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Integrated supply chain model for imperfect production and fuzzy parameters with probabilistic demand pattern, and variable production rate under the environment of inflation

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Abstract: In this paper, the researchers have developed a supply chain inventory model with an imperfect production process for deteriorating items with variable production rates. In fact, it has been observed that in the production process at the beginning of any production, the rate of production remains almost constant for a certain period of time, but after that the rate of production decreases due to law of variable proportions. This law applies due to problems like machine failure, sluggishness of workers, or delay in reaching the raw material to the production plant, etc. All these lead to uncertainty about cost and by removing it, fuzzy set theory has been used to understand the model naturally and the parameters that affect the model and those who have some uncertainty about them have been taken as triangular fuzzy numbers. In developing the model, production has been taken demand dependent with probabilistic demand patterns.

Key words: *multi echelon, deteriorating items, imperfect production, probabilistic demand pattern, fuzzy, inflation.*

Introduction:

A supply chain model normally consists of many types of facilities like, the handling of the entire production flow of goods or services which starts from the raw components all the way to deliver the final product to the user. Concept of inventory optimization is to have the reasonable quantity of inventory in the storage, at the correct time in order to successfully meet customers' requirements with demand at all steps of the supply chain. Multi echelon inventory optimization (MEIO) plays important role to give the solution for optimization. A MEIO is one that relies heavily on hierarchy of suppliers distributed across multiple distribution places and that is based on outsourced manufacturing. Jauhari et al [1] discussed about cooperative inventory model in unequally sized shipment for defective goods with carbon emission cost. They considered deterministic demand for vendor and buyer with carbon emission cost added in their model and investigated its impact on environment. Yadav et. al [2] proposed an inventory model in which they considered multivariable demand under PT for multi constraints supply chain model. In this supply chain model trade credit plays an important part, where manufacturer provide trade credit to its retailer with demand depending on trade credit. Green supply chain management is the proper flow of services or information from the manufacturers and suppliers to the end customer in the context of the environment. Hence, the emphasis is laid on the development of new eco-friendly models by the supply chain managers. Manna et. al [3] presented a two-layer green supply chain inventory model for imperfect production in which three tier credit period is taken. Ullah et al [4] established a two-echelon supply chain model under the effect of PT on waste production. The investment in the PT used by the researchers to control the amount of deterioration is assumed to be variable taken in this paper. Mashud et. al [5] discussed an inventory model for imperfect production item to control carbon emission under deterioration. In the present era, there is need of smart product for use in daily life. One of the main reasons people buy smart products is the energy savings they make. Bhuniya et. al [6] presented a smart production inventory system in which production process of items under supply chain management for maximum energy consumption is considered. Rout et. al [7] developed an inventory model for deteriorating and imperfect items keeping in mind the carbon emission norms under sustainable supply chain. Through this paper they developed a model to reduce the emission of carbon caused by various sources of SCM like transportation, warehousing by implimenting various carbon reduction policies. Daryanto and Wee [8] discussed a green supply chain inventory model having deteriorated imperfect quality items.

In the inventory system, the deterioration of goods cannot be ignored as it is their natural property, which results in decline in the usefulness or price of that commodity, such as pharmaceutical substances, food items and chemical substances etc. The concept of deterioration of items plays a very important role in developing the inventory model, so deterioration cannot be ignored while developing the inventory model. The deterioration rate of stock during storage has attracted the attention of the researchers. Ghare and Schrader [9] were the first who introduced deteriorating inventory model with exponential demand. The amount of delay in payment depends on the quantity of the order, when the quantity of the order is less than the estimated quantity of delay on payment, payment must be made for the item. Agrawal and Jaggi [10] presented deterioration inventory model under the permissible delay in payment. Mishra et al [11] proposed an inventory model having time dependent demand where deterioration is taken as proportional to time, shortage is also considered for this model. Sheikh and Patel [12] presented inventory model for deteriorating items under shortage and assumed different deterioration rates in cycle considering holding cost a function of time. Demand is an important factor in the success of any inventory model, is of many types and depends on many factors. Many research papers have been published on different types of demand. Bishi and Sahu [13] presented a deteriorating item inventory model with quadratic demand under shortage in which the deterioration rate is constant for the model. Khurana et. al [14] discussed the production inventory model for deteriorating item having variable demand and shortage. Mahapatra et. al [15] developed the fuzzy EOQ model for deteriorating items with the effect of learning and promotional effect with a finite time interval. Tayal et. al [16] discussed a two-warehouse inventory model for deteriorating items having different demand under shortage, they considered the warehouse as a rented warehouse. Various research papers based on discount policy have been published by many researchers. Saren et. al [17] described a price discount policy for deteriorating item in advanced inventory model having delay in payment. There are some products whose demand increases for a certain period and then stabilizes after attaining the highest level, this type of demand is called ramp type demand. Shaikh et. al [18] presented an inventory model for deteriorating items with trade credit facility under preserved technology, where the demand is taken to be ramp type and shortage is allowed for the model. Due to the deterioration of perishable goods, their utility and value keeps on decreasing throughout their lifetime. Considering this fact, preserving the deteriorating goods becomes a challenging problem. Mishra et. al [19] presented an EPQ model under preservation technology for the deteriorating items under fully backlogged shortage.

