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Inferential Estimation of Reliability Function for the Inverse Family of Distributions

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Abstract: An inverse family of distributions is recommended. To derive UMVUES and MLES considered the problem of estimating R(t) and P. The estimators are developed when X and Y many follow same, over and above, dissimilar distributions to estimating P. A relative study has been done for the enactment of the two procedures of estimation. The enactment of estimators has been inspected by the Simulation approach.

Key words: Reliability function; stress-strength set-up; uniformly minimum variance unbiased estimators; maximum likelihood estimators; bootstrap methods.

1. Introduction

In the area of reliability estimation, much work has been done with inverse distributions. Inverse Weibull distribution based on various flops of mechanical mechanisms obtained by Keller and Kanath (1982). This model provided the suitable data to the failure exhibited by Keller, Giblin, and Farnwortt (1985). The reciprocal Weibull distribution is referred to by Mudholkar and Kollia (1994). The inverse Weibull distribution provided a decent fitting showed by Erto (1989). Wampler and Stablein (1983). The statistical properties of inverse Weibull distribution for the maximum likelihood estimator of the reliability function investigated by Calabria and Pulcini (1989).The maximum likelihood estimator of the parameters and reliability function was analyzed by Calabria and Pulcini (1990). The integrity time for the reliability function of inverse Weibull distribution were constructed by Calabria and Pulcini (1992). The inverse Weibull model based on progressive type-II censoring for the structure parameter constructed confidence interval and achieved a first-order BLUE by Nigam and Abo-eleneen (2007). The role of inverse gamma model as a life-time distribution has been exhibited by Lin et al. (1989). The moments of inverse Burr distribution were obtained by Tadikamalla (1980). To analyzing an epidemic outbreak data used inverse Burr as a survival distribution by Brock and Heestorbeek (2007).

An inverse family of inverse distributions was recommended by Chaturvedi and Sharma (2007). In specific cases, it's covering five distributions. The complications of estimating two measures of reliability are considered by them. Based on the regression approach they were derived uniformly minimum variance unbiased estimators (UMUVES), maximum likelihood estimators (MLES). They imagined when X and Y observed the similar distributions to estimate ‘P’.

We recommend a group of distributions, which lids some inverse distributions as special cases in the present paper. To derived UMVUES and MLES considered the problems of estimating R(t) and P. The estimators are developed when X and Y many follow same, over and above, dissimilar distributions and to estimating 'P', expanding the outcomes of Chaturvedi and Sharma (2007). The enactment of the estimators has inspected by the Simulation approach.

In section 2 the inverse family of distributions is acquainted. In section 3 the UMVUES of R(t) and 'P' are derived. In section 4 we achieved the MLES of R(t) and 'P'. The exploration of simulated approach and physical life records have been awarded for demonstrative goals in section 5.

2. The Inverse Family of Distributions

Suppose a random variable (rv) Y having pdf

$$f(y;\alpha,\beta,\underline{\theta})=\frac{\alpha^{\beta}G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G^{'}\left(y^{-1};\underline{\theta}\right)}{y^{2}\Gamma(\beta)}\exp\left(-\alpha G\left(y^{-1};\underline{\theta}\right)\right);$$

$$y>0,\ \alpha>0,\beta>0. \qquad \text{------ (2.1)}$$

Where, $G\left(y^{-1};\underline{\theta}\right)$, is depend on $\underline{\theta}$ and a function of y. Furthermore, $G\left(y^{-1};\underline{\theta}\right)$ real-valued, rigorously reducing function of y with $G(\infty;\underline{\theta})=\infty$ and $G'\left(y^{-1};\underline{\theta}\right)$ stances for the derivative of $G(y;\underline{\theta})$ by y^{-1}.
The equation (2.1) shows that the inverse family of distributions, can be converted in the following inverse family of distributions as special cases:

i. If $G(y;\underline{\theta})=y^{p}$, $p>0,\beta>0$, gives the inverse generalized gamma distribution.

ii. If $G(y;\underline{\theta})=y^{2}$, $\beta=k+1(k\geq 0)$, $\left(k=\frac{-1}{2}\right)$ gives the inverse Half-normal distribution and (k = 0) the inverse Rayleigh distribution.

iii. If $G(y;\underline{\theta})=log\left(1+\frac{y^{b}}{v^{b}}\right)$, $b>0,\ v=1,\ \beta>1$,gives the inverse Burr distribution.

iv. If $G(y;\underline{\theta})=log\left(1+\frac{y^{b}}{v^{b}}\right)$, $b=1,\ v>1,\ \beta>1$,gives the inverse Lomax distribution.

v. If $G(y;\underline{\theta})=\log\left(\frac{y}{a}\right)$ and $\beta=1$, we get inverse Pareto distribution.

vi. If $G(y;\underline{\theta})=y^{r}\exp(ay)$, $r>0,a>0,\beta=1$, we get inverse modified Weibull distribution.

vii. If $G(y;\underline{\theta})=\mu y+\frac{vy^{2}}{2}$, $\alpha=\beta=1$, we get inverse linear exponential distribution.

viii. If $G(y;\underline{\theta})=\log y$, we get inverse of the log-gamma distribution.

3. Uniformly Minimum Variance Unbiased Estimators for Reliability Function R(t) AND 'P'

Let β and $\underline{\theta}$ are known and α is unknown throughout this section. Suppose $Y_1,Y_2,\ldots\ldots\ldots\ldots\ldots,Y_n$ be a random sample of size n from (2.1).

Theorem 3.1: Let $Z=\sum_{i=1}^{n}G\left(y_i^{-1};\underline{\theta}\right)$. Therefore, Z is complete sufficient for the inverse family of distributions. And, the probability density function of Z is

$$k(z;\alpha,\beta,\underline{\theta})=\frac{\alpha^{n\beta}s^{n\beta-1}}{\Gamma(n\beta)}\exp(-\alpha z); \qquad z>0$$

Proof: Suppose $f^{\#}\left(y_1,y_2,\ldots\ldots\ldots y_n;\alpha,\beta,\underline{\theta}\right)$, is the likelihood of $y_1,y_2,\ldots\ldots\ldots\ldots\ldots,y_n$,

$$f^{\#}\left(y_1, y_2, \ldots\ldots\ldots y_n\right) = \frac{\alpha^{n\beta}}{\left\{\Gamma(\beta)\right\}^n} \prod_{i=1}^{n} \left\{ \frac{G^{\beta-1}\left(y_i^{-1};\underline{\theta}\right) G'\left(y_i^{-1};\underline{\theta}\right)}{y_i^2} \right\} \exp\left(-\alpha z\right) \qquad \text{--- (3.1)}$$

Z is sufficient $f\left(y;\alpha,\beta,\underline{\theta}\right)$ by Fisher-Neyman factorization theorem [21].

In (3.1), put $Q = G\left(y^{-1};\underline{\theta}\right)$, the pdf of Q is

$$k\left(q;\alpha,\beta,\underline{\theta}\right) = \frac{\alpha^{\beta} q^{\beta-1}}{\Gamma(\beta)} \exp\left(-\alpha q\right); \qquad q > 0$$

Z go by the additive property of gamma distribution [15]. Meanwhile, Z belongs to the exponential family of distributions, it remains complete.

