

JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)

An International Scholarly Open Access, Peer-reviewed, Refereed Journal

ON Q-FUZZY SOFT RING AND Q-FUZZY SOFT IDEAL

DR. B. N. YADAV* & DR. R. K. DAS**

*Deptt. of Mathematics , S.P COLLEGE , DUMKA

**Deptt. of mathematics, S. K. M. University, Dumka.

ABSTRACT

As a new algebraic structure , Q-fuzzy groups were introd<mark>uced by</mark> Solairajee and Nagaranjan in 2009 . Fuzzy soft rings and fuzzy soft ideals were introduced by Ghosh , Dinda and Samanta in 2011 .

In this paper , we introduce Q-fuzzy soft ring and Q- fuzzy soft ideal by using fuzzy soft sets . Several algebraic properties together with homomorphic image and pre image are investigated.

KEY WORD : Fuzzy soft sets, Fuzzy soft ring, Fuzzy soft ideal, Q-fuzzy soft ring, Q-fuzzy soft ideal

1. INTRODUCTION

Fuzzy set theory developed by Zadeh [3] is considered as a special case of soft sets. Due to the lack of parameterization, Zadeh's fuzzy set theory was not successful in some cases . It is for this reason that Molodstov [5] introduced the concept of soft sets in 1999 as a new mathematical tool for dealing with uncertainties.

Ahamad and Kharal [6] introduced the definition of fuzzy soft set and studied some of their basic properties . Zhiming Zhang [7] studied intuitionistic fuzzy soft ring.

Solairajee and Nagaranjan [1] and [9] analyzed Q- fuzzy groups and introduced fuzzy left R-subgroups of near rings with respect to t-norm. Ghosh, Dinda and Samanta [2] introduced the concept of fuzzy soft rings and fuzzy soft ideals.

In our paper , we have developed the concept of Q-fuzzy soft ring and Q-fuzzy soft ideal by the help of fuzzy soft sets . Some algebraic properties as well as homomorphic image and pre image have also been studied .

2. PRELIMINARIES :

DEF(2.1) : A fuzzy set \tilde{A} on a non empty set X is characterized by its membership function , $\mu_{\tilde{A}}: X \rightarrow [0,1]$ where

 $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element of X in fuzzy set \tilde{A} for all $x \in X$. The complement of fuzzy set \tilde{A} on X is defined by

 $\tilde{\mathbf{A}}^{\mathrm{c}} = 1 - \mu_{\tilde{\mathbf{A}}}(x) \ .$

The null fuzzy set $\tilde{0}$ and the whole fuzzy set $\tilde{1}$ are constant mapping from X to {0}and {1} respectively. [3]

[6]

DEF(2.2) The standard intersection of two fuzzy sets \tilde{A} and \tilde{B} is represented in general by binary operation $i:[0,1]\times[0,1] \rightarrow [0,1]$ on unit interval called t-norm which satisfies the following conditions.

(i) i is commutative and associative.

(ii) i is continuous.

(iii) i(a,1) = a ; $\forall a \in [0,1]$

(iv)
$$a \le c$$
; $b \le d \implies i(a,b) \le i(c,d)$; $\forall a,b,c,d \in [0,1]$

[4]

DEF(2.3) : The standard union of two fuzzy sets \tilde{A} and \tilde{B} is represented in general by binary operation $u:[0,1]\times[0,1]\to[0,1]$ on unit interval called t- co norm which satisfied the following condition .

(i) *u* is a commutative and associative

- (ii) *u* is a continuous
- (iii) u(a,0) = a ; $\forall a \in [0,1]$

(iv) $a \le c \; ; b \le d \implies u(a,b) \le u(c,d) \quad \forall a,b.c.d \in [0,1]$ [4]

DEF(2.4): Let X be the universal set of discourse, E be the set of parameter and P(X) be the power set of X. Then the ordered pair (F,E), where $F: E \rightarrow P(X)$, is a mapping, is called a soft set over X.

In fact a soft set is a parameterized family of subsets of X , where each

set $\{F(p): p \in E\}$ of the family is a collection of p element of the soft set (F,E). [5]

DEF(2.5) : Let X be the universal set , E be the set of parameter and $A \subseteq E$

Then the pair (F,A) where $F: A \rightarrow I^X$ is a mapping and I=[0,1] is called a fuzzy soft

set on X , I^x being the collection of all fuzzy subsets of X .

