



Determination of Poison Distribution of Queuing Systems in Five Manufacturing Companies

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Abstract. The poison distribution of arrival rate and service rate was determined. The poison distribution model was developed to handle the queuing systems in the selected manufacturing companies. Data were collected using arrival rate and service rate. The tools for data collection were questionnaire, supervisory control and sourcing of existing literatures from these companies. The model was evaluated and results were obtained. The obtained result shows that the distributions were within acceptable range of probability distribution. For first Aluminum, P_λ is 0.282 and P_μ is 0.55 and M_λ is 2.282, M_μ is 5.55. The probability distribution of Eagle cement is given as: P_λ is 0.352, P_μ is 0.75; M_X is 3.52, M_N is 7.5. For Cocacola company, the probability distribution obtained as: P_λ is 0.47, P_μ is 0.571; M_λ is 4.7; M_μ is 5.71. 7up bottling company has its probability values as P_λ is 0.634 and P_μ is 0.494; M_λ 6.34, M_μ is 4.94. Results obtained showed that the probability distributions are within the expectant range which is between 0 and 1. This range could be used to make predictions. This implies that the model can be used as a good working tool for decisions making and predictions.

1. Introduction

In all facets of life queues occur. This is because there are persons or materials or customers that are waiting for purpose of obtaining service. These queues naturally emanate when customers are waiting to be served.

Almost all individuals must have had the experience of having to wait in line for some sort of services rendered by limited number of servers.

If cost factor is brought into consideration, it will be discovered that providing more service facilities incurs more cost resulting into drop in profit margin, whereas excessive long queue also reduces profit margin due to lost sales and lost customers. Hence the management faces a peculiar problem of

balancing costs associated with waiting against the cost associated with prevention of waiting to maximize profit.

Queue arises whenever several customers/materials are waiting for some sort of service provided by a limited number of servers. Queues are generally formed because of limited resources. Customers can be people, telephone calls, orders and requests. Also queues can be machines, parcels or anything else that arrives at a facility and requires services.

When there is a queue of customers waiting to be served, servers sometimes device a rule for selecting the next customer waiting on the line. The rule for selecting the next customer waiting on the line is the queue discipline. Some organisations use the “first come-first served” discipline (FCFS). Example of the queue with this discipline is the queue of customers waiting to board a car at a bus stop. Others use the “last in first out” (LIFO), as in a situation where a teacher wants to grade the assessment scripts submitted by some students. The one on top is collected and graded first, but that was the one submitted last, and the others that were initially submitted would be attended to later.

There are other variations including pre-emptive repeat, in which the lower priority customer repeats the entire service from beginning. In a non-preemptive priority queuing system, the newly arrived customer waits until customer in service completes service before gaining access to the service facility. This type of system is called a head-of-the line system, HOL.

In any type of system, which can be modeled as a queuing system, there are trade-offs to be considered. If the service facility of the system has such a large capacity that queues rarely form, then the service facility is likely to be idle for a large fraction of the time so that unused capacity exist. Contrarily, if most customers must join the queue and servers are rarely idle, there may be customer dissatisfaction and possibly loss of customers. Service channels are characterized by the configuration of the service system and the pattern of service time. The service systems are classified in terms of their number of channels and number of phases. Hence we have the “single server single phase” system. The single server single—phase system is defined as a queuing system whereby a customer or unit passes through an only service facility and leaves. The system is widely used in many companies and organizations. The First aluminum Company of Nigeria Plc employs this method to reduce idle time.

Panta etal (2021) studied the extensive survey of the queuing system from its birth in 1909 to till date. The study emphasizes that old techniques could not handle current problems as such new methods should emanate to proffer solutions to these current problems. The research reported the developed policies and also cited fantastic methods to proffer solutions to queuing problems.

According to Sergio etal (2021), feedback calls could organize an obit of repeated calls known as r-calls. When R-calls arrive a system, if the system is busy, the system could be left with some state dependent probability or with a complementary probability returns to of bit. The study developed the method of calculating steady-state probabilities of appropriate three-dimensional Markov chain performance measures were developed. Obtained results were appropriate handle.

