



## THE NÖRLUND SPACE AND NÖRLUND ORLICZ SPACE OF BIQUADRATIC ENTIRE SEQUENCES

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**Abstract:** In the present paper, we begin by defining  $\Gamma^4$ ,  $\Lambda^4$ ,  $\Gamma_\pi^4$  and  $\Lambda_\pi^4$ , the spaces of all biquadratic entire sequences, biquadratic analytic sequences, biquadratic entire rate sequences and biquadratic analytic rate sequences respectively. We proved some theorem related with the properties of Nörlund space of biquadratic entire sequences and Nörlund Orlicz space of biquadratic entire rate sequences.

**Key Words:** Entire Sequence, Analytic Sequence, Biquadratic Sequence, Nörlund Spaces, Orlicz Sequence.  
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### I. INTRODUCTION

Class of all complex biquadratic sequence is denoted by  $\omega^4$

A sequence  $x = (x_{mnkl})$  is said to be biquadratic analytic sequence if [7]

$$\sup_{m,n,k,l} |x_{mnkl}|^{\frac{1}{m+n+k+l}} < \infty$$

We denote the vector space of all biquadratic analytic sequences by  $\Lambda^4$ .

A sequence  $x = (x_{mnkl})$  is said to be biquadratic entire sequence if [7]

$$|x_{mnkl}|^{\frac{1}{m+n+k+l}} \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty$$

We denote the vector space of all biquadratic entire sequences by  $\Gamma^4$ .

A sequence  $x = (x_{mnkl})$  is said to be biquadratic chi sequence if [7]

$$(m+n+k+l)! |x_{mnkl}|^{\frac{1}{m+n+k+l}} \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty$$

We denote the vector space of all biquadratic chi sequences by  $\chi^4$ .

The space  $\Lambda^4$  and  $\Gamma^4$  are metric spaces with the metric

$$d(x, y) = \sup_{m,n,k,l} \{|x_{mnkl} - y_{mnkl}|^{\frac{1}{m+n+k+l}}; m, n, k, l = 1, 2, 3, \dots\}, \quad (1.1)$$

for all  $x = (x_{mnkl})$ ,  $y = y_{mnkl}$  in  $\Lambda^4$  and  $\Gamma^4$ .

A sequence  $x = (x_{mnkl})$  is said to be biquadratic analytic rate sequence if

$$\sup_{m,n,k,l} \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} < \infty$$

We denote the vector space of all biquadratic analytic rate sequences by  $\Lambda^4_\pi$ .

A sequence  $x = (x_{mnkl})$  is said to be biquadratic entire rate sequence if

$$\left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty$$

We denote the vector space of all biquadratic entire rate sequences by  $\Gamma^4_\pi$ .

A sequence  $x = (x_{mnkl})$  is said to be biquadratic chi rate sequence if

$$(m+n+k+l) \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty$$

We denote the vector space of all biquadratic chi rate sequences by  $\chi^4_\pi$ .

The space  $\Lambda^4_\pi$  and  $\Gamma^4_\pi$  are metric spaces with the metric

$$d(x, y) = \left\{ \sup_{m,n,k,l} \left| \frac{x_{mnkl} - y_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} : m, n, k, l = 1, 2, 3, \dots \right\}, \tag{1.2}$$

for all  $x = (x_{mnkl})$ ,  $y = y_{mnkl}$  in  $\Lambda^4_\pi$  and  $\Gamma^4_\pi$ .

## II. PRELIMINARIES

**Definition 2.1:** Let  $(P_{mnkl})_{m,n,k,l=0}^\infty$  be a sequence of non-negative real numbers with  $p_{0000} > 0$ . Consider the transformation

$$y_{mnkl} = \frac{1}{\sum_p^m \sum_q^n \sum_r^k \sum_s^l P_{pqrs}} \sum_p^m \sum_q^n \sum_r^k \sum_s^l P_{pqrs} x_{m-p, n-q, k-r, l-s}$$

for  $m, n, k, l \in \mathbb{N}$ .