Theorem 3.2 : For $p \in \left(-\infty, \infty\right)$, the UMVUE of α^{-p} is

$$\hat{\alpha}^{-p} = \begin{cases} \dfrac{\Gamma(n\beta)}{\Gamma(n\beta + p)} Z^{p}, & n\beta + p > 0 \\ 0, & otherwise \end{cases}$$

Proof : From theorem 3.1,

$$E\left(Z^{p}\right) = \frac{\alpha^{n\beta}}{\Gamma(n\beta)} \int_0^{\infty} z^{n\beta+p-1} \exp\left(-\alpha z\right) dz \quad = \left\{ \frac{\Gamma(n\beta + p)}{\Gamma(n\beta)} \right\} \alpha^{-p}$$

and the theorem observe from Lehmann-Scheffe theorem [21].

Theorem 3.3 : The UMVUE of $f\left(y;\alpha,\beta,\underline{\theta}\right)$ for a stipulated point 'y'

$$f\left(y;\alpha,\beta,\underline{\theta}\right) = \begin{cases} \dfrac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right) G'\left(y^{-1};\underline{\theta}\right)}{y^2 Z^{\beta} B((n-1)\beta, \beta)} \left[1 - \dfrac{G\left(y^{-1};\underline{\theta}\right)}{Z} \right]^{(n-1)\beta-1}, & G\left(y^{-1};\underline{\theta}\right) < Z \\ 0, & otherwise \end{cases}$$

Proof : Any function $K\left(Z\right)$ of Z gratifying $E\left[K\left(Z\right)\right] = f\left(y;\alpha,\beta,\underline{\theta}\right)$ will be the UMVUE of $f\left(y;\alpha,\beta,\underline{\theta}\right)$. Z complete sufficient statistic for the inverse family of distributions $f\left(y;\alpha,\beta,\underline{\theta}\right)$.

From (2.1) and theorem1, we get

$$\frac{\alpha^{n\beta}}{\Gamma(n\beta)} \int_0^{\infty} K(z) z^{n\beta-1} \exp\left(-\alpha z\right) dz \quad = \frac{\alpha^{\beta} G^{\beta-1}\left(y^{-1};\underline{\theta}\right) G'\left(y^{-1};\underline{\theta}\right)}{y^2 \Gamma(\beta)} \exp\left(-\alpha G\left(y^{-1};\underline{\theta}\right)\right),$$

or

or $\frac{\alpha^{(n-1)\beta}}{\Gamma(n\beta)}\int_0^\infty K(z) z^{n\beta-1}\exp\left(-\alpha\left(z-G\left(y^{-1};\underline{\theta}\right)\right)\right)dz = \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{y^2\Gamma(\beta)}$

or,

$$\frac{\alpha^{(n-1)\beta}}{\Gamma(n\beta)}\int_{-G\left(y^{-1};\underline{\theta}\right)}^{\infty} K\left(z+G\left(y^{-1};\underline{\theta}\right)\right)\left(z+G\left(y^{-1};\underline{\theta}\right)\right)^{n\beta-1}\exp(-\alpha z)dz$$

$$= \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{y^2\Gamma(\beta)} \qquad \text{-----------} \qquad (3.2)$$

Equation (3.2) is satisfied if we choose

$$K\left(z+G\left(y^{-1};\underline{\theta}\right)\right) = \begin{cases} \frac{\Gamma(n\beta)}{y^2\Gamma(\beta)\Gamma((n-1)\beta)} \cdot \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{\left[z+G\left(y^{-1};\underline{\theta}\right)\right]^{n\beta-1}} u^{(n-1)\beta-1}, & z>0 \\ 0, & -G\left(y^{-1};\underline{\theta}\right)\le z\le 0 \end{cases}$$

and the theorem holds.

Remarks 1 : We can write (2.1) as

$$f\left(y;\alpha,\beta,\underline{\theta}\right) = \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{y^2\Gamma(\beta)}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}G'\left(y^{-1};\underline{\theta}\right).\alpha^{i+\beta}$$

From Chaturvedi and Tomer (2002) [Lemma 1] and theorem 2, for integer-valued β, the UMVUE of $f\left(y;\alpha,\beta,\underline{\theta}\right)$ for stipulated point 'y'

$$\hat{f}\left(y;\alpha,\beta,\underline{\theta}\right) = \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{y^2\Gamma(\beta)}\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}G'\left(y^{-1};\underline{\theta}\right)\hat{\alpha}^{i+\beta}$$

$$= \frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G'\left(y^{-1};\underline{\theta}\right)}{y^2\Gamma(\beta)}\sum_{i=0}^{(n-1)\beta-1}(-1)^i\binom{(n-1)\beta-1}{i}\left\{\frac{G\left(y^{-1};\underline{\theta}\right)}{T}\right\}^i,$$

This concur with theorem 3.3, we derive the UMVUE of $f\left(y;\alpha,\beta,\underline{\theta}\right)$ for specified point 'y' using the UMVUE of the power of α, If β is integer-value.

Theorem 3.4 : The UMVUE of Reliability function

$$\hat{R}(t)=\begin{cases}1-I_{\frac{G\left(t^{-1};\underline{\theta}\right)}{z}}\left(\left(n-1\right)\beta,\beta\right), & G\left(t^{-1};\underline{\theta}\right)<Z\\ 0, & Otherwise\end{cases}$$

Where $I_z\left(s,q\right)=\frac{1}{\beta\left(s,q\right)}\int_0^z x^{s-1}\left(1-x\right)^{q-1}dx$ [The incomplete beta function]

Proof : Now, let us suppose the expectation

$$1-\int_0^t \hat{f}\left(y;\alpha,\beta,\underline{\theta}\right)dy$$

The integration with respect to Z

$$=1-\int_0^\infty\left\{\int_0^t \hat{f}\left(y;\alpha,\beta,\underline{\theta}\right)k\left(z;\alpha,\beta,\underline{\theta}\right)dz\right\}dy$$

$$=1-\int_0^t\left[E\left\{\hat{f}\left(y;\alpha,\beta,\underline{\theta}\right)\right\}\right]dy$$

$$=1-\int_0^t f\left(y;\alpha,\beta,\underline{\theta}\right)dy$$

$$=R\left(t\right) \qquad (3.3)$$

We deduce from (3.3) that $\hat{f}\left(y;\alpha,\beta,\underline{\theta}\right)$ can achieve the UMVUE of R(t). $G^{\#}\left(.\right)$, the inverse function of $G\left(.\right)$, by theorem 3.3.

$$\hat{R}(t)=1-\frac{1}{Z^{\beta}B\left(\left(n-1\right)\beta,\beta\right)}\int_{\left[G^{*}(Z)\right]^{-1}}^{t}\frac{G^{\beta-1}\left(y^{-1};\underline{\theta}\right)G^{'}\left(y^{-1};\underline{\theta}\right)}{y^2}\left[1-\frac{G\left(y^{-1};\underline{\theta}\right)}{Z}\right]^{(n-1)\beta-1}dy;$$

$$=1-\frac{1}{B\left(\left(\left(n-1\right)\beta\right),\beta\right)}\int_{\frac{G\left(t^{-1};\underline{\theta}\right)}{z}}^{1}x^{\beta-1}\left(1-x\right)^{\left(\left(n-1\right)\beta\right)-1}dx; \qquad \left(G\left(t^{-1};\underline{\theta}\right)<Z\right)$$

and the theorem proved.