DEF(2.6) : Let G be the group and (F,A) be the soft set over G. Then (F,A) is said to be soft group over G iff F(p) < G for each $p \in A$

DEF(2.7) : Let G be group and (F,A) be a fuzzy soft set over G. Then (F,A) is said to be a fuzzy soft group over G, iff for each $p \in A$ and $x, y \in G$

(i)
$$F_{p}(x, y) \ge i[F_{p}(x), F_{p}(y)]$$

(ii) $F_{p}(x^{-1}) \ge F_{p}(x)$

DEF(2.8) : Let f and g be any two fuzzy subsets of a ring . Then u(f,g) is also a fuzzy subset of R

defined by $u(f,g)(z) = \begin{cases} \bigvee[\min\{f(x),g(y)\}] & ; \quad z = xy. \\ 0 & ; \quad z \neq xy. \end{cases}$ where $x, y, z \in R$.

DEF(2.9) : Let (F,A) be a soft set over a ring $(R,+,\cdot)$. Then (F,A) is called a soft ring over R iff F(p) is a subring of R for all $p \in A$

DEF(2.10) : Let R be the soft ring . Then a fuzzy set $\mu_{\tilde{A}} : \mathbb{R} \to [0,1]$ on R is called a fuzzy soft ring on R if

(i)
$$\mu_{\widetilde{A}}(x+y) \ge i\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{A}}(y)\}$$

(ii)
$$\mu_{\tilde{A}}(-x) \geq \mu_{\tilde{A}}(x)$$

(iii) $\mu_{\tilde{A}}(xy) \ge i\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}\$, for all $x, y, \in R$ xy stands for x.y and i is t-norm

DEF(2.11) : Let $(f,g): X \to Y$ be a fuzzy soft function. if $f: X \to Y$ be a homomorphism,

then (f,g) is called a fuzzy soft homomorphism and if $f: X \rightarrow Y$ be an isomorphism and

 $g: X \to Y$ be one one mapping from A onto B, then (f,g) is called a fuzzy soft isomorphism, where $A \subset X$, $B \subset Y$

3: Q-FUZZY SOFT RINGS

DEF(3.1) : Let R be the soft ring. A fuzzy set $\mu_{\tilde{A}}(x)$ where $\tilde{A} = \{x, \mu_{\tilde{A}}(x) : x \in R\}$ on R

is called Q-fuzzy soft ring in R if following conditions hold.

$$Q_{l}: \quad \mu_{\tilde{A}}(x+y,q) \geq i \left\{ \mu_{\tilde{A}}(x,q), \mu_{\tilde{A}}(y,q) \right\}$$

$$Q_2: \quad \mu_{\tilde{A}}(-x,q) \geq \mu_{\tilde{A}}(x,q)$$

 $Q_3: \quad \mu_{\tilde{A}}(x,y,q) \geq i \quad \{\mu_{\tilde{A}}(x,q), \mu_{\tilde{A}}(y,q)\} \quad \text{for all } x, y \in R \text{ and } q \in Q$

Theorem (3.2) : Every assumed Q-fuzzy soft ring $\mu_{\tilde{A}}(x)$ is a Q-fuzzy soft ring of R .

Proof : Let $\mu_{\tilde{A}}$ be an assumed Q-fuzzy soft ring

$$Q_{l}: \quad \mu_{\tilde{A}}(x+y,q) \geq i \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\}$$

$$Q_{2}: \quad \mu_{\tilde{A}}(-x,q) \geq \mu_{\tilde{A}}(x,q)$$

$$Q_{3}: \quad \mu_{\tilde{A}}(x,y,q) \geq i \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} \text{ for all } x, y \in R \text{ and } q \in Q$$
Since $\mu_{\tilde{A}}(x)$ is assumed
$$\wedge \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} = i \left[\wedge \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\}; \wedge \mu_{\tilde{A}}(x,q), \wedge \mu_{\tilde{A}}(y,q)\right]$$

$$\leq i \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\}$$

 $\leq \wedge \{\mu_{\tilde{A}}(x,q), \mu_{\tilde{A}}(y,q)\}$

And hence,

$$i\{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} = \wedge \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\}$$

$$\Rightarrow \mu_{\tilde{A}}(x+y,q) \ge i \quad \{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\}$$

$$= \min\{\mu_{\tilde{A}}(x,q), \mu_{\tilde{A}}(y,q)\} \text{ for all } x, y \in R \text{ and } q \in Q$$

Hence $\mu_{\tilde{A}}(x)$ is a Q-fuzzy soft ring of R

Theorem (3.3) Let $\mu_{\tilde{A}}(x)$ be the Q- fuzzy soft ring of R and ϕ be an endomorphism of R,

then $\mu_{\tilde{A}}(\phi)$ is also Q-fuzzy soft ring R.