The space of all  $(x_{mnkl})$  for which  $(y_{mnkl}) \in \Gamma^4$  is called the Nörlund space of Biquadratic entire sequences. The Nörlund space of Biquadratic entire sequences is denoted by  $\eta(\Gamma^4)$ .

Similarly, the space of all  $(x_{mnkl})$  for which  $(y_{mnkl}) \in \Lambda^4$  is called the Nörlund space of Biquadratic analytic sequences. The Nörlund space of Biquadratic analytic sequences is denoted by  $\eta(\Lambda^4)$ .

**Definition 2.2:** Let  $(p_{mnkl})_{m,n,k,l=0}^\infty$  be a sequence of non-negative real numbers with  $p_{0000} > 0$ . Consider the transformation

$$y_{mnkl} = \frac{1}{\sum_p^m \sum_q^n \sum_r^k \sum_s^l P_{pqrs}} \sum_p^m \sum_q^n \sum_r^k \sum_s^l P_{pqrs} \frac{x_{m-p, n-q, k-r, l-s}}{\pi_{m-p, n-q, k-r, l-s}}$$

for  $m, n, k, l = 0, 1, 2, \dots$

The space of all  $(x_{mnkl})$  for which  $(y_{mnkl}) \in \Gamma^4_\pi$  is called the Nörlund space of Biquadratic entire rate sequences. The Nörlund space of Biquadratic entire rate sequences is denoted by  $\eta(\Gamma^4_\pi)$ .

Similarly, The space of all  $(x_{mnkl})$  for which  $(y_{mnkl}) \in \Lambda^4_\pi$  is called the Nörlund space of Biquadratic analytic rate sequence. The Nörlund space of Biquadratic analytic rate sequence is denoted by  $\eta(\Lambda^4_\pi)$ .

**Definition 6.2.3:** [6]The space consisting of all those sequences  $x = (x_{mnkl})$  in  $\omega^4$  s. t.  $M \left( \frac{|x_{mnkl}|^{\frac{1}{m+n+k+l}}}{\rho} \right)$  as  $m, n, k, l \rightarrow \infty$  for some arbitrary fixed  $\rho > 0$  is denoted by  $\Gamma^4_M$ ,  $M$  being a modulus function.  $\Gamma^4_M$  is called the Orlicz space of entire sequences.

**Definition 6.2.4:** [6] The space consisting of all those sequences  $x = (x_{mnkl})$  in  $\omega^4$  s. t.  $M\left(\frac{|x_{mnkl}|^{\frac{1}{m+n+k+l}}}{\rho}\right) < \infty$  for some arbitrary fixed  $\rho > 0$  is denoted by  $\Lambda_M^4$ ,  $M$  being an modulus function.  $\Lambda_M^4$  is called the Orlicz space of analytic sequences.

**Definition 6.2.5:** [1] An absolutely convex absorbent closed subset of locally convex topological vector space  $X$  is called a barrel.  $X$  is called a Barreled space if each barrel is a neighbourhood of zero.

**Definition 6.2.6:** [1] A locally convex topological vector space  $X$  is said to be semi reflexive if each bounded closed set in  $X$  is  $\sigma(X, X')$ -compact.

Let  $\omega^4$  be the space of all complex biquadratic sequences  $x = (x_{mnkl})_{m,n,k,l=1}^\infty$  and  $f: [0, \infty) \rightarrow [0, \infty)$  be an Orlicz function,

$$\Gamma_f^4 = \{x \in \omega^4: \left[ f\left(\frac{|x_{mnkl}|^{\frac{1}{m+n+k+l}}}{\rho}\right) \right] \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty \text{ for some } \rho > 0\}$$

and

$$\Lambda_f^4 = \{x \in \omega^4: \left[ f\left(\frac{|x_{mnkl}|^{\frac{1}{m+n+k+l}}}{\rho}\right) \right] < \infty \text{ for some } \rho > 0\}$$

The spaces  $\Lambda_f^4$  and  $\Gamma_f^4$  are metric spaces with the metric

$$d(x, y) = \inf\{\rho > 0: \sup_{m,n,k,l \geq 1} [f\left(\frac{|x_{mnkl} - y_{mnkl}|}{\rho}\right)] \leq 1\} \\ \forall x, y \in \Gamma_f^4 \text{ and } \Lambda_f^4.$$