Corollary 1 : If β = 1, then the distribution

$$\hat{R}(t)=\begin{cases}1-\left[1-\frac{G\left(t^{-1};\underline{\theta}\right)}{Z}\right]^{(n-1)}, & if\ \mathrm{G}\left(t^{-1};\underline{\theta}\right)<Z\\ 0, & otherwise\end{cases}$$

Suppose two independent rv's X and Y follow the inverse families of distributions $f_1\left(x;,\alpha_1,\beta_1,\underline{\theta}_1\right)$ and $f_2\left(y;\alpha_2,\beta_2,\underline{\theta}_2\right)$, sequentially,

$$f_1\left(x;\alpha_1,\beta_1,\underline{\theta}_1\right)=\frac{\alpha_1^{\beta_1}G^{\beta_1-1}\left(x^{-1};\underline{\theta}_1\right)G'\left(x^{-1};\underline{\theta}_1\right)}{x^2\Gamma\left(\beta_1\right)}\exp\left(-\alpha_1 G\left(x^{-1};\underline{\theta}_1\right)\right);$$

$$x>0,\ \alpha_1>0,\ \beta_1>0$$

$$f_2\left(y;\alpha_2,\beta_2,\underline{\theta}_2\right)=\frac{\alpha_2^{\beta_2}H^{\beta_2-1}\left(y^{-1};\underline{\theta}_2\right)H'\left(y^{-1};\underline{\theta}_2\right)}{y^2\Gamma\left(\beta_2\right)}\exp\left(-\alpha_2 H\left(y^{-1};\underline{\theta}_2\right)\right);$$

$$y>0,\ \alpha_2>0,\ \beta_2>0$$

Where $a_1,a_2,\beta_1,\beta_2,\underline{\theta}_1$ and $\underline{\theta}_2$ are known but α_1 and α_2 are unknown. Suppose a random sample $X_1,X_2,\ldots\ldots\ldots\ldots,X_n$ of size n from $f_1\left(x;\alpha_1,\beta_1,\underline{\theta}_1\right)$ and another random sample $Y_1,Y_2,\ldots\ldots\ldots\ldots,Y_m$ of size m from $f_2\left(y;\alpha_2,\beta_2,\underline{\theta}_2\right)$. And notation by $S=\sum_{i=1}^{n}G\left(x_i^{-1};\underline{\theta}_1\right)$ and $T=\sum_{i=1}^{n}H\left(y_i^{-1};\underline{\theta}_2\right)$.

Theorem 3.5: The UMUVE of 'P' is

$$\hat{P}=\begin{cases}\dfrac{1}{B\left(\left(n-1\right)\beta_1,\beta_1\right)B\left(\left(m-1\right)\beta_2,\beta_2\right)}\displaystyle\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{(n-1)\beta_1-1}{i} \\ \displaystyle\int_0^{\frac{H(G^*(S))}{T}} w^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}\left\{\frac{G\left(H^*(Tw)\right)}{S}\right\}^{\beta_1+i}dw, \\ \quad if\ \ G^*\left(S\right)<H^*\left(T\right) \\ \dfrac{1}{B\left(\left(n-1\right)\beta_1,\beta_1\right)B\left(\left(m-1\right)\beta_2,\beta_2\right)}\displaystyle\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{(n-1)\beta_1-1}{i} \\ \displaystyle\int_0^{1} w^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}\left\{\frac{G\left(H^*(Tw)\right)}{S}\right\}^{\beta_1+i}dw, \\ if\ \ G^*\left(S\right)>H^*\left(T\right)\end{cases}$$

The summations range from 0 to $(n-1)\beta_1-1$ in case $(n-1)\beta_1$ is an integer.

Proof : From theorem 3.3

$$\hat{f}_1(x;\alpha_1,\beta_1,\underline{\theta}_1)=\begin{cases}\dfrac{G^{\beta_1-1}(x^{-1};\underline{\theta}_1)G'(x^{-1};\underline{\theta}_1)}{x^2S^{\beta_1}B((n-1)\beta_1,\beta_1)}\left[1-\dfrac{G(x^{-1};\underline{\theta}_1)}{S}\right]^{(n-1)\beta_1-1}, & \mathrm{G}(x^{-1};\underline{\theta}_1)<S\\ 0, & otherwise\end{cases} \quad \ldots(3.4)$$

$$\hat{f}_2(y;\alpha_2,\beta_2,\underline{\theta}_2)=\begin{cases}\dfrac{H^{\beta_2-1}(y^{-1};\underline{\theta}_2)H'(y^{-1};\underline{\theta}_2)}{y^2T^{\beta_2}\beta((m-1)\beta_2,\beta_2)}\left[1-\dfrac{H(y^{-1};\underline{\theta}_2)}{T}\right]^{(m-1)\beta_2-1}, & \mathrm{H}(y^{-1};\underline{\theta}_2)<T\\ 0, & otherwise\end{cases} \quad \ldots.(3.5)$$

The UMVUES of $f_1(x;\alpha_1,\beta_1,\underline{\theta}_1)$ and $f_2(y;\alpha_2,\beta_2,\underline{\theta}_2)$ for specified pints 'x' and 'y' respectively, similarly, from theorem 3.4, we get the UMVUE of P

$$\hat{P}=\int_{y=0}^{\infty}\int_{x=y}^{\infty}\hat{f}_1(x;\alpha_1,\beta_1,\underline{\theta}_1)\,\hat{f}_2(y;\alpha_2,\beta_2,\underline{\theta}_2)\,dxdy$$