Proof : for all $x, y \in R$, we have

$$Q_{1} : \qquad \mu_{\bar{A}}(\phi)\{(x+y,q)\} = \mu_{\bar{A}}\{\phi(x+y,q)\} \\ \geq \mu_{\bar{A}}\{\phi(x,q), \phi(y,q)\} \\ \geq i \{\mu_{\bar{A}}\phi(x,q), \mu_{\bar{A}}\phi(y,q)\} \\ \geq i \{(\mu_{\bar{A}}\phi)(x,q), (\mu_{\bar{A}}\phi)(y,q)\} \\ Q_{2} : \mu_{\bar{A}}(\phi)(-x,q) = \mu_{\bar{A}}\{\phi(-x,q)\} \\ \geq \mu_{\bar{A}}\{\phi(x,q)\} \\ \geq \mu_{\bar{A}}(\phi)(x,q)$$

$$Q_{3}: \quad \mu_{\bar{A}}(\phi)\{(x.y,q)\} = \quad \mu_{\bar{A}}\{\phi(xy,q)\}$$
$$= \quad \mu_{\bar{A}}\{(\phi x,q),(\phi y,q)\}$$
$$\geq \quad i\{\mu_{\bar{A}}(\phi x,q),\mu_{\bar{A}}(\phi y,q)\}$$
$$\geq \quad i\{\mu_{\bar{A}}\phi(x,q),\mu_{\bar{A}}\phi(y,q)\}$$

 $\Rightarrow \ \mu_{ ilde{A}}(\phi)$ is a Q-fuzzy soft ring of R

Theorem (3.4) : Let R and R' be two rings and $\phi: R \to R'$ be a soft homomorphism.

if $\mu_{\tilde{A}}$ and F_p be Q-fuzzy soft rings of R and R' respectively. Then the pre image $\phi^{-1}(F_p)$ is a Q-fuzzy soft ring of R

Proof : Suppose that F_p is a Q – fuzzy soft ring of R' , Such that $x, y \in R$ and $q \in Q$

$$\begin{pmatrix} \phi \\ (i) \\ (i)$$

$$\mu_{\tilde{A}}(x_{0},q) = \sup_{(h,q)\in\phi^{-1}(x')} \{\mu_{\tilde{A}}(h,q)\}$$

and $\mu_{\tilde{A}}(y_{0},q) = \sup_{(h,q)\in\phi^{-1}(y')} \{\mu_{\tilde{A}}(h,q)\}$ respectively.

Then we can deduce that

$$Q_{1} : \mu_{\tilde{A}}^{\prime} \{ (x' + y', q) \} = \sup_{\substack{(z,q) \in \phi^{-1}(x' + y', q)}} \{ \mu_{\tilde{A}}(z,q) \}$$

$$\geq i \{ \mu_{\tilde{A}}(x_{0},q), \mu_{\tilde{A}}(y_{0},q))$$

$$\geq i \{ \sup_{(h,q) \in \phi^{-1}(x',q)} \mu_{\tilde{A}}(h,q) , \sup_{(h,q) \in \phi^{-1}(y',q)} \mu_{\tilde{A}}(h,q) \}$$

$$\geq \min \{ \mu_{\tilde{A}}^{\prime}(x',q), \mu_{\tilde{A}}^{\prime}(y',q) \}$$

$$Q_{2} : \mu_{\tilde{A}}^{/}(-x',q) = \sup_{\substack{(z,q)\in\phi^{-1}(-x',q)}} \{\mu_{\tilde{A}}(z,q)\} \\ \ge \mu_{\tilde{A}}(x_{0},q) = \\ \ge \sup_{(h,q)\in\phi^{-1}(x',q)} \mu_{\tilde{A}}(h,q) \\ = \mu_{\tilde{A}}(x',q)$$

$$Q_{3} : \mu_{\tilde{A}}^{/}\{(x'y',q)\} = \sup_{\substack{(z,q)\in\phi^{-1}(x'y',q)}} \{\mu_{\tilde{A}}(z,q)\}$$

$$\geq i \{\mu_{\tilde{A}}(x_{0},q),\mu_{\tilde{A}}(y_{0},q)\}$$

$$\geq i \{\sup_{(h,q)\in\phi^{-1}(x',q)}\mu_{\tilde{A}}(h,q), \sup_{(h,q)\in\phi^{-1}(y',q)}\mu_{\tilde{A}}(h,q)$$

$$= \min\{\mu_{\tilde{A}}^{/}(x',q),\mu_{\tilde{A}}^{/}(y',q)\}$$

$$= \min\{\mu_{\tilde{A}}(x,q), \mu_{\tilde{A}}(y,q)\}$$

Hence , $\mu_{\tilde{\mathrm{A}}}$ is a Q-fuzzy soft ring of R

Theorem (3.6) : Let
$$\mu_{\tilde{A}}$$
 be a Q – fuzzy soft set of R and $\mu_{\tilde{A}}^*$ be a Q-fuzzy set in