### III. THE NÖRLUND SPACE OF BIQUADRATIC SEQUENCES

**Theorem 3.1:**  $\eta(\Gamma^4) = (\Gamma^4)$

**Proof:** Firstly we shall show that

$$\eta(\Gamma^4) \subset \Gamma^4.$$

Let  $x = (x_{mnkl}) \in \eta(\Gamma^4)$ . Then  $y \in (\Gamma^4)$  so that for every  $\epsilon > 0$ , we have a positive integer  $m_0$  s.t.

$$\left| \frac{P_{0000}x_{mnkl} + \dots + P_{mnkl}x_{0000}}{P_{mnkl}} \right| < \epsilon^{m+n+k+l} \quad \forall m, n, k, l \geq m_0$$

Take  $P_{0000} = 1; P_{1111} = \dots = P_{mnkl} = 0$ .

We then have  $|x_{mnkl}| < \epsilon^{m+n+k+l}, \forall m, n, k, l \geq m_0$ .

Therefore,  $x = (x_{mnkl}) \in \Gamma^4$ .

Hence

$$\eta(\Gamma^4) \subset \Gamma^4 \tag{3.1}$$

In the second part, we shall show that

$$\Gamma^4 \subset \eta(\Gamma^4)$$

Let  $x = (x_{mnkl}) \in \eta(\Gamma^4)$ . But for any given  $\epsilon > 0$ ,  $\exists$  a positive integer  $m_0$  s.t.  $|x_{mnkl}| < \epsilon^{m+n+k+l}, m, n, k, l > m_0$ . We have

$$|y_{mnkl}| \leq \left| \frac{P_{0000}x_{mnkl} + \dots + P_{mnkl}x_{0000}}{P_{mnkl}} \right| \\ \leq \frac{1}{P_{mnkl}} [p_{0000}|x_{mnkl}| + \dots + p_{mnkl}|x_{0000}|]$$

$$\begin{aligned} &\leq \frac{1}{P_{mnkl}} [p_{0000}\epsilon^{m+n+k+l} + \dots + p_{mnkl}\epsilon^{0+0+0+0}] \\ &\leq \frac{\epsilon^{m+n+k+l}}{P_{mnkl}} [p_{0000} + \dots + p_{mnkl}] \\ &\leq \frac{\epsilon^{m+n+k+l}}{P_{mnkl}} P_{mnkl} \\ &\leq \epsilon^{m+n+k+l} \quad \forall m, n, k, l \geq m_0 \end{aligned}$$

Therefore,  $y_{mnkl} \in \Gamma^4$ . Consequently  $x_{mnkl} \in \eta(\Gamma^4)$ .

Hence

$$\Gamma^4 \subset \eta(\Gamma^4) \tag{3.2}$$

From equation (3.1) and (3.2), we get

$$\Gamma^4 = \eta(\Gamma^4)$$

**Theorem 3.2:**  $\eta(\Lambda^4) = (\Lambda^4)$

**Proof:** Firstly we shall show that

$$\Gamma^4 \subset \eta(\Gamma^4).$$

Let  $x = (x_{mnkl}) \in \eta(\Lambda^4)$ . Then  $\exists$  a positive constant  $K$  s.t.

$$\begin{aligned} |x_{mnkl}| &\leq K^{m+n+k+l} \quad m, n, k, l = 0, 1, 2, \dots \\ |y_{mnkl}| &\leq \frac{|p_{0000}K^{m+n+k+l} + \dots + p_{mnkl}K^{0+0+0+0}|}{P_{mnkl}} \\ &\leq \frac{K^{m+n+k+l}}{P_{mnkl}} \left[ p_{0000} + \dots + \frac{p_{mnkl}}{K^{m+n+k+l}} \right] \\ &\leq \frac{K^{m+n+k+l}}{P_{mnkl}} P_{mnkl} \\ &\leq K^{m+n+k+l} \quad \forall m, n, k, l \geq m_0 \end{aligned}$$

Hence  $(y_{mnkl}) \in \Lambda^4$ . But then  $(x_{mnkl}) \in \eta(\Lambda^4)$ .