Akhil, Sreelatha, Ushakumari, (2021) discussed application of queuing theory to a railway ticket window. The research described queue as a form of waiting time and that it is a common problem in all the service disciplines.

The single channel queuing model was handled and this followed a passion arrival and an exponential service. Queue management at the railway ticket counter with a single server was studied. Data were collected and analyzed. The results from M/M/I and m/d/1 did not conform real life situation and such an M/G/I model was applied and the service time was calibrated based on normal distribution. The result from performance measure showed conformity with the observed data, and this makes the M/M/I model more effective that the M/M/1 model.

Yousry and Maram, (2011) examined new measures of performance based on single server markovian queuing system. The performance measures are dependent upon moment of order of statistics. Presentations were made based on variance of number of customers and variance based on waiting time. The M/M/I was applied using the performance measures to ensure its applicability and effectiveness.

A state-dependent approximation method for estimating truck queue length at marine terminals was approached, (Qu, Tao, Xie, Qi; 2021). The study discussed port congestion that lead to long truck queues at marine terminals. The counter measures were proposed to handle this problem. A model was developed in this research and it was based on simulation of the truck queuing system. Queue formation and dispersion was handled by the created model.

The study conducted a simulation based case study to evaluate the prediction accuracy of the developed model. Results from this model and other four existing models were compared and the new model performed better in terms of accuracy. This makes the model more useful in assessing the effectiveness of the counter measures that are necessary in peak-hour congestion reduction in marine terminals.

Computing and analysis of multi server queuing model using pentagonal fuzzy members was studied by Mridula and Anamika (2021). Multiple server queue miniature using pentagonal fuzzy number were applied. Input parameters were converted from crisp to fuzzy so as to conform with real life situation. The system characteristics were calculated with DSW algorithms with alpha-cut at various levels. The performance measures were calculated directly as PFNs with pentagonal number arithmetic. The authenticity of the model was attempted using arrival rate and service rate as PFN. In terms of validation, effects of different parameters were showcased. **Multi-server formulae:**

As can be seen below, the formulae for multi-server performance measures are quite formidable, particularly for large values of S. Fortunately, there are easier methods for obtaining values: we can use table or count the formulae in spread sheets. The best idea is to use professional software.

In the sea parts, congestion of the parts causes a lot of delays and this makes it difficult for ships to use it. Pruyn and Groenveld (2018) studied queues using Markov chains to analyze days delay due to port congestion. In the past, these delays have been handled using queuing theory that are in form of prediction done by modeling and simulation. The study determines the suitability in predicting days delays in bulk shipping companies as a result of port congestion. This is done by using Markov chain analysis to predict future situation based on the current situation. The obtained data have shown that ship users can predict congestion so as to enable prediction of services.

Park et al (2020) determined analysis of minimum speed control effect using queue model focusing on Busan port. The study emphasized two ways in which efficiency of maritime traffic could be increased. Decrease in marine accident and increase in infrastructure utilization rate were the two basic ways that improved efficiency. The study approached the qualitative analysis of the effects of speed regulation under safe conditions for effective use of water ways. A queue approach was used to develop a model and simulation was used for the analysis considering waiting time according to minimum speed. The results from simulation showed a maximum reduction in congestion of 6.1% at 6 knots minimum speed and 2.5 ships passed safely.

Krishnamoorthy (2020) determined analysis of a batch arrival batch service queuing-inventory system with processing of inventory while on vacation. The study considered a single server queuing – inventory system in which arrivals are governed by a batch Markovian arrival process and successive arrival batch sizes forming a finite first-order Markov chain. It was observed that the time required for processing one unit of inventory followed a phase-type distribution. The effects of correlation in inter-arrival times and service times on performance measured were numerically demonstrated. It considered optimization problem as well.

