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Numerical Solution For Non-Linear Undamped Duffing Equation By Using Leapfrog Method

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Abstract : The main objective of this article is to propose a Leapfrog Method it has been developed for exclusively for solving Non-Linear undamped duffing equations. The methods taken from the literature have been extended and modified to suit the needs of the discussed problems. The numerical technique has applied for the determination of discrete solutions, and a well composed analysis has been carried out for the problems. The solution efficiency and error is demonstrated in a graphical form.

I. Introduction

The Mathematical modelling aims to describe the different aspects of the real world, their interaction, and their dynamics through mathematics. Now days, the mathematical simulation has a very important role also in the field of science and engineering. In this article, the periodic and oscillatory problems was considered (discussed by Sekar et al. [1]) and present a different approach using the Leapfrog method with more accuracy for periodic and oscillatory problems. Several numerical methods based on the use of polynomial basis functions have been developed for solving this class of important problems (see Lambert [2, 3], Hairer et al in [4], Hairer [5], and Sommeijer [6]). The motivation governing the exponentially-fitted methods is inherent in the fact that if the frequency or a reasonable estimate of it is known in advance, these methods will be more advantageous than the polynomial based methods [7].

1.1 Singular System (Time-Invariant):

Consider the following time-invariant singular system of the form

$$K\dot{x}(t) = A\,x(t) + B\,u(t) \text{ with } x(0) = x_0,$$

Where $x \in R^n$ and $u \in R^m$ are its state vector and control input respectively. $K, A \in R^{n\times n}$ and $B \in R^{m\times n}$. Here K is a singular matrix.

1.2 Singular System (Time Varying):

Consider a time varying singular system of the form

$$K(t)\dot{x}(t) + F(t)\,x(t) = f(t)$$

Where K is assumed to be singular for all t. Let $K, F \in R^{n\times n}$, $I = [t_0, t_1]$ and assume that K, F are at least 2n-times differentiable.

1.3 Solvability of nonlinear singular systems

Many of the important singular systems are nonlinear. The nonlinearity, of course, will be much more difficult. Consider the nonlinear singular system of the form [8].

$Kx'(t) + A(x) = f$, Where K is a singular $n\times n$ matrix and A is a vector valued function of x that is possibly nonlinear. Let A_1 be the vector gradient of A evaluated at x_0 .

II. The Leapfrog Method

In this section, we modified the method to solve the Non-Linear undamped Duffing Equation as follows; The Eulers Method for the derivative is of the form

$$f(t, y) = y', \quad y_0 = y(t_0), \quad y \in R^d$$

$$\frac{[y(t+h)-y(t)]}{h} \approx y'. \text{ With } t_n + h = t_{n+1},\ n = 0,1,\ldots t_0 .$$

Hence, $N = 1$. Modifying the distinction remainder gives

$$y_n + hf(t_n, y_n) = y_{n+1},\ n = 0,1,\ldots t_0 .$$

The proposed method we define t_n as

$$t_n + h = t_{n+1},\ n = 0,1,..t_0,\ hy'\left(t+\frac{h}{2}\right) \approx y(t+h) - y(t)$$

$$\frac{[y(t+2h)..\ y(t)]}{h} \approx y'(t+h)$$

and then define it as follows: $y_{n-1} + 2hf(t_n, y_n) = y_{n+1}$, Where $n = 0,1,..,t_0$.

The proposed method is a linear $m = 2-$step method, with $a_0 = 0,\ b_0 = 2\ \ a_1 = 1,\ b_1 = 0$ and $b_{-1} = -1$,

Which must be tended to when we examine them two qualities y_0 and y_1, are

$$y_{n-1} + 2hf(t_n, y_n) = y_{n+1},\ n = 0,1,2,\ldots,t_0\ ;\ f(t,y) = y', y_0 = y(t_0),\ \ y \in R^d$$

The practical use of this method we apply the above technique to the following examples of Non-linear Duffing Equation problems.

III. NON-LINEAR UNDAMPED DUFFING EQUATION

Consider the nonlinear undamped Duffing equation

$y'' + y + y^3 = B\cos(wx)$; Where B = 0:002 and ω = 1:01.

The solution of the above equation is given by $y(x) = \sum_{i=0}^{3} A_{2i+1} \cos[(2i+1)\omega x]$

Where, A_1=0.200179477536, $A_3 = 0.246946143\times10^{-3}$, $A^5 = 0.304016\times10^{-6}$ and $A_7 = 0.374\times10^{-9}$.

The above equation has been solved numerically with boundary conditions of the form

$$y(0) = A_1 + A_3 + A_5 + A_7,\ y_0(0) = 0$$

The results obtained (with step size time t = 1) using the Leapfrog and STHWS methods along with exact solutions and absolute errors between them are calculated and are presented in the following table. A graphical representation is given for Duffings equation in Figure 1, using three-dimensional effect. It is inferred that, the Leapfrog method gives better solution for the non-linear undamped Duffing equation when compared to STHWS method.

