\alpha_p) \\ (b_1,\beta_1),.......,(b_q,\beta_q) \end{matrix} \right]$$

$$= \frac{1}{2\pi i}\int_L z^s \phi(s)\,ds, \qquad (z \neq 0) \qquad (11)$$

where,

$$\phi(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)} \qquad (12)$$

Here m, n, p, q are integers satisfying $0 \le m \le q,\ 0 \le n \le p$; $a_j(j=1,......,p)$ and $b_j(j=1,........q)$ are complex parameters, $\alpha_j \ge 0(j=1,........,p), \beta_j \ge 0(j=1,......,q)$ are positive numbers. The contour integral (9) converges absolutely if,

$$T = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j > 0$$

and $|\arg z| < \frac{1}{2}\pi T$

$\bar{H}$-function:

The $\bar{H}$-function was introduced by Inayat Hussain [1] and studied by Bushman and Shrivastava [2] is defined and represented in the following manner,

$$\bar{H}_{p,q}^{m,n}[z] = \bar{H}_{p,q}^{m,n}\left[z \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},\ (a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m},\ (b_j,\beta_j;B_j)_{m+1,q} \end{matrix} \right]$$

$$= \frac{1}{2\pi i}\int_L z^s \bar{\phi}(s)\,ds, \quad (z \neq 0) \qquad (13)$$

where,

$$\bar{\phi}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j s)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j s)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j s)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j s)}, \qquad (14)$$

Here L is a contour starting at the point c-i∞ and terminating at the point c+i∞, $a_j(j=1,......,p)$ and $b_j(j=1,........q)$ are complex parameters, $\alpha_j \geq 0(j=1,........,p)$, $\beta_j \geq 0(j=1,......,q)$, (not all zero simultaneously) and the exponents $A_j(j=1,........,n)$, $B_j(j=m+1,......,q)$ can take integer values.

Sufficient condition for absolute convergence of the contour integral in (11) established by Buschman and Shrivastava [2] is as follows;

$$T=\sum_{j=1}^{m}\beta_j+\sum_{j=1}^{n}\left|A_j\alpha_j\right|-\sum_{j=m+1}^{q}\left|B_j\beta_j\right|-\sum_{j=n+1}^{p}\alpha_j \;>\; 0,$$

and $\left|\arg z\right|<\frac{1}{2}\pi T$,

Generalized Galue Type Struve Function:

Galue [17] introduced a generalization of the Bessel function of order h given by;

$$_{\mu}J_h(z)=\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma(\mu k+h+1)\,k!}\left(\frac{z}{2}\right)^{2k+h}, \qquad z\in R,\ \mu\in N \tag{15}$$

Struve investigated the Struve function of order p, which is given by;

$$H_p(z)=\sum_{k=0}^{\infty}\frac{(-1)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+p+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2k+p+1}, \qquad \text{for all } z\in\square \tag{16}$$

which is a particular solution of non-homogeneous differential equation,

$$z^2y''(z)+zy'(z)+(z^2-p^2)y(z)=\frac{4\left(\frac{z}{2}\right)^{p+1}}{\sqrt{\pi}\,\Gamma\left(p+\frac{1}{2}\right)}$$

and its homogeneous part is Bessel's equation. For several generalization of the Struve function, see [8,9,10,11]

Nisar [12] defined the generalized Galue type Struve function which is the generalized form of Struve function, as:

$$_{a}W_{p,b,c,\delta}^{\mu,\nu}(t)=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{t}{2}\right)^{2k+p+1} \tag{17}$$

Where $a\in N$; $p, b, c\in C$ and $\mu>0,\ \delta>0,\ \nu$ is an arbitrary parameter.

Main Results

Theorem 1: If $\alpha,\alpha',\beta,\beta',\gamma,\rho\in\square$, $x>0,\ T>0,\ \left|\arg z\right| < \frac{1}{2}\pi T$,such that $\mathrm{Re}(\gamma)>0$ and

$\mathrm{Re}\left[\rho+2k\sigma+p\sigma+\sigma+\lambda\xi\right] > \max\left[0,\ \mathrm{Re}(\alpha+\alpha'+\beta-\gamma),\ \mathrm{Re}(\alpha'-\beta')\right]$, then;

$$I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}\,(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\ (f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\Delta-\alpha-\alpha'+\gamma-1}\times\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times\bar{H}_{P+3,Q+3}^{M,N+3}\left[\omega x^{\lambda}\left|\begin{matrix}(1-\Delta,\lambda;1),(1-\Delta-\gamma+\alpha+\alpha'+\beta,\lambda;1),\\ (f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-\Delta-\beta'+\alpha',\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\ (1-\Delta-\beta',\lambda;1),(1-\Delta-\gamma+\alpha+\alpha',\lambda;1),(1-\Delta-\gamma+\alpha'+\beta,\lambda;1)\end{matrix}\right] \qquad (18)$$

where $\Delta=\rho+2k\sigma+p\sigma+\sigma$

Proof: Applying equation (5), (13) and (17) to the left-hand side of (18) and then interchanging the order of summation and integration we have,

$$I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}\,(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\ (f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times\frac{1}{2\pi i}\int_L \omega^{\xi}\theta(\xi)\ \left\{I_{0,+}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho+2k\sigma+p\sigma+\sigma+\lambda\xi-1}\right\}(x)\,d\xi$$

Now applying the Saigo Maeda operator (7) we obtain the right hand side of (18).