The Poisson distribution occurs in cases where P is very small and n is very large. Where n = number of trials, p = probability of trials. It is a limited form of binomial distribution. As n moves towards infinity, p moves towards zero and np or mean remains constant. The following are obtainable, parameter, range, mean, variance, mode and MLE.

Pozuelo, Amo-salas and Casero (2021) described the effect of probability distribution of the response variable in optimal experimental design with application in medicine. Practically, it is assumed that response variable follow a normal distribution with constant variance. This is obtained in an optimal experimental design theory. The study is on the effect in terms of efficiency as is attributed to misspecification in the probability distribution of the response variable. A generalized Fisher information matrix was provided by the elemental information matrix. Normal distribution was compared with gamma or Poisson distribution. Firstly, analytical results were obtained and were applied to some real illustrative instances. Misspecification influence was studied by nonlinear 4-parameter Hill Model. The analysis portrayed a behavior of efficiency of the design gotten in the presence of misspecification. This assumed a heteroscedastic normal distributions, with respect to D-optimal design for the gamma or Poisson distribution.

Arum and Ayesha (2015) studied the extended Poisson exponential distribution. The new mixture of modified exponential and Poisson distribution was approached. The sample size follows a zero truncated Poisson distribution maximum likelihood procedure was employed to obtain parameter estimation.

2. Materials and Methods

2.1 Materials

Tool applied in this study are:

Supervisory control, questionnaires, acquisitions of data, case studies, developed models application and software application.

2.2 Methods

Relevant data were collected using arrival rate and service rate and the heuristic algorithm was developed using flow chart that captured optimization with dynamic programming as a working tool. The distributions of the queuing systems were determined, and the performance evaluations of the queuing systems were carried out. The correlation of the arrival rate and the service rate was computed, followed by the application of optimization using dynamic programming.

2.3 Procedure for Data Collection and Analysis

The data were collected at First Aluminum Company Port Harcourt and four other companies. The following procedures were employed:

2.4 Direct Investigation

This involves critical observation of the operational process in the production lines. As the aluminum coils arrived at the production lines their inter arrival times and the service times were recorded. This information was sourced at the company. Also, the queues that occur at Eagle cement, Vinal Aluminum, Coca Cola bottling Company and 7up bottling company were be sourced.

2.5 Interview

Questions were asked based on the causes of queues in the production lines. These questions were based on power failure, pretreatment of tanks, cleaning down, mechanical work, etc.

2.6 Questionnaire

Questionnaires were distributed to a good number of the workers on the causes of queues.

2.7 Collection of Unpublished data:

Unpublished data that give details of queues and the productivity of the company were collected. The data helped in the critical examination of the causes of queues in the companies and how to proffer adequate solution towards these causes.

2.8 To Determine the Poison Distribution

To determine the Poison distribution of the arrival rate and the service rate, according to DASS (2008), the poison distribution occurs in cases where P is very small and N is very large

Where N = no of trials

P = probability of trials

The poison distribution is a limited form of binomial distribution as it moves towards infinity and P moves towards zero and np or mean remains constant.

$$P = \frac{M}{N} \quad 3.10$$

where p = probability

M = mean

N = no of trials

Here the means of the arrival rates and service rates was computed

Mathematically, $\frac{\sum M}{N}$ mean of the service rates 3.11

From the above the probability of the arrival rate and the service rate was calculated.

2.9 The Poisson Distribution

The poison distribution occurs in cases where p is very small and n is very large. Such cases arise in connection with rare cases.

Where n = number of trials.

P = probability of trials

The poison distribution is a limited form of binomial distribution as n moves towards infinity and p moves towards zero and np or mean remains constant.

Table 1. Poisson Distribution

Number (x) success	Probabilities p(x)
0	e^{-m}
1	me^{-m}
2	$\frac{m^2e^{-m}}{2!}$
3	$\frac{m^3e^{-m}}{3!}$
R	$\frac{m^r e^{-m}}{r!}$
N	$\frac{m^n e^{-m}}{n!}$

Table 2. Poisson Distribution of Dependent Variable and Independent Variable

Computation using Data from First Aluminum Port Harcourt.