Consequently,

$$\Lambda^4 \subset \eta(\Lambda^4) \tag{3.3}$$

In the second part, we shall show that

$$\eta(\Gamma^4) \subset \Gamma^4$$

Let  $(x_{mnkl}) \in \eta(\Lambda^4)$ . Then  $(y_{mnkl}) \in (\Lambda^4)$ .

Hence  $\exists$  a positive constant  $K$  such that  $|y_{mnkl}| < T^{m+n+k+l}$  for  $m, n, k, l = 0, 1, 2, \dots$

This implies that

$$\begin{aligned} &\left| \frac{p_{0000}x_{mnkl} + \dots + p_{mnkl}x_{0000}}{P_{mnkl}} \right| < K^{m+n+k+l} \\ \Rightarrow &\frac{1}{P_{mnkl}} |p_{0000}x_{mnkl} + \dots + p_{mnkl}x_{0000}| < K^{m+n+k+l} \\ \Rightarrow &|p_{0000}x_{mnkl} + \dots + p_{mnkl}x_{0000}| < K^{m+n+k+l} P_{mnkl} \end{aligned}$$

Take  $p_{0000} = 1; p_{1111} = \dots = p_{mnkl} = 0$ .

Then it follows that  $P_{mnkl} = 1$  and so  $|x_{mnkl}| < K^{m+n+k+l}$  for all  $m, n, k, l$ .

Consequently  $x_{mnkl} \in (\Lambda^4)$ .

Hence

$$\eta(\Lambda^4) \subset (\Lambda^4) \tag{3.4}$$

From equation (3.3) and (3.4), we get

$$\Lambda^4 = \eta(\Lambda^4)$$

**Theorem 3.3:**  $\Gamma^4$  is not a barreled space.

**Proof:** Let

$$A = \left\{x \in \Gamma^4: |x_{mnkl}|^{1/m+n+k+l} \leq \frac{1}{m+n+k+l}, \forall m, n, k, l\right\}.$$

Then A is an absolutely convex, closed in  $\Gamma^4$ . But A is not a neighbourhood of zero. Hence  $\Gamma^4$  is not barreled.

**Theorem 3.4:**  $\Gamma^4$  is not semi-reflexive.

**Proof:** Let  $\{\delta_{mnkl}\} \in U$  be the unit closed ball in  $\Gamma^4$ . Clearly no subsequence of  $\{\delta_{mnkl}\}$  can converge weakly to any  $y \in \Gamma^4$ . Hence  $\Gamma^4$  is not semi-reflexive.

#### IV. THE NÖRLUND ORLICZ SPACE OF BIQUADRATIC RATE SEQUENCES

Let  $\omega^4$  be the space of all complex biquadratic sequences  $x = \left(\frac{x_{mnkl}}{\pi_{mnkl}}\right)_{m,n,k,l=1}^{\infty}$  and  $f: [0, \infty) \rightarrow [0, \infty)$  be an Orlicz function,

$$\Gamma_{f\pi}^4 = \left\{x \in \omega^4: \left[ f \left( \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} \right) \right] \rightarrow 0 \text{ as } m, n, k, l \rightarrow \infty \right\}$$

and

$$\Lambda_{f\pi}^4 = \left\{x \in \omega^4: \sup_{m,n,k,l \geq 1} \left[ f \left( \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} \right) \right] < \infty \right\}$$

The spaces  $\Lambda_{f\pi}^4$  and  $\Gamma_{f\pi}^4$  are metric spaces with metric

$$d(x, y) = \inf \left\{ \sup_{m,n,k,l \geq 1} \left[ f \left( \frac{|x_{mnkl} - y_{mnkl}|}{\pi_{mnkl}} \right) \right] \leq 1 \right\} \\ \forall x, y \in \Gamma_{f\pi}^4 \text{ and } \Lambda_{f\pi}^4.$$

**Theorem 4.1:**  $\eta(\Gamma_{f\pi}^4) = (\Gamma_{f\pi}^4)$

**Proof:** Firstly we shall show that

$$\eta(\Gamma_{f\pi}^4) \subset \Gamma_{f\pi}^4$$

Let  $x = (x_{mnkl}) \in \eta(\Gamma_{f\pi}^4)$ . Then  $y = (y_{mnkl}) \in (\Gamma_{f\pi}^4)$  so that for every  $\epsilon > 0$ , we have a positive integer  $m_0$  s.t.