Inflation has strained the most vulnerable section of the consumers (the daily wage earners), who mostly demand goods in short run and in small quantities. Hence inflation plays an important role in developing any model. Singh and Singh [20] discussed a production inventory model for imperfect production having exponential demand and inflation. Uncertainty is defined as the ambiguity that is characteristic of fuzzy set theory. Singh et. al [21] formulated an imperfect production model for storage problem under the effect of inflation and fuzzy environment and used triangular fuzzy number to fuzzify the total cost of the inventory model with shortage by considering a realistic assumption that production rate is dependent upon demand. Volume flexibility is a flexible system that allows production to be adjusted up or down at the time of manufacturing. Jain et. al [22] presented a volume flexible inventory model for imperfect production and deterioration under inflation. Singh et. al [23] discussed an inflationary production inventory model for deteriorating items under shortage. They assume linear demand under the effect of time value of money with constant deterioration rate and time dependent holding cost. Ramp type demand is demand which increases for a certain time interval and then becomes constant. Kumar and Rajput [24] discussed inventory model by taking inflation under probabilistic deterioration and shortage having ramp type demand. Singh et. al [25] described an inventory model for deteriorating items having trade credit shortage with replenishment policy. Based on trade credit different cases have been studied under the inflationary environment. Barman et. al [26] described a cloudy fuzzy inventory model for deteriorating items having shortage under the effect of inflation. The total cost is defuzzified by ranking index method of fuzzy as well as cloudy fuzzy number.

In real life, there is ambiguity that many concepts are neither fully true nor completely false. There have been different types of experiments to tell such arguments efficiently of which fuzzy logic has made an important place of itself. The main function of fuzzy set theory is how to deal with objects related to uncertainty. So the fuzzy set theory is playing a very important role for inventory research. Researchers have developed models by relating fuzzy and inventories among themselves. In developing the inventory models deterioration plays an important place and by using this concept Jaggi et al. [27] analysed an inventory model for time varying demand of deteriorating items and shortage in which the total cost is discussed in fuzzy sense. For effective demand the key factors are viz. price, advertisement, quality of products etc. Sharmila and Uthayakumar [28] formulated the model for the deteriorating items on exponential demand, in which deteriorating cost, holding cost, shortage cost and purchasing cost are taken as triangular fuzzy number. De and Beg [29] explained the triangular dense fuzzy sets and gave different method of solving the problem based on optimization. Saha [30] developed a fuzzy model in which the demand is price dependent and used the singed distance method to defuzzify the cost function. Karmakar et al. [31] discussed the EOQ model in cloudy fuzzy demand rate. Saranya and Varadarajan [32] developed an inventory model in which the demand is constant with acceptable shortage and used the graded mean integration method to defuzzify the cost function. Over the past few years, various inventory models have been shown by various researchers with imperfect production. A model for imperfect quality items in cloudy fuzzy sense has been introduced by De and Mahata [33]. Sharma et al. [34] described a reverse logistics inventory model in which production and remanufacturing both are discussed in fuzzy environment.

The performance of individuals engaged in an organization or business can be improved over time. This learning effect reduces the cost of the product, it also increases the quality of the product and there is a possibility of getting more profit. Jawla et al [35] discussed an EPQ model for imperfect items using the concept of learning under the effect of inflation. Saha and Chakrabarti [36] presented supply chain model taking different types of demands for deteriorating items in imperfect production system under trade credit. A flexible manufacturing system is a system of production in which the type and quantity of the product being manufactured can be changed. Machines are designed for changing levels during production with the help of computerized system. Dem et. al [37] proposed an inventory model for perfect as well as imperfect items having different demands based on volume flexible manufacturing system. Sanjai and Periyasamy [38] discussed an imperfect production model where the manufacturing system provides the imperfect quality items. These imperfect quality items can be reworked and repaired, hence the total cost reduces significantly. Mahata and Mahata [39] considered an imperfect production model under fuzzy environment with discount cash flow policies. They have taken credit policies in this model where supplier provides cost discount to retailer and retailer provides permissible delay to its customer. Raut et. al [40] presented in EPQ model having imperfect production under rework and shortage for deteriorating items. Two types of inspection errors namely type one and type two are taken in this model. In many inventory models, demand is taken as constant or linear. But in real life, demand depends on many factors, so different demands have been envisaged by researchers while expanding the inventory model. Kumar et. al [41] described an inventory model for imperfect production process having different demands under fuzzy environment.

The rest of the paper is presented as follows: Research gap analysis are presented in Section 2. In Section 3, there are notation and assumptions for the proposed model. Mathematical model of this problem with subsections of producer and buyer presented in Section 4. The fuzzy model is developed in section 5. In section 6 the solution methodology of numerical análisis is presented. Sensitivity analysis of this model is in Section 7. Graphical representation, Analysis and Discussion of this study in section 8. Conclusion of this model is shown in Section 9.

2. Research gap analysis:

The difference between the work done and the work of this paper can be understood through Table 1. Many researchers have developed inventory models very effectively in different conditions, but very few have been able to develop the multi echelon inventory model with probabilistic demand pattern and variable production rate under inflationary and fuzzy environment.