Using (3.4) and (3.5) we get

$$\hat{P}=\frac{1}{B((n-1)\beta_1,\beta_1)B((m-1)\beta_2,\beta_2)S^{\beta_1}T^{\beta_2}}$$

$$.\int_{y=[H^*(T)]^{-1}}^{\infty}\int_{x=y}^{\infty}\left\{\frac{G^{\beta_1-1}(x^{-1};\underline{\theta}_1)G'(x^{-1};\underline{\theta}_1)}{x^2}\right\}\left[1-\frac{G(x^{-1};\underline{\theta}_1)}{S}\right]^{(n-1)\beta_1-1}$$

$$\left\{\frac{H^{\beta_2-1}(y^{-1};\underline{\theta}_2)H'(y^{-1};\underline{\theta}_2)}{y^2}\right\}\left[1-\frac{H(y^{-1};\underline{\theta}_2)}{T}\right]^{(m-1)\beta_2-1} dxdy$$

$$=\frac{1}{B((n-1)\beta_1,\beta_1)\beta((m-1)\beta_2,\beta_2)T^{\beta_2}}\int_{y=[H^*(T)]^{-1}}^{\infty}\left\{\frac{H^{\beta_2-1}(y^{-1};\underline{\theta}_2)H'(y^{-1};\underline{\theta}_2)}{y^2}\right\}$$

$$.\left[1-\frac{H(y^{-1};\underline{\theta}_2)}{T}\right]^{(m-1)\beta_2-1}\int_{z=0}^{\frac{G(y^{-1};\underline{\theta}_1)}{S}} z^{\beta_1-1}(1-z)^{(n-1)\beta_1-1}\,dzdy$$

$$=\frac{1}{B((n-1)\beta_1,\beta_1)B((m-1)\beta_2,\beta_2)T^{\beta_2}}\left[\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{\{(n-1)\beta_1\}-1}{i}\right]$$

$$\cdot \int_{y=\max\left\{\left[G^*(S)\right]^{-1},\left[H^*(T)\right]^{-1}\right\}}^{\infty} \left\{\frac{H^{\beta_2-1}\left(y^{-1};\underline{\theta}_2\right)H'\left(y^{-1};\underline{\theta}_2\right)}{y^2}\right\}^i$$

$$\cdot\left[1-\frac{H\left(y^{-1};\underline{\theta}_2\right)}{T}\right]^{(m-1)\beta_2-1}\left\{\frac{G\left(y^{-1};\underline{\theta}_1\right)}{S}\right\}^{\beta_1+i} dy \qquad \text{----------(3.6)}$$

Let us suppose a case when $G^*(S) < H^*(T)$. Now, from (3.6),

$$\hat{P} = \frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)T^{\beta_2}}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{(n-1)\beta_1-1}{i}$$

$$\int_0^{\frac{h(G^*(S))}{T}} (Tw)^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}T\left\{\frac{G\left(H^*(Tw)\right)}{S}\right\}^{\beta_1+i} dw \qquad \text{-----(3.7)}$$

From (3.6), Now, if $G^*(S) > G^*(T)$. We get,

$$\hat{P} = \frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)T^{\beta_2}}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{(n-1)\beta_1-1}{i}$$

$$\int_0^1 (Tw)^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}T\left\{\frac{g\left(h^*(Tw)\right)}{S}\right\}^{\beta_1+i} dw \qquad \text{------(3.8)}$$

Then the theorem proved on combining (3.7) and (3.8)

Corollary 2 : If $\underline{\theta}_1 = \underline{\theta}_2 = \underline{\theta}$, and $G(x;\underline{\theta}) = H(x;\underline{\theta})$,

$$\hat{P}=\begin{cases}\dfrac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\dfrac{S}{T}\right)^{\beta_2}\sum_{i=0}^{\infty}\dfrac{(-1)^i}{(\beta_1+i)}\dbinom{\{(n-1)\beta_1\}-1}{i} \\ \quad\sum_{j=0}^{\infty}\dfrac{(-1)^j}{(\beta_1+\beta_2+i+j)}\dbinom{(m-1)\beta_2-1}{j}\left(\dfrac{S}{T}\right)^j, \qquad if\ S<T \\ \dfrac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\dfrac{T}{S}\right)^{\beta_1}\sum_{i=0}^{\infty}\dfrac{(-1)^i}{(\beta_1+i)}\dbinom{\{(n-1)\beta_1\}-1}{i} \\ \quad\left(\dfrac{T}{S}\right)^i B\left(\beta_1+\beta_2+i,(m-1)\beta_2\right), \qquad if\ S>T\end{cases}$$

The summation over *i* ranges from 0 to $\{(n-1)*\beta_1\}-1$, if $(n-1)\beta_1$ is an integer and the summation over *j* ranges from 0 to $(m-1)\beta_2$, if $(m-1)\beta_2$ is an integer.

Proof: we get From Theorem 3.5 for S < T,

$$\hat{P}=\left\{\frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\right\}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{\{n-1\}\beta_1-1}{i}$$

$$.\int_0^{\frac{S}{T}} w^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}\left(\frac{Tw}{S}\right)^{\beta_1+i}dw$$

$$=\frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\frac{S}{T}\right)^{\beta_2}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{\{n-1\}\beta_1-1}{i}$$

$$.\int_0^1 u^{\beta_1+\beta_2+i-1}\left(1-\frac{S}{T}u\right)^{(m-1)\beta_2-1}du$$

$$=\frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\frac{S}{T}\right)^{\beta_2}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\binom{(n-1)\beta_1-1}{i}$$

$$.\sum_{i=0}^{\infty}(-1)^j\binom{(m-1)\beta_2-1}{j}\left(\frac{S}{T}\right)^j\int_0^1 u^{\beta_1+\beta_2+i+j-1}du$$

and for S > T, from Theorem 3.2,

$$\hat{P} = \frac{1}{\beta((n-1)\beta_1, \beta_1)\beta((m-1)\beta_2, \beta_2)} \left(\frac{T}{S}\right)^{\beta_1} \sum_{i=0}^{\infty} \frac{(-1)^i}{(\beta_1 + i)}$$

$$\cdot \binom{\{n-1\}\beta_1 - 1}{i} \left(\frac{T}{S}\right)^i \int_0^1 w^{\beta_1+\beta_2+i-1} (1-w)^{(m-1)\beta_2 - 1} dw$$

and the second contention proved.

Remarks 2:

(i) We can see in the theorem 3.4 and Theorem 3.5, the UMVUES of R(t) and 'P are estimated separately using sampled pdf of UMVUES R(t) and 'P. Therefore, we observed two estimation problems that established the inter-relationship.

(ii) When X and Y follow the identical distribution, maybe with different parameters or maybe with the similar parameters other than and when X and Y follow unalike distributions using all the three situations, obtained the UMVUES of P'.

(iii) In theorem 3.5, if $n \to \infty$ then $Var(\hat{\alpha}) = \left(\frac{\alpha^2}{n\beta} - 2\right) \to 0$. We know that, $\hat{f}(y;\alpha,\beta,\underline{\theta})$, $\hat{R}(t)$ and $\hat{P}$ are continuous functions of consistent estimators of $f(y;\alpha,\beta,\underline{\theta})$, $R(t)$ and 'P', respectively. So, $\hat{\alpha}$ is a consistent estimator of α.

4. MLES OF R(t) AND 'P',WHEN ALL THE PARAMETERS ARE UNKNOWN

The log-likelihood function of equation (3.1)

$$\log L(\alpha,\beta,\underline{\theta}/\underline{y}) = n\beta\log\alpha - n\log\Gamma(\beta) + (\beta-1).\sum_{i=1}^{n}\log\{G(y_i^{-1};\underline{\theta})\}$$

$$+\sum_{i=1}^{n}\log\{G'(y_i^{-1};\underline{\theta})\} - 2\sum_{i=1}^{n}\log y_i - \alpha\sum_{i=1}^{n}G(y_i^{-1};\underline{\theta}) \qquad \text{-----(4.1)}$$

Theorem 4.1 : The $\tilde{f}(y;\alpha,\beta,\underline{\theta})$ is the MLE of $f(x;\alpha,\beta,\underline{\theta})$ for a specified point 'y'.

$$\tilde{f}(y;\alpha,\beta,\underline{\theta}) = \frac{(\tilde{\alpha})^{\tilde{\beta}} G^{\tilde{\beta}-1}(y^{-1};\tilde{\underline{\theta}})}{y^2\Gamma(\tilde{\beta})} \exp\left(-\alpha G(y^{-1};\tilde{\underline{\theta}})\right)$$

Proof : We can obtained from (4.1) using one-to-one property of the MLE.