S/N	$t_a(\text{min}) = \lambda$	$t_b(\text{min}) = \mu$
1	1.3	3.5
2	2.3	4.1
3	2.5	6.4
4	1.7	5.7
5	4.4	4.9
6	1.8	3.5
7	2.4	6.5
8	2.9	1.3
9	3.7	9.7
10	5.2	9.9

In this calculation,

$$P = \frac{m}{n}, \quad \text{Where } p = \text{probability}$$

$m = \text{mean}$

$n = \text{no. of trials}$

$$m_\lambda = \frac{\sum \lambda}{n}$$

Where $n = 10$

$$m_\mu = \frac{\sum \mu}{n}$$

$$= \frac{28.2}{10} = 2.82 \text{ per min} = m\lambda$$

$$= \frac{28.2}{10} = 0.282 = p\lambda$$

$$= \frac{mb}{n} = \rho\mu$$

$$= \frac{55.5}{10} = 5.55 = m\mu$$

$$= \frac{55.5}{10} = 0.55 = \rho\mu$$

Table 3. Poisson Distribution of Dependent Variable and Independent Variable

Computation using Data from Vinal Aluminum

S/N	$t_a(\text{min}) = \lambda$	$t_b(\text{min}) = \mu$
1	5.2	9.9
2	3.1	1.4
3	2.4	3.2
4	1.6	4.4
5	4.5	2.5
6	2.6	3.7
7	1.9	9.9
8	1.8	6.0
9	3.7	6.7
10	3.9	9.8

$$m\lambda = \frac{\sum \lambda}{n}$$

$$m\lambda = \frac{\sum \mu}{n}$$

$$m\mu = \frac{3.05}{10} = 3.05 \text{ per mins}$$

$$p\lambda = \frac{3.05}{10} = 0.305$$

$$m\mu = \frac{54.1}{10} = 5.14 \text{ per mins}$$

$$p\mu = \frac{51.4}{10} = 0.514$$

**Table 4. Poison Distribution of Dependent Variable and Independent Variable
Computation using Data from Eagle Cement**

S/N	$t_a(\text{min}) = \lambda$	$t_b(\text{min}) = \mu$
1	2.7	7.6
2	3.8	3.9
3	3.6	8.4
4	1.9	9.7
5	3.7	9.3
6	3.8	6.5
7	4.1	9.7
8	3.7	6.3
9	2.6	7.5
10	5.3	6.1