Table 1. Key features of the inventory model developed in earlier researches

Author's	Model	Demand	Deteriorating item	Inflation	Fuzzy
Singh and Singh [42]	Multi echelon	Exponential	No	Yes	Yes
Shastri et al. [43]	Multi echelon	Constant	Yes	Yes	No
Urvashi et al [44]	Multi echelon	Stock dependent	No	Yes	No
Singh et al. [45]	Multi echelon	Constant	No	No	No
Yadav et al. [2]	Multi echelon	Function of selling price	No	No	No
Dai et al. [46]	Multi echelon	Ramp type, and traipezoidal	Yes	No	No
Yadav and Singh [47]	Multi echelon	Function of selling price	Yes	No	No
Lu et al. [48]	Multi echelon	Function of selling price	No	No	No
This model	Multi echelon	Probabilistic demand	Yes	No	Yes

3. Notation and Assumptions:

3.1. Notations:

In this subsection, some notation can be used for development the model.

D	Demand rate
P	Production rate
θ_1	Deteriorating rate for producer
θ_2	Deteriorating rate for buyer
T_1	Constant production period
T_2	Cycle length of total production
T_3	Buyer cycle length for one shipment given by producer
T	Total cycle length of this model
P_S	Setup cost for producer
Z_S	Development cost of producer
P_P	Production cost for producer
h_P	Holding cost for producer
d_P	Deteriorating cost for producer
O_b	Ordering cost for buyer
h_b	Holding cost for buyer
d_b	Deteriorating cost for buyer
Ψ	total number of shipments given by producer to buyer

3.2. Assumption:

1. Demand rate follows uniform distribution, so demand rate as D =f [(x)]= $\frac{(\sigma+\zeta)}{2}$, σ > 0, ζ > 0 and σ < ζ.
2. shortage is not allowed in any part of supply chain
3. production rate is variable which is

$$P=\begin{cases} P_0, & for\ 0 \le t \le T_1 \\ P_0 e^{-\mu(t-T_1)}, & for\ T_1 \le t \le T_2 \end{cases}$$

Where μ is constant $0< \mu < 1$

4. that in the production process at the beginning the rate of production remains almost constant for a certain period of time, but after that it decreases due to some inherent problems as machine failure, sluggishness of workers to work, or delay in reaching the raw material to the production plant, etc. Such undesired conditions can adversely affect the profitability of the model. In present time the business of the market has become so competitive that it is a big challenge to maintain the existence of the company in the field of business. And that's why no businessman can ignore such unwanted situations. Therefore, to guard against such unforeseen situations, proper strategy is required for them and for doing so one has to pay additional cost, which is known as development cost.

Here considered the development cost as the function of initial production rate

$Z_S=\delta P_0$ where δ is constant

4. Mathematical Model

4.1 Producer inventory model:

The rate of production is fixed (p_0) in the interval [0, T_1], then it starts decreasing with time, and this condition continues till time t = T_2. And in the time interval $[T_2, T]$ there is no production of any kind in the production plant, only demand is supplied, the inventory levels of producer reach to zero at t = T due to a mixed effect of deterioration and demand**.** Producer's inventory level can be represented by the following first order linear differential equations.

$$\frac{dI_P(t)}{dt}+\theta_1 I_P(t)=\begin{cases} P_0-D & ;\ 0 \le t \le T_1 \\ P_0 e^{-\mu(t-T_1)}-D & ; T_1 \le t \le T_2 \\ -D & ; T_1 \le t \le T \end{cases} \tag{1}$$

With the condition

$I_P(T_1)$=Q_{P1}, I_P(T)=0, $I_P(T_2)$=Q_{P2}

Solution of above equations are;

$$I_{P_1}(t)=\frac{P_0-D}{\theta_1}-\frac{(P_0-D)}{\theta_1}e^{-\theta_1 t} \tag{2}$$

$$I_{P_2}(t)=\frac{P_0 e^{-\mu(t-T_1)}}{\theta_1-\mu}-\frac{D}{\theta_1}+\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{-\theta_1(T_1-T)} \tag{3}$$

$$I_3(t)=\frac{-D}{\theta_1}-\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{-\theta_1(T_2-t)} \tag{4}$$

Using the boundary condition

I_P(T)=0, and $I_P(T_1)$=Q_{P1}, then

$$Q_{P1}=\frac{P_0-D}{\theta_1}-\frac{(P_0-D)}{\theta_1}e^{-\theta_1 T_1} \tag{5}$$

$$Q_{P2}=\frac{D}{\theta_1}\left(e^{\theta_1(T-T_2)}-1\right) \tag{6}$$

Now Producer's total cost depends on the following factors;

(a) Setup cost;

Before producing the item in the production plant, the set-up of the machine to prepare the production, labour, etc., is included in the setup cost.

$$S_P=P_S \tag{7}$$

(b) Production cost;

Production cost is the cost involved in producing the items in the production plant which includes material cost, labour cost and energy cost, therefore production cost for the producer is:

$$U_P=P_P\int_0^{T_2} Pe^{-rt}dt$$
$$=P_P\left[P_0\left(\frac{1-e^{-rT_1}}{r}\right)+P_0 e^{-\mu T_1}\left(\frac{e^{-(\mu+r)T_1}-e^{-(\mu+r)T_2}}{\mu+r}\right)\right] \tag{8}$$

(c) Holding cost;