N ={1,2,3.....} defined by
$$\mu_{\tilde{A}}^{*}(x,q) = \mu_{\tilde{A}}(x,q) + 1 - \mu_{\tilde{A}}(o,q)$$
, for all $x \in N$

Then $\mu^*_{\tilde{A}}$ is a normal Q –fuzzy subgroup of R

Proof : - For any $x, y \in R$ and $q \in Q$ we have ,

$$Q_{1} : \mu_{\tilde{A}}^{*}\{(x+y),q\} = \mu_{\tilde{A}}(x+y,q) + 1 - \mu_{\tilde{A}}(o,q)$$

$$\geq i\{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} + 1 - \mu_{\tilde{A}}(o,q)$$

$$\geq i\{\mu_{\tilde{A}}(x,q) + 1 - \mu_{\tilde{A}}(o,q),\mu_{\tilde{A}}(y,q) + 1 - \mu_{\tilde{A}}(o,q)\}$$

$$= i\{\mu_{\tilde{A}}^{*}(mx,q),\mu_{\tilde{A}}^{*}(my,q)\}$$

$$Q_{2} : \mu_{\tilde{A}}^{*}(-x,q) = \mu_{\tilde{A}}(-x,q) + 1 - \mu_{\tilde{A}}(o,q)$$

$$\geq \{\mu_{\tilde{A}}(x,q) + 1 - \mu_{\tilde{A}}(o,q)\}$$

$$= \mu_{\tilde{A}}^{*}(x,q)$$

$$= i\{\mu_{\tilde{A}}^{*}(mx,q),\mu_{\tilde{A}}^{*}(my,q)\}$$

$$Q_{3} : \mu_{\tilde{A}}^{*}(xy,q) = \mu_{\tilde{A}}(xy,q) + 1 - \mu_{\tilde{A}}(o,q)$$

$$\geq i[\{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} + 1 - \mu_{\tilde{A}}(o,q)]$$

$$\geq i[\{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(y,q)\} + 1 - \mu_{\tilde{A}}(o,q)]$$

$$\geq i[\{\mu_{\tilde{A}}(x,q),\mu_{\tilde{A}}(my,q)\}$$

So $\mu^{*}_{\tilde{A}}$ is a normal Q fuzy subgroup of R

4 : Q – FUZZY SOFT IDEALS

We shall first concentrate ourselves to fuzzy soft

ideals and then Q –fuzzy soft ideals .

DEF(4.1) : Let $(R,+,\cdot)$ be a ring and E be the set of parameter such that $A \subseteq E$.

- Let $F: A \rightarrow I^X$ is a mapping where I=[0,1], I^X being the collection of all fuzzy subsets of X.
- Then (F,A) is called a fuzzy soft left ideals over R iff for each $p \in A$, the corresponding

fuzzy subset $\widetilde{F}_p: R \to [0,1]$ is a fuzzy left ideal of R. i.e

(i)
$$\widetilde{F}_{p}(x-y) \ge i\{\widetilde{F}_{p}(x), \widetilde{F}_{p}(y)\}$$

(ii) $\widetilde{F}_{p}(xy) \ge \widetilde{F}_{p}(y) \quad \forall x, y \in K$

DEF(4.2): (F,A) is called a fuzzy soft right ideals over R iff for each $p \in A$, the corresponding