Theorem 4.2 : $\tilde{R}(t)$ is the MLE of R(t)

$$\tilde{R}(t) = J_{\tilde{\alpha}G(t^{-1};\tilde{\underline{\theta}})}(\tilde{\beta}), \quad \text{and}$$

$$J_y(p) = \frac{1}{\Gamma(p)}\int_0^{\infty} x^{p-1}e^{-x}dx$$

Where the above integral is the incomplete gamma function.

Proof : The invariance property of the MLE, and using theorem 4.1

$$\tilde{R}(t)=1-\int_0^t \tilde{f}\left(y;\alpha,\beta,\underline{\theta}\right)dy$$

$$=1-\frac{(\tilde{\alpha})^{\tilde{\beta}}}{\Gamma(\tilde{\beta})}\int_0^t \frac{G^{\tilde{\beta}-1}\left(y^{-1};\tilde{\underline{\theta}}\right)G^{'}\left(y^{-1};\tilde{\underline{\theta}}\right)}{y^2}\exp\left(-\tilde{\alpha}G\left(y^{-1};\tilde{\underline{\theta}}\right)\right)dy$$

$$=1-\frac{1}{\Gamma(\tilde{\beta})}\int_{\tilde{\alpha}G\left(t^{-1};\tilde{\underline{\theta}}\right)}^{\infty} x^{\tilde{\beta}-1}e^{-x}dx$$

and the theorem proved.

Corollary 4.3 : For $\tilde{\beta}$ taking integer values,

$$\tilde{R}(t)=1-\sum_{i=0}^{\beta-1}\frac{\exp\left(-\tilde{\alpha}G\left(t^{-1};\tilde{\underline{\theta}}\right)\right)\left[\tilde{\alpha}G\left(y^{-1};\tilde{\underline{\theta}}\right)\right]^i}{i!}$$

Proof : If ‘p’ is an integer values then using the result of Patel, Kapadia and Owen (1974, p. 244) proved that,

$$\frac{1}{\Gamma(p+1)}\int_0^x x^p e^{-x}dx=1-\sum_{i=0}^{p}\frac{e^{-y}y^i}{i!}$$

Corollary 4.4 : The distributions for which β = 1,

$$\tilde{R}(t)=1-\exp\left(-\tilde{\alpha}G\left(t^{-1};\tilde{\underline{\theta}}\right)\right)$$

Theorem 4.3 : $\tilde{P}$ is the MLE of ‘P’

$$\tilde{P}=\frac{(\tilde{\alpha}_2)^{\tilde{\beta}_2}}{\Gamma(\tilde{\beta}_1)\Gamma(\tilde{\beta}_2)}\int_{y=0}^{Y_{(m)}}\left[\int_{z=\left(\tilde{\alpha}_1 g\left(X_{(n)}^{-1};\tilde{\underline{\theta}}_1\right)\right)}^{\tilde{\alpha}_1 g\left(y^{-1};\tilde{\underline{\theta}}_1\right)} e^{-z}z^{\beta_1-1}dz\right].$$

$$\frac{H^{\tilde{\beta}_2-1}\left(y^{-1};\tilde{\underline{\theta}}_2\right)H^{'}\left(y^{-1};\tilde{\underline{\theta}}_2\right)}{y^2}\exp\left(-\tilde{\alpha}_2 H\left(y^{-1};\tilde{\underline{\theta}}_2\right)\right)dy$$

Proof : Using one-to-one property of the MLE and theorem 4.1,

$$\tilde{P}=\int_{y=0}^{y_{(m)}}\int_{x=y}^{X_{(n)}}\tilde{f}_1\left(x;\alpha_1,\beta_1,\underline{\theta}_1\right)\tilde{f}_2\left(y;\alpha_2,\beta_2,\underline{\theta}_2\right)dxdy$$

$$=\frac{(\tilde{\alpha}_1)^{\tilde{\beta}_1}(\tilde{\alpha}_2)^{\tilde{\beta}_2}}{\Gamma(\tilde{\beta}_1)\Gamma(\tilde{\beta}_2)}\int_{y=0}^{y_{(m)}}\int_{x=y}^{X_{(n)}}\left\{\frac{G^{\tilde{\beta}_1-1}\left(x^{-1};\underline{\tilde{\theta}}_1\right)G'\left(x^{-1};\underline{\tilde{\theta}}_1\right)}{x^2}\right\}$$

$$.\exp\left(-\tilde{\alpha}_1 G\left(x^{-1};\underline{\tilde{\theta}}_1\right)\right)\left\{\frac{H^{\tilde{\beta}_2-1}\left(y^{-1};\underline{\tilde{\theta}}_2\right)H'\left(y^{-1};\underline{\tilde{\theta}}_2\right)}{y^2}\right\}\exp\left(-\tilde{\alpha}_2 H\left(y^{-1};\underline{\tilde{\theta}}_2\right)\right)dxdy$$

$$=\frac{(\tilde{\alpha}_1)^{\tilde{\beta}_1}(\tilde{\alpha}_2)^{\tilde{\beta}_2}}{\Gamma(\tilde{\beta}_1)\Gamma(\tilde{\beta}_2)}\int_{y=0}^{y_{(m)}}\frac{H^{\tilde{\beta}_2-1}\left(y^{-1};\underline{\tilde{\theta}}_2\right)H'\left(y^{-1};\underline{\tilde{\theta}}_2\right)}{y^2}\exp\left(-\tilde{\alpha}_2 H\left(y^{-1};\underline{\tilde{\theta}}_2\right)\right)$$

$$.\left\{\int_{z=\tilde{\alpha}_1 G\left(X_{(n)}^{-1};\underline{\tilde{\theta}}_1\right)}^{\tilde{\alpha}_1 G\left(y^{-1};\underline{\tilde{\theta}}_1\right)} e^{-z}\left(\frac{z}{\tilde{\alpha}_1}\right)^{\tilde{\beta}_1-1}\frac{dz}{\tilde{\alpha}_1}\right\}dy$$

and the theorem proved.