$$m\lambda = \frac{\sum\lambda}{n}$$

$$m\mu = \frac{\sum\mu}{n}$$

$$m\lambda = \frac{3.52}{10} = 3.52 \text{ per mins}$$

$$p\lambda = \frac{3.52}{10} = 0.352$$

$$m\mu = \frac{\sum t_{a3}}{n} = \frac{75}{10} = 7.5$$

$$p\lambda = \frac{7.5}{10} = 0.75$$

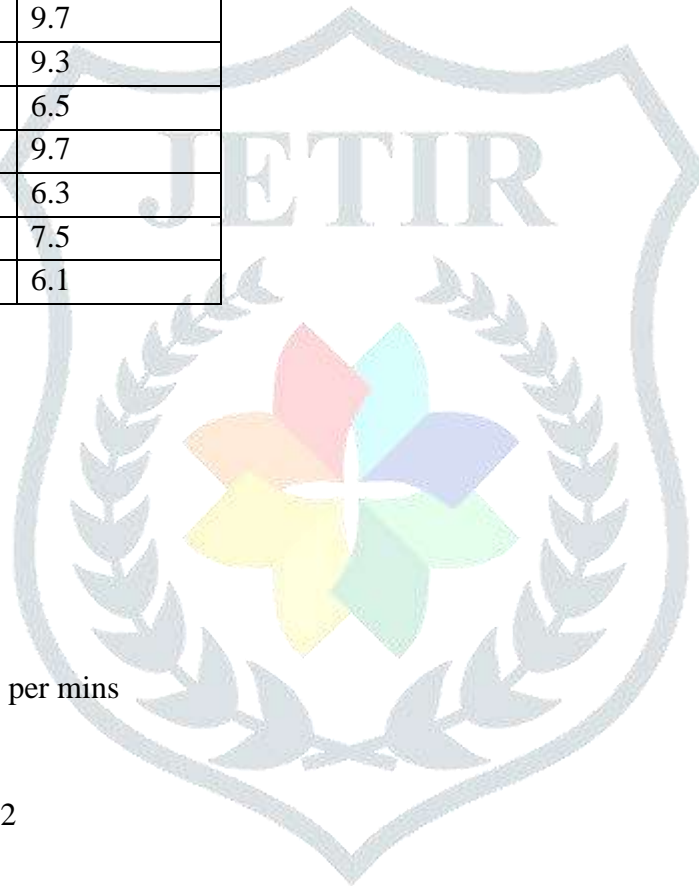


Table 5. Poison Distribution of Dependent Variable and Independent Variable

Computation using Data from 7up Bottling company Aba

S/N	$t_a(\text{min}) = \lambda$	$t_b(\text{min}) = \mu$
1	5.2	6.5
2	6.4	7.1
3	7.8	8.4
4	8.1	3.3
5	8.3	6.1
6	8.4	1.7
7	9.0	7.3
8	2.7	2.8
9	3.6	2.9
10	3.9	3.3

$$m\lambda = \frac{\sum \lambda}{n} = \frac{63.4}{10} = 6.34/\text{min}$$

$$p\lambda = \frac{6.34}{10} = 0.634$$

$$m\mu = \frac{\sum t_b}{n} = \frac{49.4}{10} = 4.94$$

$$p\mu = \frac{4.94}{10} = 0.494$$

Table 6. Poison Distribution of Dependent Variable and Independent Variable

Computation using Data from Coca cola Bottling Company, Aba

S/N	$t_a(\text{min}) = \lambda$	$t_b(\text{min}) = \mu$
1	2.2	1.7
2	2.4	3.3
3	3.6	5.5
4	4.7	6.1
5	5.3	5.3
6	7.1	6.2
7	2.6	7.4
8	3.9	8.3
9	7.3	6.1
10	7.9	7.2

$$m\lambda = \frac{\sum \lambda}{n} = \frac{47}{10} = 4.7/\text{min}$$

$$p\lambda = \frac{4.7}{10} = 0.47$$

$$m\mu = \frac{\sum\lambda}{n} = \frac{57.1}{10} = 5.71$$

$$p\lambda = \frac{5.71}{10} = 0.571$$

3.0 Result and Discussion

Result from First Aluminum Company shows that the correlation coefficient was calculated to be 0.93. Also the correlation coefficient for Vinal Aluminum was found to be 0.98. Again the correlation coefficient of Eagle Cement Port Harcourt was found to be 0.92. For Coca cola Bottling Company Aba, the correlation coefficient was found to be 0.93. Finally, the correlation coefficient for 7up Bottling Company was calculated to be 0.85.

The M_{λ} was calculated to be 2.82 and the probability (P_{λ}) was found to be 0.282. Also the Mean of the Service rate (M_{λ}) was determined to be 5.55 and its probability (P_{μ}) equals 0.55).

For Vinal Aluminum, we have M_{λ} to be 3.05, P_{λ} to be 0.305. Again, the M_{μ} is 5.14 and P_{μ} is 0.514. Eagle cement has M_{λ} as 3.52. P_{λ} as 0.352 and M_{μ} as 7.5 and P_{μ} as 0.75.

Coming to the Coca cola Bottling Company, Aba, the M_{λ} is 4.7 and P_{λ} is 0.47, also M_{μ} is 5.71 and P_{μ} is 0.571.