Corollary1: If $\alpha,\beta,\gamma,\in$□, $x>0,\ T>0,\ |\arg z|\ <\ \frac{1}{2}\pi T$,such that $\mathrm{Re}(\alpha)>0$ and $\mathrm{Re}\left[\rho+2k\sigma+p\sigma+\sigma+\lambda\xi\right]\ >\max\left[0,\ \mathrm{Re}(\beta-\gamma)\right]$, then;

$$I_{0+}^{\alpha,\beta,\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}\,(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\ (f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\Delta-\beta-1}\times\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times\bar{H}_{P+2,Q+2}^{M,N+2}\left[\omega x^{\lambda}\left|\begin{matrix}(1-\Delta,\lambda;1),(1-\Delta-\gamma+\beta,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\ (f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},(1-\Delta+\beta,\lambda;1),(1-\Delta-\alpha-\gamma,\lambda;1)\end{matrix}\right.\right] \qquad (19)$$

Theorem 2: If $\alpha,\alpha',\beta,\beta',\rho\in\mathbb{C}$, $x>0,\ T>0,\ |\arg z| < \frac{1}{2}\pi T$, such that $\mathrm{Re}(\gamma)>0$ and $\mathrm{Re}[\rho+2k\sigma+p\sigma+\sigma-\lambda\xi] < 1+\min[\ \mathrm{Re}(-\beta),\ \mathrm{Re}(\alpha+\alpha'-\gamma),(\alpha+\beta'-\gamma)]$, then;

$$I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$= x^{\Delta-\alpha-\alpha'+\gamma-1}\times\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times\bar{H}_{P+3,Q+3}^{M,N+3}\left[\omega x^{-\lambda}\left|\begin{matrix}(\Delta+\gamma-\alpha-\alpha',\lambda;1),(\Delta-\alpha-\beta'+\gamma,\lambda;1), & (\Delta+\beta,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q}, & (\Delta,\lambda;1),(\Delta-\alpha-\alpha'-\beta+\gamma',\lambda;1),(\Delta-\alpha+\beta,\lambda;1)\end{matrix}\right.\right]$$

(20)

where $\Delta=\rho+2k\sigma+p\sigma+\sigma$

Proof: Applying equation (6), (13) and (17) to the left-hand side of (20) and then interchanging the order of summation and integration we have,

$$I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$=\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times\frac{1}{2\pi i}\int_L\omega^{\xi}\theta(\xi)\ \left\{I_{0,-}^{\alpha,\alpha',\beta,\beta',\gamma}\ t^{\rho+2k\sigma+p\sigma+\sigma-\lambda\xi-1}\right\}(x)\,d\xi$$

Now applying the Saigo Maeda operator (8) we obtain the right-hand side of (20).

Corollary 2: If $\alpha,\beta,\gamma,\in\mathbb{C}$, $x>0,\ T>0,\ |\arg z| < \frac{1}{2}\pi T$, such that $\mathrm{Re}(\alpha)>0$ and $\mathrm{Re}[\rho+2k\sigma+p\sigma+\sigma-\lambda\xi] < 1+\min[\ \mathrm{Re}(\beta),\ \mathrm{Re}(\gamma)]$, then;

$$I_{-}^{\alpha,\beta,\gamma}\left(t^{\rho-1}\ {}_aW_{p,b,c,\delta}^{\mu,\nu}(\eta t^{\sigma})\ \bar{H}_{P,Q}^{M,N}\left[\omega t^{-\lambda}\left|\begin{matrix}(e_j,E_j,A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j,B_j)_{M+1,Q}\end{matrix}\right.\right]\right)(x)$$

$$x^{\Delta-\beta-1}\times\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma(\mu k+\nu)\,\Gamma\left(ak+\frac{p}{\delta}+\frac{b+2}{2}\right)}\left(\frac{\eta}{2}\right)^{2k+p+1}$$

$$\times \bar{H}^{M,N+2}_{P+2,Q+2}\left[\omega x^{-\lambda}\left|\begin{matrix}(\Delta-\beta,\lambda;1),(\Delta-\gamma,\lambda;1),(e_j,E_j;A_j)_{1,N},(e_j,E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(f_j,F_j;B_j)_{M+1,Q},(\Delta,\lambda;1),(\Delta-\alpha-\beta-\gamma,\lambda;1)\end{matrix}\right.\right] \quad (21)$$

where $\Delta=\rho+2k\sigma+p\sigma+\sigma$

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