Remarks 3 :

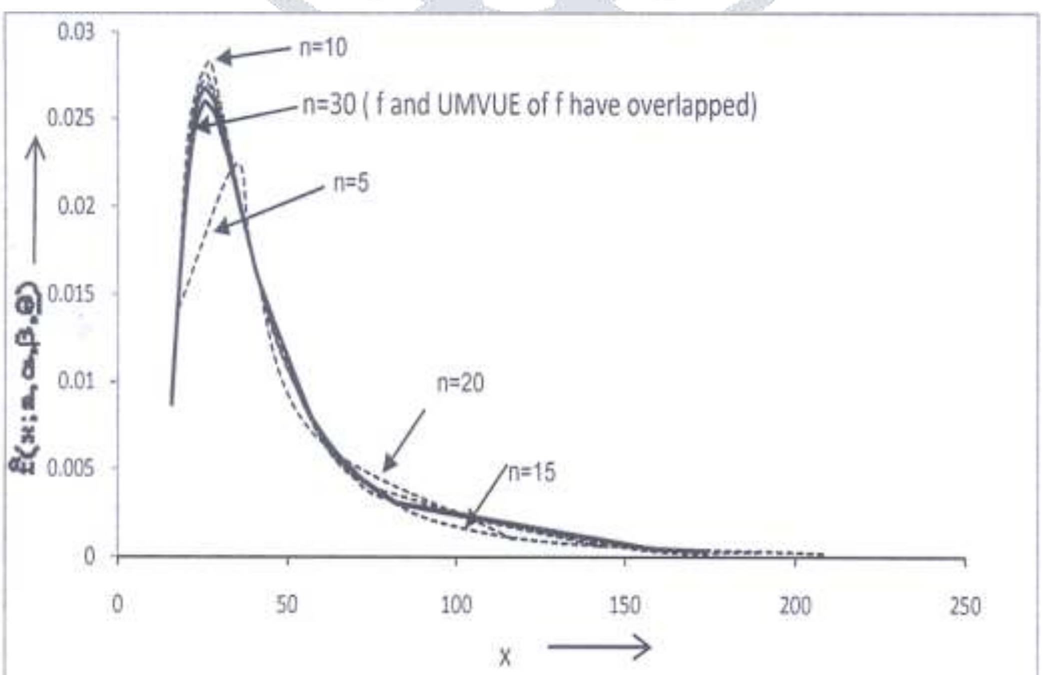
(i) UMVUES are tenable for the MLES under Remarks 2.

(ii) No need for the expressions of R(t) and 'P' to obtaining UMVUES and MLES.

5. SIMULATION APPROACH

We can see in remarks 2(iii) where $\hat{\alpha}, \hat{f}(y;\alpha,\beta,\underline{\theta}), \hat{R}(t)$ and $\hat{P}$ are consistent estimators. Fig. 1 plotted $\hat{f}(y;\alpha,\beta,\underline{\theta})$ for different sample sizes and conclude that the curves of $\hat{f}(y;\alpha,\beta,\underline{\theta})$ come closer to the curve of $f(y;\alpha,\beta,\underline{\theta})$ as sample size increases. And samples were drawn of size n = 5(5)20 and 30 From equation (2.1) if $G(y;\underline{\theta})=y^2$, $\beta=1$ and $\alpha=1000$ to verify the results . For n = 30, validates the consistency property of the estimators, because the curves overlap.

Fig. 1 : for $f(y;\alpha,\beta,\underline{\theta})$ -------- and for $\hat{f}(y;\alpha,\beta,\underline{\theta})$ **..........**



We have showed a simulation experiment using the bootstrap resampling technique for sample sizes n = 5(5)15 and 50 when α is unknown but other parameters are known. The samples are generated from (2.1) if $G\left(y^{-1};\underline{\theta}\right)=y^{2}$, β = 1, and α = 1000. Table 1 shows computation using 500 bootstrap replications with 95% confidence coefficient obtained the estimated value of UMVUES and MLES $R\left(t\right)$, bias, variance, mean sum of squares (MSES), For different values of t.

We have showed simulation trials using the bootstrap resampling technique for sample size (n, m) = (5, 5), (5, 10), (10, 10), (15, 15), (15, 25), (25, 25), (25, 30), (30, 30) when α_1 and α_2 are unknown but the other parameters are known to estimate P. The samples are produced from (2.1) if $G\left(x^{-1};\underline{\theta}\right)=\log\left(x\right)$, $\beta_1=\beta_2=1$, $G\left(y^{-1},\underline{\theta}\right)=\log\left(y\right)$, $a_1=a_2=0.01$, $\alpha_1=1$ and $\alpha_2=\frac{1}{2},\frac{1}{4},\frac{1}{6}$ and $\frac{1}{8}$. Table 2 shows computations using 500 bootstrap replications with 95% confidence coefficient obtained the estimated value of UMVUES and MLES P, bias, variance, mean sum of squares (MSES). In Section 4, we have developed a demonstration of the application for the theory.

Example:

Suppose Y is an inverse gamma distribution with unknown parameters

$$f_1\left(y;\alpha_1,\beta_1\right)=\frac{\alpha_1^{\beta_1}}{\Gamma\left(\beta_1\right)y^{\beta_1+1}}\exp\left(-\frac{\alpha_1}{y}\right);\ 0<y<\infty,\ \alpha_1>0,\ \beta_1=0 \qquad (5.1)$$

The log-likelihood is

$$\log L\left(\alpha_1,\beta_1\mid \underline{y}\right)=n\beta_1\log\alpha_1-n\log\Gamma\left(\beta_1\right)-\left(\beta_1+1\right)\sum_{i=1}^{n}\log y_i-\alpha_1\sum_{i=1}^{n}\frac{1}{y_i} \qquad (5.2)$$

If the MLEs of α_1 and β_1 are $\tilde{\alpha}_1$ and $\tilde{\beta}_1$ respectively, the likelihood equations becomes

$$\tilde{\alpha}_1=\frac{n\tilde{\beta}_1}{\sum_{i=1}^{n}\frac{1}{y_i}} \qquad \text{---------------} (5.3)$$

and $$n\log\tilde{\alpha}_1-n\frac{\Gamma'\left(\tilde{\beta}_1\right)}{\Gamma\left(\tilde{\beta}_1\right)}-\sum_{i=1}^{n}\log y_i=0 \qquad \text{----------------} (5.4)$$

From (5.1) with $\alpha_1=\beta_1=2$ generated a random sample of size 50:

0.9726528, 0.8059554, 0.9352156, 4.2241548, 1.3192561, 0.5556804, 1.2648771, 3.3811031, 2.7107683, 0.5362628, 1.3909402, 0.3591156, 0.5971763, 0.6479612, 1.7958025, 0.6308603, 0.4490559, 0.7657341, 0.7416295, 1.3797955, 0.3705804, 0.3887627, 1.2881025, 3.7087061, 6.5013019, 0.6301667, 1.5416644, 1.3730011, 5.4757502, 0.9938479, 1.6936616, 1.9595257, 15.8202208, 1.4133541, 3.1385684, 0.6545082, 1.2066657, 1.571548, 2.8287169, 1.1241934, 1.3001034, 6.2154783, 1.4956520, 1.5852830, 1.8836915, 1.2248313, 0.6209354, 0.8027393, 0.4348088, 1.3095457.