Finally, the 7Up Bottling Company Aba has its M_{λ} as 6.34, P_{λ} as 0.634 and M_{μ} as 4.94 and P_{μ} as 0.494.

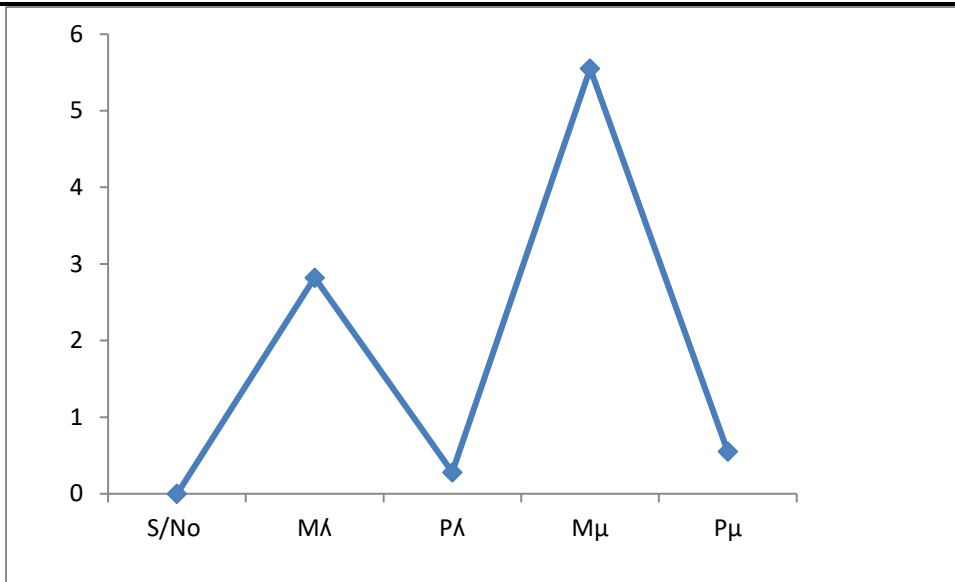


Figure 4.11: Poisson Distribution for First Aluminium

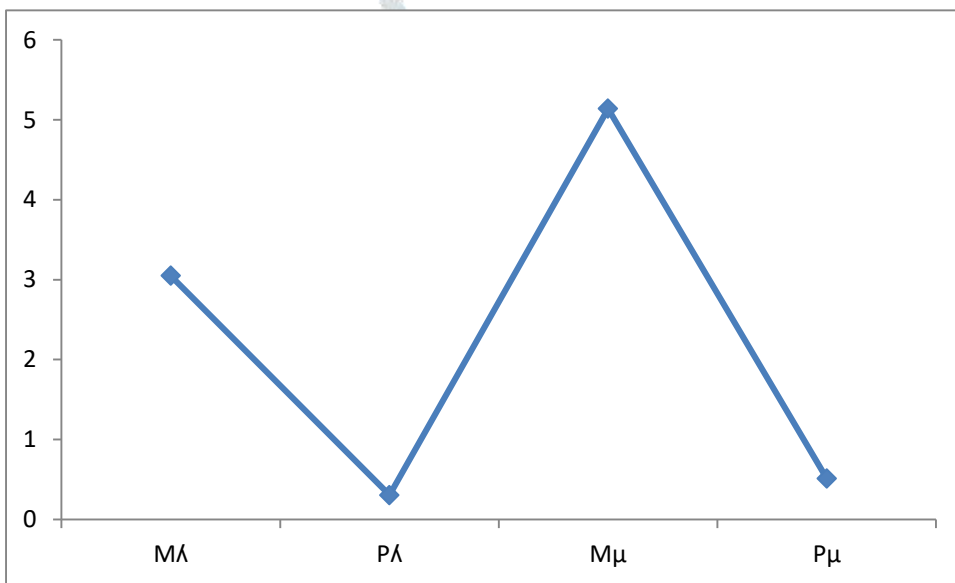
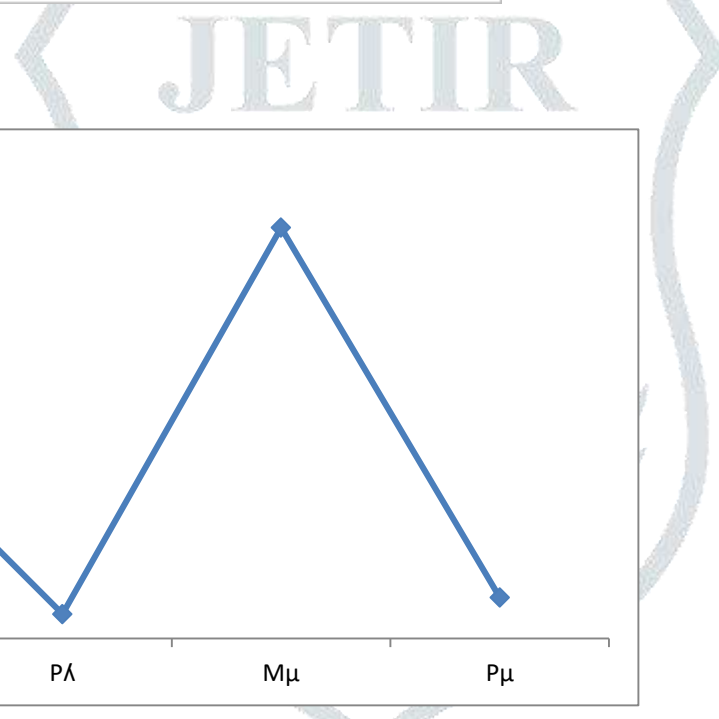