Holding cost involved in carefully storage and maintenance of inventory including hardware equipment, material handling equipment, IT software applications. Finally, the total holding cost for producer is;

$$H_P=h_p\left[\int_0^{T_1} I_{P1}(t)e^{-rt}dt+\int_{T_1}^{T_2} I_{P2}(t)e^{-rt}dt+\int_{T_2}^{T} I_{P3}(t)e^{-rt}dt\right]$$

$$=h_p\left[\begin{array}{c} \left(\frac{P_0-D}{\theta_1}\right)\left[\frac{1-e^{-rT_1}}{r}+\left(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\right)\right] \\ +\left\{\begin{array}{c} \frac{P_0}{\theta_1-\mu}e^{\mu T_1}\left(\frac{(e^{-(\mu+\theta_1)T_1}-e^{-(\mu+\theta_1)T_2})}{\mu+\theta_1}\right)+\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right) \\ +\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{\theta_1 T_1}\left(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T_2}}{\theta_1+r}\right) \end{array}\right\} \\ -\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT}}{r}\right)+\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{\theta_1 T_2}\left[\frac{e^{-(\theta_1+r)T_2}-e^{-(\theta_1+r)T}}{\theta_1+r}\right] \end{array}\right] \tag{9}$$

(d) Deteriorating cost;

Deterioration cost differently after the item is damaged for example preparing the item for re-production, or destroys the item that becomes useless so the deterioration cost for this model is:

$$D_P=\theta_1 d_p\left[\int_0^{T_1} I_{P1}(t)e^{-rt}dt+\int_{T_1}^{T_2} I_{P2}(t)e^{-rt}dt+\int_{T_2}^{T} I_{P3}(t)e^{-rt}dt\right]$$

$$=\theta_1 d_p\left[\begin{array}{c}\left(\frac{P_0-D}{\theta_1}\right)\left[\frac{1-e^{-rT_1}}{r}+\left(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\right)\right]\\ +\left\{\begin{array}{c}\frac{P_0}{\theta_1-\mu}e^{\mu T_1}\left(\frac{\left(e^{-(\mu+\theta_1)T_1}-e^{-(\mu+\theta_1)T_2}\right)}{\mu+\theta_1}\right)+\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right)\\ +\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{\theta_1 T_1}\left(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T_2}}{\theta_1+r}\right)\end{array}\right\}\\ -\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT}}{r}\right)+\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{\theta_1 T_2}\left[\frac{e^{-(\theta_1+r)T_2}-e^{-(\theta_1+r)T}}{\theta_1+r}\right]\end{array}\right] \qquad (10)$$

Total cost of producer in this model is

$$\Pi_p=\frac{1}{T}\left[S_P+U_P+H_P+D_P+\delta P_0\right] \quad =\frac{1}{T}\left[\begin{array}{c}\delta P_0+P_P\left[P_0\left(\frac{1-e^{-rT_1}}{r}\right)+P_0e^{-\mu T_1}\left(\frac{e^{-(\mu+r)T_1}-e^{-(\mu+r)T_2}}{\mu+r}\right)\right]+P_S\\ +\left(h_p+\theta_1 d_p\right)\left[\begin{array}{c}\left(\frac{P_0-D}{\theta_1}\right)\left[\frac{1-e^{-rT_1}}{r}+\left(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\right)\right]\\ +\left\{\begin{array}{c}\frac{P_0}{\theta_1-\mu}e^{\mu T_1}\left(\frac{\left(e^{-(\mu+\theta_1)T_1}-e^{-(\mu+\theta_1)T_2}\right)}{\mu+\theta_1}\right)+\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right)\\ +\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{\theta_1 T_1}\left(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T_2}}{\theta_1+r}\right)\end{array}\right\}\\ -\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT}}{r}\right)+\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{\theta_1 T_2}\left[\frac{e^{-(\theta_1+r)T_2}-e^{-(\theta_1+r)T}}{\theta_1+r}\right]\end{array}\right]\end{array}\right]$$

(11)

4.2 Buyer inventory model:

Buyer inventory model can be written by the differential equation

$$\frac{dI_B(t)}{dt}+\theta_2 I_B(t)=-D \qquad ;\ 0\le t\le T_3 \qquad (12)$$

With boundary condition $I_B(T_3)=0$

Solution of the equation (12) is;

$$I_B(t)=\frac{D}{\theta_2}\left(e^{\theta_2(T_3-t)}-1\right) \qquad (13)$$

Buyer's total cost depends on following factors

(a) Ordering cost;

Ordering cost is the total cost involved in ordering and purchasing the inventory, finding producer, and inspection of the inventory, finally total ordering cost for buyer is

$$O_B=C_B \qquad (14)$$

(b) Holding cost;

Holding cost involved in carefully storage the inventory and its maintenance, including hardware equipment, material handling equipment, IT software applications etc. Finally, the total holding cost for buyer is;

$$H_B=h_B\left[\int_0^{T_3} I_B(t)e^{-rt}dt\right]$$

$$H_B=h_B\left[\frac{D}{\theta_2}\left\{e^{\theta_2 T_3}\left(\frac{1-e^{-(\theta_2+r)T_3}}{\theta_2+r}\right)+\left(\frac{e^{-rT_3}-1}{r}\right)\right\}\right] \qquad (15)$$

(c) Deterioration cost:

After getting the inventory to the buyer from the production plant, the challenge with the buyer is to maintain that inventory well till the demand of the inventory reaches the consumer completely and for this he makes many efforts. But many times, it has been seen that inventory has a time period of its own and after that period it starts deteriorating. Due to which that inventory can be reused. For this it is sent again to the production plant, and eliminating the inventories that have been completely destroyed, all cost are included in deterioration cost;

$$D_B=d_B\theta_2\left[\int_0^{T_3} I_B(t)e^{-rt}dt\right]$$

$$D_B=d_B\theta_2\left[\frac{D}{\theta_2}\left\{e^{\theta_2 T_3}\left(\frac{1-e^{-(\theta_2+r)T_3}}{\theta_2+r}\right)+\left(\frac{e^{-rT_3}-1}{r}\right)\right\}\right] \qquad (16)$$

Total cost for buyer in ψ shipment is;

$$\Pi_B=\frac{\psi}{T}\left[O_B+H_B+D_B\right]$$

$$\Pi_B=\frac{\psi}{T}\left[O_B+(h_B+\theta_2 d_B)\left[\frac{D}{\theta_2}\left\{e^{\theta_2 T_3}\left(\frac{1-e^{-(\theta_2+r)T_3}}{\theta_2+r}\right)+\left(\frac{e^{-rT_3}-1}{r}\right)\right\}\right]\right] \qquad (17)$$

Total cost for this supply chain

TC= $\Pi_P + \Pi_B$

$$TC=\frac{1}{T}\left[\begin{array}{c} \delta P_0 + P_P\left[P_0\left(\frac{1-e^{-rT_1}}{r}\right)+P_0e^{-\mu T_1}\left(\frac{e^{-(\mu+r)T_1}-e^{-(\mu+r)T_2}}{\mu+r}\right)\right]+P_S \\ +\left(h_p+\theta_1 d_p\right)\left[\begin{array}{c} \left(\frac{P_0-D}{\theta_1}\right)\left[\frac{1-e^{-rT_1}}{r}+\left(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\right)\right] \\ +\left\{\begin{array}{c} \frac{P_0}{\theta_1-\mu}e^{\mu T_1}\left(\frac{\left(e^{-(\mu+\theta_1)T_1}-e^{-(\mu+\theta_1)T_2}\right)}{\mu+\theta_1}\right)+\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right) \\ +\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{\theta_1 T_1}\left(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T_2}}{\theta_1+r}\right) \end{array}\right\} \\ -\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT}}{r}\right)+\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{\theta_1 T_2}\left[\frac{e^{-(\theta_1+r)T_2}-e^{-(\theta_1+r)T}}{\theta_1+r}\right] \end{array}\right] \\ +\psi\left[O_B+(h_B+\theta_2 d_B)\left[\frac{D}{\theta_2}\left\{e^{\theta_2 T_3}\left(\frac{1-e^{-(\theta_2+r)T_3}}{\theta_2+r}\right)+\left(\frac{e^{-rT_3}-1}{r}\right)\right\}\right]\right] \end{array}\right] \quad (18)$$

5. Fuzzy model;

To understand this model in fuzzy environment we need some important definitions, which are as follows**:**

Definition 5.1.

A fuzzy set $\tilde{\beta}$ on the interval $(-\infty, \infty)$ is called a fuzzy point if its membership function (MF) is

$$\mu_{\tilde{\beta}}(y) = \begin{cases} 1, & y=\beta \\ 0, & y\neq\beta \end{cases}$$

Where β is the support point of fuzzy set.

Definition 5.2.

A fuzzy set $[U_\beta, V_\beta]$ where $0 \le \beta \le 1$. U, V ϵ R and U$< V$, is called a level of fuzzy interval if its MF is

$$\mu_{[U_\beta,V_\beta]}(x) = \begin{cases} \beta, & U\le y\le V \\ 0, & otherwise \end{cases}$$

Definition 5.3.

A fuzzy number M = (ξ, σ, ρ) where $\xi < \sigma < \rho$ and ξ, σ, ρ ϵ R, is called triangular fuzzy number (TFN) if its MF is:

$$\mu_M = \begin{cases} \frac{y-\xi}{\sigma-\xi}, & \xi\le y\le\sigma \\ \frac{\rho-y}{\rho-\sigma}, & \sigma\le y\le\rho \\ 0, & otherwise \end{cases}$$

When ξ = σ = ρ, we have fuzzy point, (ρ, ρ, ρ) = $\tilde{\rho}$.

The family of all TFN on R is denoted as:

$$E_N = \{(\xi,\sigma,\rho) \mid \xi<\sigma<\rho\ \forall\ \xi,\sigma,\rho \in R\}$$

The β cut of M= (ξ, σ, ρ)$\in E_N$, $0\le \beta \le 1$ is

$$M(\beta) = [M_L(\beta), M_R(\beta)].$$

Where $M_L(\beta) = \xi + (\sigma-\xi)\beta$ and $M_R(\beta) = \rho - (\rho-\sigma)\beta$ are the left and right endpoint of M(β).

Definition 5.4. The centroid method on the triangular fuzzy number M = (ξ, σ, ρ) is define as

$$C(M) = \frac{\xi+\sigma+\rho}{3}$$

Due to uncertainly it is not easy to define the parameters precisely, we assumed holding cost, deteriorating cost of buyer and producer may change within some limit.

Let $d_p = (d_{p_1}, d_{p_2}, d_{p_3})$, $h_p = (h_{p_1}, h_{p_2}, h_{p_3})$, $h_B = (h_{B_1}, h_{B_2}, h_{B_3})$, $d_B = (d_{B_1}, d_{B_2}, d_{B_3})$ are triangular fuzzy number.