We get $\tilde{\alpha}_1 = 2.002446$ and $\hat{\beta}_1 = 2.030299$ Solving (5.3) and (5.4) simultaneously. Then, we obtain R(0.74) = 0.7352000 and $\tilde{R}(0.74)$=0.726702.

Let X follow inverse Weibull distribution

$$f_2(x;\alpha_2,\beta_2) = \frac{\alpha_2\beta_2}{x^{\beta_2+1}}\exp\left(-\frac{\alpha_2}{x^{\beta_2}}\right), 0 < y < \infty,\ \alpha_2 > 0,\ \beta_2 > 0 \quad \text{---(5.5)}$$

The log-likelihood of (5.5) is

$$\log L(\alpha_2,\beta_2 \mid \underline{x}) = n\log\alpha_2 - n\log\beta_2 - (\beta_2+1)\sum_{i=1}^{n}\log x_i - \alpha_2\sum_{i=1}^{n}\frac{1}{(x_i)^p}$$

The MLEs of α_2 and β_2 denoting by $\tilde{\alpha}_2$ and $\tilde{\beta}_2$, respectively, then

$$\tilde{\alpha}_2 = \frac{n}{\sum_{i=1}^{n}\frac{1}{(x_i)^{\tilde{\beta}_2}}} \quad \text{---------------- (5.6)}$$

and

$$\frac{n}{\tilde{\beta}_2} - \sum_{i=1}^{n}\log x_i + \tilde{\alpha}_2\sum_{i=1}^{n}\frac{\log x_i}{(x_i)^{\tilde{\beta}_2}} = 0 \quad \text{--------------- (5.7)}$$

$$P = \int_0^\infty\int_{y=x}^\infty f_1(y;\alpha_1,\beta_1)\, f_2(x;\alpha_2,\beta_2)\,dydx$$

$$= \int_0^\infty\left[1-\exp\left(-\frac{\alpha_1}{x^{\beta_1}}\right)\right]\frac{(\alpha_2)^{\beta_2}}{\Gamma(\beta_2)x^{\beta_2+1}}\exp\left(-\frac{\alpha_2}{x}\right)dx.$$

Thus, $\tilde{P} = \int_0^\infty\left(1-\exp\left(-\frac{\tilde{\alpha}_1}{x^{\beta_1}}\right)\right)\frac{(\tilde{\alpha}_2)^{\tilde{\beta}_2}}{\Gamma(\tilde{\beta}_2)x^{\beta_2+1}}\exp\left(-\frac{\tilde{\alpha}_2}{x}\right)dx$

Put $\alpha_2 = 1$ and $\beta_2 = 2$, generated a sample of size 50 from (5.5)

2.0610061, 0.8768468, 1.6200659, 2.0055902, 1.1885818, 1.4876399, 2.5082242, 1.8627984, 1.0388745, 3.1222151, 1.0401347, 1.0033122, 1.1967154, 0.5352343, 0.8513860, 0.7181698, 0.6042370, 1.3026825, 0.8082968, 1.1752178, 2.4336687, 1.6297682, 1.3234489, 0.7522204, 1.7227175, 2.0796265, 1.2977617, 0.6506584, 3.6656974, 1.2090752, 0.8330044, 0.5908377, 2.9358808, 1.1747845, 1.4442243, 0.7608844, 2.7082828, 2.2248628, 1.0062166, 0.7421728, 3.2906883, 0.7398104, 2.9310752, 0.9199286, 1.2173181, 1.9784526, 6.9812447, 1.8577832, 0.8661783, 1.2434324.

Find $\tilde{\alpha}_2 = 1.110087$ and $\tilde{\beta}_2 = 2.185285$ by solving (5.6) and (5.7) simultaneously. We get P = 0.5157443, $\tilde{P} = 0.5174784$, Using the data of X and Y simultaneously and theorem 4.3.

Table 1: Estimate of R(t) Using Simulation Approach

N		5		10		15		50	
1	$R(t)$	$\tilde{R}(t)$	$\hat{R}(t)$	$\tilde{R}(t)$	$\hat{R}(t)$	$\tilde{R}(t)$	$\hat{R}(t)$	$\tilde{R}(t)$	$\hat{R}(t)$
15	0.988256	0.975000 –0.013256 0.001254 0.109726 75.150500	0.975577 –0.012679 0.001535 0.113457 72.196300	0.982199 –0.006058 0.000342 0.050549 79.221600	0.987858 –0.000399 0.000271 0.042173 73.581400	0.985162 –0.003095 0.000167 0.039335 83.633800	0.989151 0.000894 0.000134 0.034016 80.072600	0.987757 –0.000500 0.000040 0.020675 87.371500	0.988996 0.000740 0.000038 0.019854 86.957600
20	0.917915	0.903480 –0.01435 0.008234 0.287885 83.748200	0.885121 –0.032794 0.010621 0.322730 84.561000	0.910388 –0.007528 0.003464 0.178809 86.774200	0.914626 –0.003289 0.003984 0.188914 85.838900	0.915237 –0.002678 0.002017 0.145604 89.152000	0.918637 0.000722 0.002234 0.152525 88.850000	0.918618 0.000703 0.000565 0.079166 89.878200	0.919682 0.001767 0.000585 0.080517 89.856300
25	0.798104	0.803311 0.005207 0.019340 0.442341 87.788800	0.786440 –0.011663 0.025835 0.499316 86.995000	0.798970 0.000866 0.008834 0.295848 88.344800	0.793371 –0.004733 0.010559 0.323471 88.329400	0.801307 0.003204 0.005178 0.237045 90.018900	0.797564 –0.000540 0.005855 0.252153 90.020100	0.801190 0.003086 0.001404 0.125086 90.376500	0.799987 0.001883 0.001456 0.127418 90.380900
30	0.670807	0.698337 0.027530 0.028333 0.537555 88.952600	0.673953 0.003146 0.037555 0.611731 88.534100	0.680869 0.010061 0.012591 0.356619 88.054900	0.666155 –0.004652 0.014486 0.383180 87.934300	0.679482 0.008675 0.007144 0.278715 89.726500	0.669284 –0.001523 0.007823 0.291637 89.654700	0.675551 0.004743 0.001827 0.142550 90.436600	0.672371 0.001564 0.001874 0.144326 90.436400
45	0.389714	0.444143 0.054429 0.029392 0.548545 87.657900	0.402206 0.012492 0.031309 0.564467 86.719500	0.409555 0.019841 0.011188 0.331236 85.906000	0.387121 –0.002593 0.011276 0.331191 85.513800	0.402778 0.013064 0.005701 0.245934 88.381800	0.387598 –0.002116 0.005672 0.244903 88.226800	0.394959 0.005245 0.001285 0.118951 90.280500	0.390400 0.000686 0.001279 0.118657 90.273600
50	0.329680	0.383788 0.065108 0.025684 0.511556 87.058500	0.341782 0.012102 0.025609 0.508494 86.084500	0.349176 0.019486 0.009403 0.301375 85.327900	0.327590 –0.002090 0.009186 0.296388 84.927200	0.342179 0.012499 0.004661 0.221558 88.068400	0.327687 –0.001993 0.004541 0.218317 87.920300	0.334531 0.004851 0.001024 0.106079 90.232900	0.330208 0.000528 0.001013 0.105504 90.226100
55	0.281492	0.333453 0.051961 0.021851 0.470494 86.529100	0.292797 0.011305 0.020589 0.454089 85.572000	0.300028 0.018536 0.007747 0.271640 84.840200	0.279791 –0.001701 0.007379 0.263625 84.447800	0.293153 0.011661 0.003752 0.198166 87.816300	0.279655 –0.001837 0.003595 0.193649 87.678900	0.285911 0.004419 0.000808 0.094143 90.193600	0.281906 0.000414 0.000795 0.093416 90.187100
60	0.242535	0.291452 0.048917 0.018320 0.429515 86.073600	0.252908 0.010373 0.016455 0.404430 85.160800	0.259840 0.017305 0.006327 0.243921 84.430300	0.241133 –0.001402 0.005901 0.234210 84.055500	0.253268 0.010733 0.003004 0.176869 87.612700	0.240860 –0.001675 0.002840 0.171694 87.487200	0.246532 0.003997 0.000636 0.083515 90.161200	0.242865 0.000330 0.000625 0.082718 90.155200
70	0.184604	0.226701 0.042097 0.012663 0.355076 85.355200	0.193162 0.008558 0.010556 0.321943 84.559700	0.199307 0.014703 0.004203 0.196726 83.794200	0.183617 –0.000987 0.003797 0.185941 83.465100	0.193547 0.008943 0.001936 0.141401 87.311100	0.183228 –0.001377 0.001794 0.135939 87.207900	0.187856 0.003252 0.000401 0.066187 90.112600	0.184825 0.000221 0.000391 0.065382 90.107600
80	0.144655	0.180364 0.035709 0.008759 0.293909 84.832800	0.151691 0.007036 0.006920 0.259485 84.154400	0.157014 0.012360 0.002828 0.160079 83.336400	0.143928 –0.000727 0.002498 0.149681 83.053300	0.152079 0.007424 0.001274 0.114385 87.019400	0.143522 –0.001133 0.001165 0.109245 87.019400	0.147314 0.002660 0.000259 0.053230 90.078900	0.144811 0.000156 0.000252 0.052489 90.074700