(i)
$$\widetilde{F}_p(x-y) \ge i\{\widetilde{F}_p(x), \widetilde{F}_p(y)\}$$

(ii) $\widetilde{F}_p(xy) \ge \widetilde{F}_p(x)$; $\forall x, y \in R$

DEF(4.3): (F,A) is called a fuzzy soft ideals over R iff for each $p \in A$, the corresponding fuzzy

subset $\widetilde{F}_p: R \to [0,1]$ is a fuzzy ideal of R. i.e

(i)
$$\widetilde{F}_p(x-y) \ge i\{\widetilde{F}_p(x), \widetilde{F}_p(y)\}$$

- (ii) $\widetilde{F}_p(xy) \ge \lor \{\widetilde{F}_p(x), \widetilde{F}_p(y)\}$; $\forall x, y \in R$
- **DEF(4.4)**: Let R be a fuzzy soft ring. Then the fuzzy subset $\widetilde{F}_p: R \to [0,1]$ is called

a Q –fuzzy soft ideal in R, if

(IQ₁)
$$\widetilde{F}_p(x-y,q) \ge i\{\widetilde{F}_p(x,q),\widetilde{F}_p(y,q)\}$$

(IQ₂)
$$\widetilde{F}_p(xy,q) \ge \bigvee \{\widetilde{F}_p(x,q), \widetilde{F}_p(y,q)\}$$

 $\forall x, y \in R ; q \in Q$

THEOREM(4.5) : Let (R, +, .) be a fuzzy soft ring and E be the parameter such that $A \subseteq E$.

for each $p \in A$, the corresponding fuzzy subset $\widetilde{F}_p: R \to [0,1]$ is a Q-fuzzy soft ideal of R iff the following conditions hold

(i)
$$F_p(x-y,q) \ge i\{F_p(x,q), F_p(y,q)\}$$

(ii) $u(I_R, \tilde{F}_p) \le \tilde{F}_p$ and $u(\tilde{F}_p, I_R) \le \tilde{F}_p$ respectively.

 I_{R} being characteristic function of R and u being the fuzzy t co-norm .

Proof : Suppose that $\widetilde{F}_p: R \to [0,1]$ be a Q-fuzzy soft left ideal over R for each $p \in A$.

$$\Rightarrow 1. \quad \widetilde{F}_p(x-y,q) \ge i\{\widetilde{F}_p(x,q), \widetilde{F}_p(y,q)\}$$

2.
$$\widetilde{F}_p(xy,q) \ge \widetilde{F}_p(y,q)$$
, $\forall x, y \in R$; $q \in Q$

Let z be the element of R \cdot . Then by definition (2.8)

$$u(I_{R}, \widetilde{F}_{p})z = \bigvee_{z=(xy,q)} [\wedge \{I_{R}(x), \widetilde{F}_{p}(y,q)\}]$$
$$\bigvee_{z=(xy,q)} [\widetilde{F}_{p}(y,q)]$$
$$\leq \widetilde{F}_{p}(xy,q) \qquad \text{by condition (2)}$$
$$= \widetilde{F}_{p}(z)$$

If z be not expressed as z = (xy, q), where $x, y \in R$ and $q \in Q$, then

$$u(I_R, \tilde{F}_p)z = 0 \le \tilde{F}_p(z)$$

$$\therefore \quad u(I_R, \tilde{F}_p) \le \tilde{F}_p$$

A.

Similarly, we can prove that

$$u(\widetilde{F}_p, I_R) \leq \widetilde{F}_p$$

Also (i) follows by (ii)

Conversely, suppose that (i) and (ii) hold for any fuzzy subset (F,A) over R with $p \in A$

$$u(I_R, \widetilde{F}_n) \leq \widetilde{F}_n$$

 $\widetilde{F}_p(x-y,q) \ge i\{\widetilde{F}_p(x,q),\widetilde{F}_p(y,q)\}$

Clearly then , $\tilde{F}_p(xy,q) \ge u\{I_R, \tilde{F}_p\}(xy,q) \ge \tilde{F}_p(y,q)$

 $\Rightarrow \quad \widetilde{F}_p \text{ is a Q-fuzzy soft left ideal in R} .$

Similar proof for Q-fuzzy soft right ideal in R can be established .

This means .

$$(IQ)_{1} \qquad \widetilde{F}_{p}(x-y,q) \ge i\{\widetilde{F}_{p}(x,q),\widetilde{F}_{p}(y,q)\}$$

$$(IQ)_{2} \qquad \widetilde{F}_{p}(xy,q) \ge \vee\{\widetilde{F}_{p}(x,q),\widetilde{F}_{p}(y,q)\}$$

$$\forall \quad x, y \in R \quad ; \quad q \in Q$$

 $\Rightarrow \quad \widetilde{F}_p \text{ is a Q-fuzzy soft ideal in R}.$

5. CONCLUSION

In the present paper, the theoretical point of view of Q-fuzzy soft ring and Q-fuzzy soft ideal are discussed. These concept are basic supporting structure for development of soft set theory. one can extend this work by studying other algebraic structures

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