Figure 4.12: Poisson Distribution for Vinal Aluminium

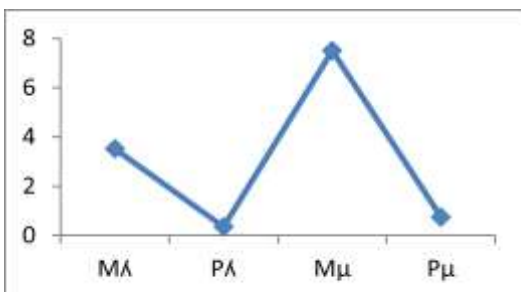


Figure 4.13: Poisson Distribution for Eagle Cement

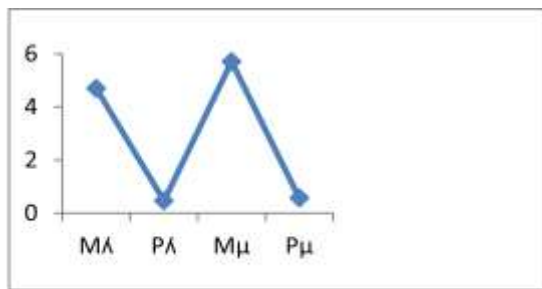


Figure 4.14::Poisson Distribution for Coca cola

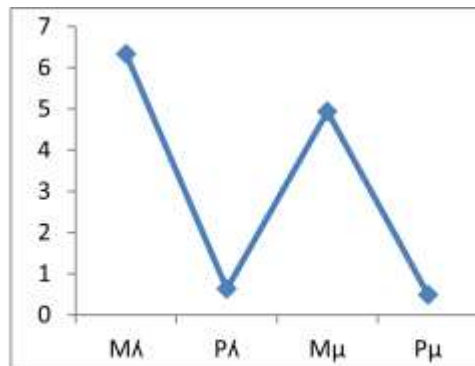


Figure4.15: Poisson Distribution for 7up

4. Conclusion

Relevant were collected using arrival rate and service rate, the distribution of the queuing system were determined using poison distribution and results obtained we between 0 and 1 which are the valid range of probability distribution. These results are god enough to make predictions. As such the poison distribution model can be said to a better tool for decision making.

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Conflict of Interest

We have no conflict of interest to declare.

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