The sequence of row indicates the estimates, the bias, the variance, length and the coverage percentage of 95% confidence .

Table 2 : Estimation of P Using Simulation Approach

α_2	$\left(\frac{1}{2}\right)$		$\left(\frac{1}{4}\right)$		$\left(\frac{1}{6}\right)$		$\left(\frac{1}{8}\right)$	
P	0.6666667		0.8		0.8571429		0.8888889	
(m, n)	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$
(5, 5)	0.66625 –0.00042 0.01310 0.37465 89.45130	0.55623 –0.11044 0.00542 0.22246 80.37040	0.79245 –0.00755 0.00825 0.29287 88.37660	0.47390 –0.32610 0.01490 0.37771 85.74140	0.85131 –0.00583 0.00521 0.21917 85.85220	0.38238 –0.47476 0.01713 0.42422 89.45480	0.88938 0.00050 0.00226 0.15231 87.89120	0.30929 –0.57960 0.01262 0.36142 88.90590
(5, 10)	0.66939 0.00272 0.01343 0.38062 89.88720	0.53600 –0.13067 0.00452 0.19540 78.37130	0.80224 0.00224 0.00617 0.25112 88.46100	0.46939 –0.33061 0.01127 0.31962 84.44310	0.85300 –0.00415 0.00525 0.23214 87.51160	0.38720 –0.46995 0.01481 0.40524 90.30860	0.88923 0.00034 0.00244 0.15582 87.72200	0.32230 –0.56659 0.01240 0.35937 89.30480
(10, 10)	0.66096 –0.00591 0.00709 0.26423 88.23800	0.65675 –0.00991 0.00550 0.21508 82.97810	0.79749 –0.00251 0.00320 0.17838 87.85950	0.70795 –0.09205 0.00192 0.11900 74.26570	0.84474 –0.01241 0.00329 0.18275 86.83540	0.65375 –0.20340 0.00603 0.24016 84.09150	0.88476 –0.00413 0.00155 0.12176 87.11550	0.58240 –0.30649 0.00975 0.32224 89.11930
(15, 15)	0.66441 –0.00226 0.00604 0.26138 89.98640	0.66743 0.00076 0.00588 0.25845 89.62380	0.80245 0.00245 0.00166 0.13777 90.89700	0.77868 –0.02132 0.00050 0.06267 75.94680	0.85492 –0.00223 0.00127 0.11497 88.74370	0.76714 –0.09000 0.00126 0.10138 77.24890	0.88795 –0.00094 0.00092 0.09689 88.14030	0.71988 –0.16901 0.00398 0.19464 85.30640
(15, 25)	0.66978 0.00311 0.00324 0.18845 90.10820	0.66943 0.00277 0.00312 0.18624 90.23030	0.79561 –0.00439 0.00234 0.15194 87.77000	0.76784 –0.03216 0.00088 0.08226 75.26710	0.85674 –0.00040 0.00111 0.10749 89.23060	0.76313 –0.09401 0.00131 0.10513 75.75390	0.88892 0.00003 0.00065 0.08171 88.63980	0.72191 –0.16698 0.00287 0.16528 85.85700
(25, 25)	0.66432 –0.00234 0.00296 0.17797 89.80140	0.66728 0.00061 0.00304 0.18026 89.77150	0.79942 –0.00058 0.00182 0.14352 90.47150	0.79972 –0.00028 0.00149 0.12584 88.23660	0.85627 –0.00088 0.00084 0.09115 88.21130	0.84052 –0.01662 0.00032 0.04498 73.12100	0.88749 –0.00140 0.00054 0.07990 90.71200	0.84112 –0.04777 0.00027 0.05119 76.80410
(25, 30)	0.66545 –0.00122 0.00278 0.17136 89.55620	0.66744 0.00077 0.00285 0.17360 89.61430	0.79985 –0.00015 0.00145 0.12950 90.80010	0.79960 –0.00040 0.00120 0.11518 89.27470	0.85393 –0.00322 0.00130 0.11215 87.34830	0.83576 –0.02138 0.00054 0.05916 72.99830	0.88492 –0.00397 0.00062 0.07973 88.83420	0.84001 –0.04888 0.00031 0.04987 74.42350
(30, 30)	0.66466 –0.00201 0.00223 0.16182 91.31110	0.67086 0.00419 0.00190 0.14227 89.39850	0.79943 –0.00057 0.00127 0.11788 89.66460	0.80168 0.00168 0.00117 0.11381 89.51520	0.85183 –0.00531 0.00115 0.10656 87.84130	0.84563 –0.01151 0.00067 0.07456 80.50970	0.88852 –0.00037 0.00048 0.07390 90.07250	0.86285 –0.02604 0.00014 0.02872 71.64800

The sequence of row indicates the estimates, the bias, the variance, length and the coverage percentage of 95% confidence.

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