$$f \left( \left| \frac{p_{0000}x_{mnkl} + \dots + p_{mnkl}x_{0000}}{\pi_{mnkl}P_{mnkl}} \right| \right) < \epsilon^{m+n+k+l} \quad \forall m, n, k, l \geq m_0$$

Take  $p_{0000} = 1; p_{1111} = \dots = p_{mnkl} = 0$ .

We then have  $f \left( \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right| \right) < \epsilon^{m+n+k+l}, \quad \forall m, n, k, l \geq m_0$ .

Therefore,  $x = (x_{mnkl}) \in \Gamma_{f\pi}^4$ .

Hence

$$\eta(\Gamma_{f\pi}^4) \subset \Gamma_{f\pi}^4 \tag{4.1}$$

In the second part, we shall show that

$$\Gamma_{f\pi}^4 \subset \eta(\Gamma_{f\pi}^4)$$

Let  $x = (x_{mnkl}) \in \eta(\Gamma_{f\pi}^4)$ . But for any given  $\epsilon > 0, \exists$  a positive integer  $m_0$  s.t.  $f \left( \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right| \right) < \epsilon^{m+n+k+l}, \quad \forall m, n, k, l \geq m_0$ . We have

$$\begin{aligned}
 f\left(\left|\frac{y_{mnkl}}{\pi_{mnkl}}\right|\right) &\leq f\left(\left|\frac{p_{0000}x_{mnkl}+\dots+p_{mnkl}x_{0000}}{p_{mnkl}\pi_{mnkl}}\right|\right) \\
 &\leq \frac{1}{P_{mnkl}}\left[p_{0000}\left(f\left|\frac{x_{mnkl}}{\pi_{mnkl}}\right|\right)+\dots+p_{mnkl}\left(f\left|\frac{x_{mnkl}}{\pi_{mnkl}}\right|\right)\right] \\
 &\leq \frac{1}{P_{mnkl}}\left[p_{0000}\epsilon^{m+n+k+l}+\dots+p_{mnkl}\epsilon^{0+0+0+0}\right] \\
 &\leq \frac{\epsilon^{m+n+k+l}}{P_{mnkl}}\left[p_{0000}+\dots+p_{mnkl}\right] \\
 &\leq \frac{\epsilon^{m+n+k+l}}{P_{mnkl}}P_{mnkl} \\
 &\leq \epsilon^{m+n+k+l} \quad \forall m, n, k, l \geq m_0
 \end{aligned}$$

Therefore,  $y_{mnkl} \in \Gamma_{f\pi}^4$ . Consequently  $x_{mnkl} \in \eta(\Gamma_{f\pi}^4)$ .

Hence

$$\Gamma_{f\pi}^4 \subset \eta(\Gamma_{f\pi}^4) \tag{4.2}$$

From equation (4.1) and (4.2), we get

$$\Gamma_{f\pi}^4 = \eta(\Gamma_{f\pi}^4)$$

**Theorem 4.2:**  $\eta(\Lambda_{f\pi}^4) = (\Lambda_{f\pi}^4)$

**Proof:** Firstly we shall show that

$$\Lambda_{f\pi}^4 \subset \eta(\Lambda_{f\pi}^4)$$

Let  $x = (x_{mnkl}) \in \eta(\Lambda_{f\pi}^4)$ . Then  $\exists$  a positive constant  $K$  s.t.

$$\begin{aligned}
 \left(f\left|\frac{x_{mnkl}}{\pi_{mnkl}}\right|\right) &< K^{m+n+k+l} \quad m, n, k, l = 0, 1, 2, \dots \\
 \left(f\left|\frac{y_{mnkl}}{\pi_{mnkl}}\right|\right) &\leq \frac{p_{0000}K^{m+n+k+l}+\dots+p_{mnkl}K^{0+0+0+0}}{P_{mnkl}} \\
 &\leq \frac{K^{m+n+k+l}}{P_{mnkl}}\left[p_{0000}+\dots+\frac{p_{mnkl}}{K^{m+n+k+l}}\right] \\
 &\leq \frac{K^{m+n+k+l}}{P_{mnkl}}P_{mnkl} \\
 &\leq K^{m+n+k+l} \quad \forall m, n, k, l = 0, 1, 2, \dots
 \end{aligned}$$