We defuzzify the total cost function in fuzzy sense by the centroid method

$$TC_{CM} = \frac{1}{3}\left[TC_{CM_1}(T_1)+TC_{CM_2}(T_1)+TC_{CM_3}(T_1)\right]$$

Where $\boldsymbol{TC_{CM_i}} = \frac{\mathbf{1}}{\boldsymbol{T}}$

$$\left[\begin{array}{c} \delta P_0 + P_P\left[P_0\left(\frac{1-e^{-rT_1}}{r}\right)+P_0e^{-\mu T_1}\left(\frac{e^{-(\mu+r)T_1}-e^{-(\mu+r)T_2}}{\mu+r}\right)\right]+P_S \\ +\left(h_{pi}+\theta_1 d_{pi}\right)\left[\begin{array}{c} \left(\frac{P_0-D}{\theta_1}\right)\left[\frac{1-e^{-rT_1}}{r}+\left(\frac{e^{-(\theta_1+r)T_1}-1}{\theta_1+r}\right)\right] \\ +\left\{\begin{array}{c} \frac{P_0}{\theta_1-\mu}e^{\mu T_1}\left(\frac{\left(e^{-(\mu+\theta_1)T_1}-e^{-(\mu+\theta_1)T_2}\right)}{\mu+\theta_1}\right)+\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT_1}}{r}\right) \\ +\left(Q_{P1}-\frac{P_0}{\theta_1-\mu}+\frac{D}{\theta_1}\right)e^{\theta_1 T_1}\left(\frac{e^{-(\theta_1+r)T_1}-e^{(\theta_1+r)T_2}}{\theta_1+r}\right) \end{array}\right\} \\ -\frac{D}{\theta_1}\left(\frac{e^{-rT_2}-e^{-rT}}{r}\right)+\left(Q_{P2}+\frac{D}{\theta_1}\right)e^{\theta_1 T_2}\left[\frac{e^{-(\theta_1+r)T_2}-e^{-(\theta_1+r)T}}{\theta_1+r}\right] \end{array}\right] \\ +\psi\left[O_B+(h_{Bi}+\theta_2 d_{Bi})\left[\frac{D}{\theta_2}\left\{e^{\theta_2 T_3}\left(\frac{1-e^{-(\theta_2+r)T_3}}{\theta_2+r}\right)+\left(\frac{e^{-rT_3}-1}{r}\right)\right\}\right]\right] \end{array}\right] \quad \mathbf{(19)}$$

And i=1, 2, 3

6. Solution procedure of numerical analysis:

6.1 Solution procedure for crisp model

Differentiate both sides of equation (18) with respect to T_1, and the necessary condition to obtain the optimal value is

$$\frac{dTC(T_1)}{dT_1}=0 \tag{20}$$

Then T_1=2.573 if the values of parameters are

p_0 = 100unit, p_s =10, P_P =5, θ_1 = 0.02, h_P = 0.5, δ = 0.03, μ = 0.1, η = 0.8, d_P = 0.3, σ = 65, ζ = 85. $T_1=\eta T_2$, D=(σ+ ζ) /2, r= 0.015 , C_B=30 d_B = 0.5, θ_2 = 0.08, h_B = 0.6, ψ=2, T=90,

Again, the second derivative of equation (18) with respect to variable T_1 and

$$\frac{d^2TC(T_1)}{dT_1^{\,2}} > 0,$$

Hence total cost of this integrated model in crisp is minimum at T_1=2.573 and minimum value of total cost is TC= 1440.04

6.2 Solution procedure for fuzzy model

Differentiate both sides of equation (19) with respect to T_1, and the necessary condition to obtain the optimal cycle length

$$\frac{dTC_{CMi}(T_1)}{dT_1}=0 \tag{21}$$

Then T_1=2.578 if the values of parameters are

p_0 = 100unit, p_s =10, P_P =5, θ_1 = 0.02, , δ = 0.03, μ = 0.1, η = 0.8, σ = 65,ζ = 85. $T_1=\eta T_2$, D=(σ+ ζ) /2, r= 0.015 , C_B=30, θ_2 = 0.08, ψ=2, T=90, d_p= (0.2, 0.42, 0.54), h_p= (0.451, 0.502, 0.542), h_B= (0.26, 0.44, 0.65), d_B= (0.45, 0.56, 0.78)

Again, the second derivative of equation (19) with respect to variable T_1 and

$$\frac{d^2TC_{CMi}(T_1)}{dT_1^{\,2}} > 0,$$

Hence total cost of this integrated model in crisp is minimum at T_1=2.578 and minimum value of total cost is TC= 1008.732.