IV. RESULT AND DISCUSSION

4.1 Technical Program:

To compute the results, programs are structured in C Language, using double procession mode and run in the CORE i5 processor under Turbo C environment. Programs are designed for general systems of non-linear singular systems using the formulae of the Single-term Haar wallet series and Leapfrog method.

Table 1: Results for the Duffings Equation at various values of *x*

Time t	Exact Solution	Solution for STHWS		Solution for Leapfrog Method	
	Solutions	Solutions	Error	Solutions	Error
0	0.2004294	0.2004294	0	0.2004294	0
0.1	0.3066526	0.3066526	0	0.3066526	0
0.2	0.2199634	0.2199634	0	0.2199634	0
0.3	0.0207932	0.0207932	0	0.0207932	0
0.4	-0.1036664	-0.1036664	0	-0.1036664	0
0.5	-0.0375666	-0.0375667	1E-07	-0.0375666	1E-09
0.6	0.1578427	0.1578428	1E-07	0.1578427	2E-09
0.7	0.2990128	0.2990129	1E-07	0.2990128	3E-09
0.8	0.2543050	0.2543051	1E-07	0.2543050	4E-09
0.9	0.0651066	0.0651069	3E-07	0.0651066	5E-09
1	-0.0919355	-0.0919358	3E-07	-0.0919355	6E-09
1.1	-0.0682613	-0.0682613	6E-08	-0.0682613	7E-09
1.2	0.1123593	0.1123593	8E-08	0.1123593	8E-09
1.3	0.2815429	0.2815434	5E-07	0.2815429	9E-09
1.4	0.2809778	0.2809783	5E-07	0.2809778	1E-08
1.5	0.1111868	0.1111875	7E-07	0.1111868	1.1E-08
1.6	-0.0689371	-0.0689377	6E-07	-0.0689371	1.2E-08
1.7	-0.0905881	-0.0905889	8E-07	-0.0905881	1.3E-08
1.8	0.0662643	0.0662653	1E-06	0.0662643	1.4E-08
1.9	0.2550834	0.2550844	1E-06	0.2550834	1E-09
2.0	0.2986884	0.2986894	1E-06	0.2986884	2E-09

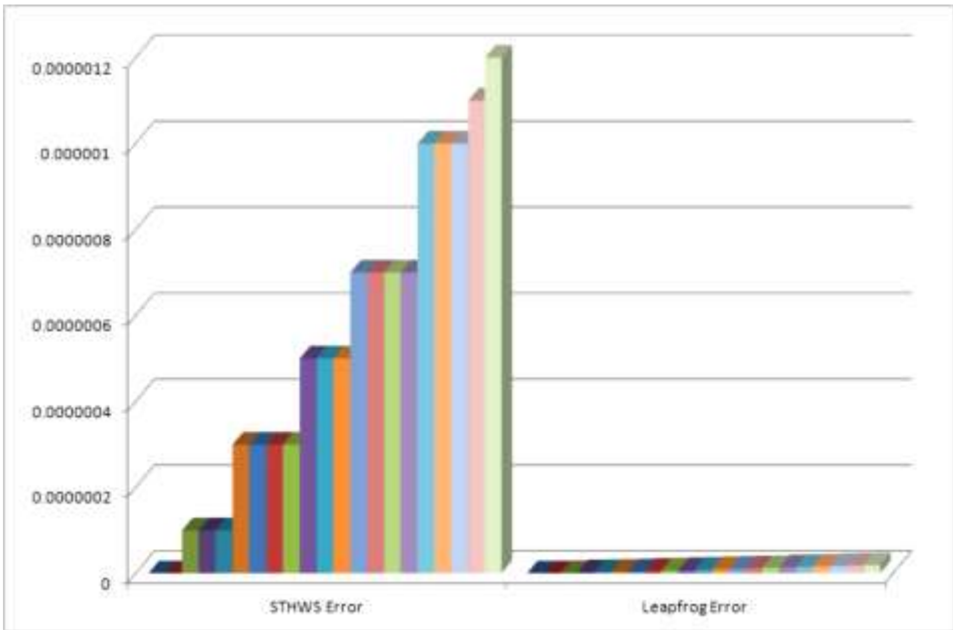

Figure: 1 Error graph for various values of *x*

V. CONCLUSION

The above observed results, the calculated value by leapfrog method is very closure with the original solution and compared with single term haar wavelet series which yields very less error. The errors are highlighted in graphical form. Hence the leapfrog method also more suitable for non-linear duffings equation.

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