Hence  $(y_{mnkl}) \in \Lambda_{f\pi}^4$ . But  $(x_{mnkl}) \in \eta(\Lambda_{f\pi}^4)$ . Consequently,

$$\Lambda_{f\pi}^4 \subset \eta(\Lambda_{f\pi}^4) \tag{4.3}$$

In the second part, we shall show that

$$\eta(\Gamma_{f\pi}^4) \subset \Gamma_{f\pi}^4$$

Let  $(x_{mnkl}) \in \eta(\Lambda_{f\pi}^4)$ . Then  $(y_{mnkl}) \in (\Lambda_{f\pi}^4)$ .

Hence  $\exists$  a positive constant  $K$  s.t.

$$\left(f\left|\frac{x_{mnkl}}{\pi_{mnkl}}\right|\right) < K^{m+n+k+l} \text{ for } m, n, k, l = 0, 1, 2, \dots$$

This implies that

$$\begin{aligned}
 \left(f\left|\frac{p_{0000}x_{mnkl}+\dots+p_{mnkl}x_{0000}}{P_{mnkl}\pi_{mnkl}}\right|\right) &< K^{m+n+k+l} \\
 \Rightarrow \frac{1}{P_{mnkl}}\left(f\left|\frac{p_{0000}x_{mnkl}+\dots+p_{mnkl}x_{0000}}{\pi_{mnkl}}\right|\right) &< K^{m+n+k+l} \\
 \Rightarrow \left(f\left|\frac{p_{0000}x_{mnkl}+\dots+p_{mnkl}x_{0000}}{\pi_{mnkl}}\right|\right) &< K^{m+n+k+l}P_{mnkl}
 \end{aligned}$$

Take  $p_{0000} = 1, p_{1111} = \dots = p_{mnkl} = 0$ .

Then it follows that  $P_{mnkl} = 1$  and so  $\left(f \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right| \right) < K^{m+n+k+l}$  for all  $m, n, k, l$ .

Consequently  $x_{mnkl} \in (\Lambda_{f\pi}^4)$ .

Hence

$$\eta(\Lambda_{f\pi}^4) \subset (\Lambda_{f\pi}^4) \quad (4.4)$$

From equation (4.3) and (4.4), we get

$$\Lambda_{f\pi}^4 = \eta(\Lambda_{f\pi}^4)$$

**Theorem 4.3:**  $\Gamma_{f\pi}^4$  is not a barreled space.

**Proof:** Let

$$A = \left\{ x \in \Gamma_{f\pi}^4 : \left( f \left| \frac{x_{mnkl}}{\pi_{mnkl}} \right|^{\frac{1}{m+n+k+l}} \right) \leq \frac{1}{m+n+k+l}, \forall m, n, k, l \right\}.$$

Then  $A$  is an absolutely convex, closed in  $\Gamma_{f\pi}^4$ . But  $A$  is not a neighbourhood of zero. Hence  $\Gamma_{f\pi}^4$  is not barreled.

**Theorem 4.4:**  $\Gamma_{f\pi}^4$  is not semi-reflexive.

**Proof:** Let  $\{\delta_{mnkl}\} \in U$  be the unit closed ball in  $\Gamma_{f\pi}^4$ . Clearly no subsequence of  $\{\delta_{mnkl}\}$  can converge weakly to any  $y \in \Gamma_{f\pi}^4$ . Hence  $\Gamma_{f\pi}^4$  is not semi-reflexive.

## V. CONCLUSION

This paper begins with the introduction of the spaces of biquadratic sequences. We discuss some properties of Nörlund space of biquadratic sequences and also obtained why it is not barreled space and semi-reflexive. In last section of the chapter we established some theorem related to Nörlund Orlicz space of biquadratic rate sequences.

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