7. Sensitivity analysis

Parameter	% change in parameter	Crisp total cost	% change in crisp total cost	Fuzzy total cost	% change in fuzzy total cost
P_0	-50	622.32	-56.7%	191	-81.05%
	-25	1031.18	-28.3%	599.8	-37.7%
	+25	1848.9	+28.3%	1417.6	+37.7%
	+50	2257.76	+56.7%	1826.5	+75.5%
P_p	-50	678.19	-52.9%	246.8	-29.5%
	-25	1059.12	-26.4%	627.8	-14.7%
	+25	1820.96	+26.4%	1389.6	+14.7%
	+50	2201.89	+52.9%	1770.6	+29.5%
P_s	-50	1440.03	0	1008.7	0
	-25	1440.04	0	1008.7	0
	+25	1440.04	0	1008.7	0
	+50	1440.04	0	1008.7	0
θ_1	-50	1424.6	-1.07%	996.02	-0.4%
	-25	1430.05	-0.69%	1000.1	-0.3%
	+25	1451.87	+0.82%	1019.23	+0.4%
	+50	1464.63	+1.7%	1030.7	+0.8%
θ_2	-50	Not exist	-------	Not exist	------
	-25	562.54	-60.9%	365.58	-63.7%
	+25	2317.54	+60.9%	1615.9	+60.2%
	+50	3195.04	+121.8%	2295.06	+127.6%
δ	-50	1440.03	0	1008.7	0
	-25	1440.03	0	1008.7	0
	+25	1440.05	0	1008.7	0
	+50	1440.06	0	1008.7	0
μ	-50	1465.53	+1.7%	1034.2	+2.5%
	-25	1456.37	+7.1%	1025.07	+1.6%
	+25	1421.66	-1.2%	990.35	-1.8%
	+50	1402.41	-2.6%	971.1	-3.7%
r	-50	Not exist	------	Not exist	------
	-25	944.39	-34.4%	513.03	-49.2%
	+25	1737.43	+20.6%	1306.16	+29.5%
	+50	1935.69	+34.4%	1504.44	+49.2%
C_B	-50	1440.01	0	1008.7	0
	-25	1440.04	0	1008.7	0
	+25	1440.04	0	1008.9	0
	+50	1440.05	0	1008.9	0

8. Graphical representation, and Analysis of this study

8.1. Graphical representation:

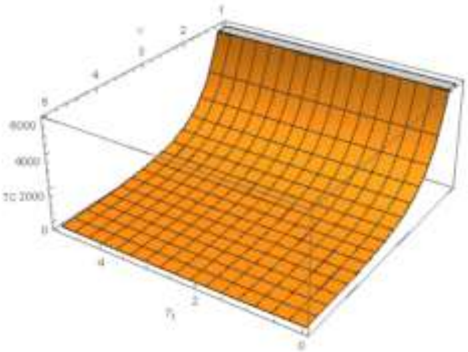

Fig. (a)

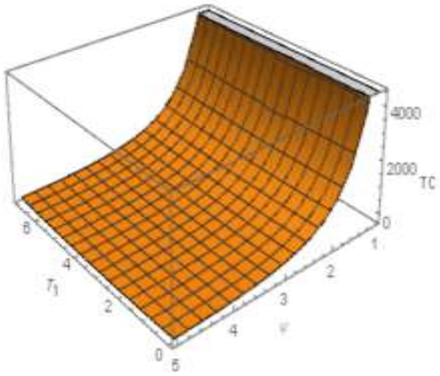

Fig. (b)

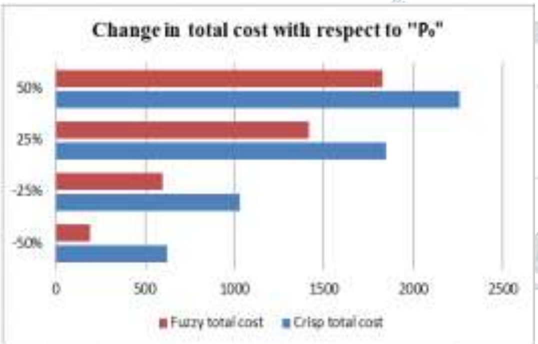

Fig. (c)

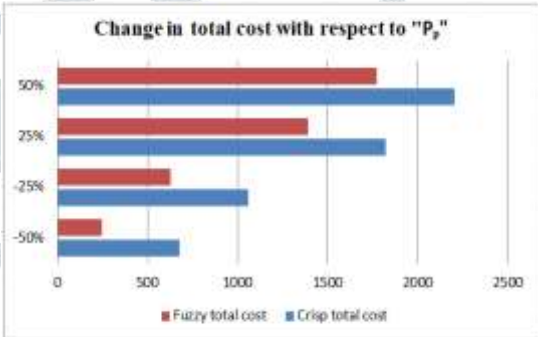

Fig. (d)

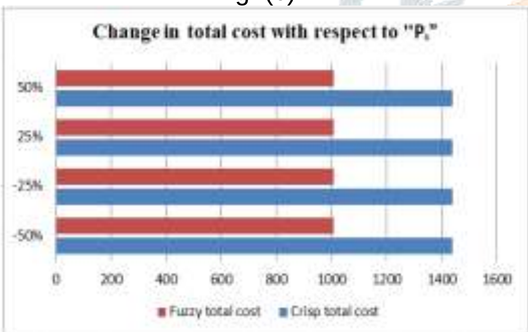

Fig. (e)

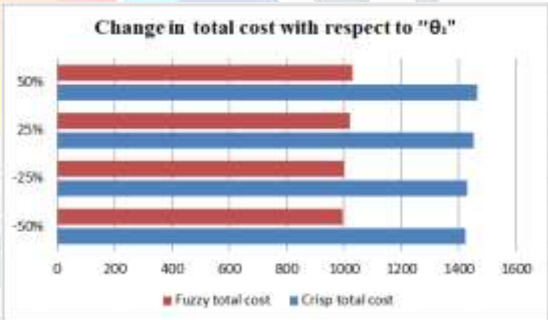

Fig. (f)

