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THEORY OF Y-EXTREMAL HYPERSURFACES IN A FINSLER SPACE

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ABSTRACT -

The purpose of this paper is to study the theory of Y-Extremal Hypersurfaces of Finsler Space equipped with h-recurrent Finsler Connection.

1. INTRODUCTION :-

The Theory of Hypersurfaces in Finsler Space has been first considered by E.Cartan [2] from two points of view. One is regard a Hypersurface as the whole of tangent line elements and then it is also a Finsler Space [6]. The other is to regard it as whole of normal line elements then it is a Riemannian Space. J.M. Wegener ([11]) has treated Hypersuface from the alter view point and dealt in particular with minimal Hypersurfaces. E.T. Davies [3] has considered subspaces from the former view point mainly, but referred a little to minimal subspaces - Both have pointed out a weak point of their theories that the minimal subspaces are characterized only by vanishing of mean curvature provided Cartan's torsion vector vanishes. To over come this weak point W. Barthel [1] has proposed a new Finsler Connection with surviving torsion tensorand obtained a satisfactory results for time being. B.Su [10] has further developed the theory of minimal subspaces based on Barthel's Stand point. In 1986 Matsumoto [7] defined cartan y-connectin from fundamental function and non zero vector field. He studied theory of Y-Extremal Hypersurfaces by considering the metrical property of connection.

2. LINEARY-CONNECTION :-

Let $F^n = (M^n, L, F\Gamma)$ be Finsler Space on n dimensional manifold M^n equipped with fundamental function L = L (x,y) and Finsler Connection $F \quad = (\Gamma, N)$ ([4], [5]).

Definition - A Finsler Connection is called h recurrent Finsler Connection with torsion and denoted by Rec. $\sqrt{(a,T)}$. If the following four conditions are satisfied

(C-1) h - recurrent with recurrence vector field a_k i.e. $g_{ij\,|k} = a_k\, g_{ij}$

(C-2) Deflection tensor = 0

(C-3) V-metrical $g_{ij}|_k = 0$

(C-4) (V) V - torsion tensor = 0

The condition (C-1) and (C-3) are respectively written as

(C-1) $g_{ij|k} = \delta_k\, g_{ij} - F_{ijk} - F_{jik} = a_k\, g_{ij}$

(C-3) $g_{ij}|_k = \partial_k\, g_{ij} - C_{ijk} - C_{jik} = 0$

Where $F_{ijk} = g_{jr}\, F^r{}_{ik}$, $C_{ijk} = g_{jr}\, C^r{}_{ik}$

$F^i_{jk} = \sqrt{\,}^{\,i}_{jk}$ $C^i_{jr} N^r_k$ and - N^r_k

(C-3) and (C-4) lead to cijk = $\frac{1}{2}$ $\partial_k g_{ij}$ (cartan's C-tensor)

It is well know [8], [9] that if the (h) h-torsion tensor.

$T^i_{jk} = F^i_{jk} - F^i_{kj}$ putting

(2.1) Aijk = $\frac{1}{2}$ $(T_{kjh}\ g^{hi} + T_{jkh}\ g^{hi} + T^i_{jk})$

(2.2) $F^i_{jk} = \gamma^i_{jk} - (C^i_{km}\ N^m_j + C^i_{jm}\ N^m_k - g^h_i\ C_{jkm}\ N^m_h)$

Where γ^i_{jk} is christoffel symbol then (2.2) gives

(2.3) $F^i_{ok} = \gamma^i_{ok} - C^i_{km}\ N^m_o - \frac{1}{2}(a_r\ \delta^i_k + a_k\ Y^i - Y_k\ a^i)\ A^i_{ok}$

(C.2) and (2.3) lead to

(2.4) $N^i_k = \gamma^i_{ok} - C^i_{km}\ (\gamma^m_{oo} + A^m_{oo} - \frac{1}{2}\ a^m\ L^2) + A^i_{ok}$

$$-\frac{1}{2}\ (a_o\ \delta^i_k + a_k\ Y^i - Y_k\ a^i)$$

The so called h-recurrent Wegener connection has played an important role in the theory of h-recurrent generalized Berwald Spaces [8]. The h(h) torsion tensor of h-recurrent Wagner space is given by semi symmetric type $T^i_{jk} = \delta^i_j\ a_k - \delta^i_k$ a_j is known as covariant vector field. We will propose a generalized h-recurrent Finsler Connection denoted by $\sqrt{(Y)}$ which may be called of the (L, y, a) structure.

The connection coefficients $\overline{\Gamma}^{\,i}_{jk}$ of $\Gamma(Y)$ are

(2.5) $\overline{\Gamma}^{\,i}_{jk}(x) = \overline{\Gamma}^{\,i}_{jk}(x,y) + C^i_{jr}$ (x,y) $\partial_k\ Y^r$

Where

(2.6) Y_k^i (x) = ∂_k Y^i + N_k^i (x,y)

are component of tensor Y_2. Since h covariant derivatives $Y^r{}_{ik}$ are given by

(2.6) Y_k^i (x) = Y_{Ik}^i (x,y)

If follows from (2.5) that torsion tensor $\bar{T}_{jk}^i$ of $\sqrt{(Y)}$ is

(2.7) $\bar{T}_{jk}^i$ (x) = T_{jk}^i (x,y) + $C^i{}_{jr}$ (x,y) $Y^r{}_k$ - $C^i{}_{kr}$ (x,y) $Y^r{}_j$

Next we introduce Riemannia metric

(2.8) $\bar{g}_{ij}$ (x) = g_{ij} (x,y)

On D which will be called Riemannian Y-metric. In general we get ordinary tensor field $\bar{k}_j^i$ (x) = $\bar{k}_j^i$ (x,y) from Finsler tensor field k_j^i (x,y) then it is easily verified that covariant derivatives $\bar{k}_{jk;k}^i$ of $\bar{k}_j^i$

with respect to $\sqrt{(Y)}$ is

(2.9) $\bar{k}_{j;k}^i$ = $K^i{}_{j|k}$ (x,y) + $K^i{}_{j|r}$ (x,y) Y_k^r

Where $K^i{}_{j|k}$ and $K^i{}_{j|}$ are component of h and v covariant derivatives ∇_k^h and ∇_k^v of K respectively

Applying (2.9) and using (C.1) (C.3) we get

$g_{ij:k} = a_k$ (x,y) g_{ij} (x,y)

= $\bar{a}_k$ (x) $\bar{g}_{ij}$ (x)

<u>Proposition (2.1)</u> :-

The Linear Y-connection associated to a recurrent Finsler Connection Rec. $\sqrt{(T,\ \iota)}$ by y is recurrent with respect to the Riemannian Y-metric and it's torsion tensor is given by (2.7).

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