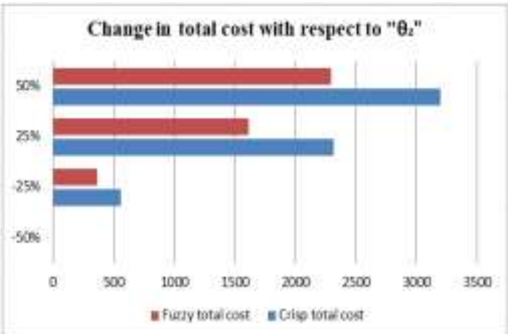

Fig. (g)

Fig. (h)

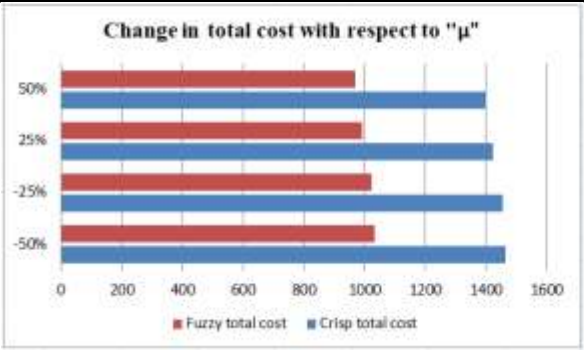

Fig. (i)

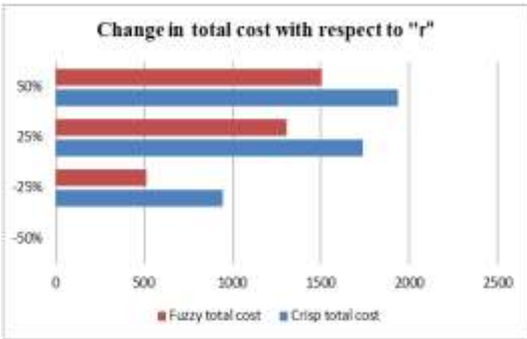

Fig. (j)

Fig. (k)

Figure (a) is convexity of total cost function w.r.t. T_1 in crisp model. **Figure (b**) is convexity of total cost function w.r.t. T_1 in fuzzy model. **Figure (c)** Variation in fuzzy and crisp total cost w.r.t.$P_{0,}$ **Figure (d)** Variation in fuzzy and crisp total cost w.r.t. $P_{P.}$ **Figure (e**) Variation in fuzzy and crisp total cost w.r.t. P_S. **Figure (f)** Variation in fuzzy and crisp total cost w.r.t. $\theta_{1.}$ **Figure (g)** Variation in fuzzy and crisp total cost w.r.t. $\theta_{2.}$ **Figure (h)** Variation in fuzzy and crisp total cost w.r.t. δ. **Figure (i)** Variation in fuzzy and crisp total cost w.r.t. μ. **Figure (j)** Variation in fuzzy and crisp total cost w.r.t. r. **Figure (k)** Variation in fuzzy and crisp total cost w.r.t. C_B.

8.2 Analysis of this study:

(1) On increasing the production rate P_0 in $[0, T_1]$ by 50%, the total cost is increasing in both the fuzzy and crisp models, but if the same production rate is reduced i.e. is reduced by 50%, in this case the total cost is decreasing for both the models.
(2) On increasing the production cost by 50%, the total cost is increasing in both the fuzzy and crisp models, but if the same production cost is reduced by -50% then the total cost is decreasing for both the models. Also, any change in the set up cost does not affect the total cost of the model.
(3) On increasing the producer's deterioration rate by 50%, the total cost is increasing for both the models fuzzy and crisp, but if the deterioration rate is reduced by -50% then in this case the total cost is decreasing for both the models. Also, the change in parameter δ is not affecting the total cost.
(4) For both the models, Fuzzy and Crisp, the minimum total cost does not exist when the buyer's deteriorating rate is reduced by -50% while the total cost is reduced by -25%, conversely, if the deterioration rate is increased by 50%. So the total cost is increasing for both the models.
(5). The minimum total cost does not exist for both the models fuzzy and crisp when the inflation rate is being reduced by -50%, while the total cost is decreasing on reduction of -25%, on the contrary, if this inflation rate is increased by 50%, then the total cost is increasing for both the models. The change in the ordering cost of the buyer does not affect the total cost.

9. Conclusion

In this research, the integrated inventory model has been developed under imprecise and inflationary environment. This model considers the variable production rate for the production plant which decreases gradually over time and this brings the present study closer to reality. Because due to sluggishness of workers, delay in supply of raw materials, etc. there is a possibility of the item being poorly produced in almost every production plant and it has been analysed that the effect of imperfect production process on total cost and the goods which have been damaged during the imperfect production process cannot be reused. Furthermore, in this model the demand rate was assumed to be probabilistic which is in uniform distribution and many other costs remain uncertain. To understand the model in reality, the model has been defuzzfied by taking all these important costs in the triangular fuzzy number. From the sensitivity analysis of this model it was concluded that if inflation rate is being reduced then the total cost is decreasing, but if inflation rate is increased then the total cost is increasing for both the models. If the production rate is kept constant and there is shortage of demand in the market along with inflation the model will not be valid. The model can be extended with time dependent holding cost and deterioration in Green suply chain with carbon emission reduction under preservation technology and cloudy fuzzy in trade credit policy.

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